CERTIFICATE IN ROCK MECHANICS

LEARNING GUIDE FOR:
PART 1: ROCK MECHANICS THEORY

SOUTH AFRICAN NATIONAL INSTITUTE OF ROCK ENGINEERING

CHAMBER OF MINES OF SOUTH AFRICA

MINING QUALIFICATIONS AUTHORITY

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CHAPTER

1

PAPER 1 LEARNING GUIDE: ROCK MECHANICS THEORY

The learning material presented in the following chapters have been developed to assist the persons studying for the Chamber of Mines Rock Mechanics Certificate, currently the only qualification that deems a rock engineering practitioner in the South African Mining Industry as 'competent'.

The material provided in the following chapters:

- is not meant to be rock engineering handbook, but a learning guide;
- is not be a complete re-write of information already captured in technical handbooks or papers but will include the minimum information to ensure that understanding can be created;
- is referenced to outcomes in the syllabi to assist the learner to ensure that all topics are covered;
- is referenced to other rock engineering handbooks that should be used by the learner to ensure complete coverage of topics;
- provide practical examples, photographs and other important or interesting information where deemed appropriate or required;
- is enhanced by exercises and case studies to illustrate the principles presented in the main text;
- animations and interactive tools on selected topics can be utilised to assist in understanding and applying specific concepts;
- includes questions for self-testing;
- is referenced to a guide in basic concepts such as units, definitions, trigonometry, physics, etc.
- diagrams and pictures in this guide should not be used to scale results since they have been re-drafted and may not be to scale. Originals can be found in the references

PAPER 1 SYLLABUS

This section contains the syllabus for Part 1 of the COM Certificate in Rock Engineering is provided, as prepared by the South African National Institute for Rock Engineering. The syllabus describes the levels of minimum knowledge and understanding and is not to be viewed as a complete list of rock engineering knowledge.

PREAMBLE

1. TOPICS COVERED

This is a general paper covering basic rock mechanics theory applicable in all types of mining environments. The theoretical rock mechanics knowledge required here is thus of a fundamental nature, and is not specific to any particular type of mining.

2. CRITICAL OUTCOMES

The examination is aimed at testing the candidate's abilities in the six cognitive levels; knowledge, comprehension, application, analysis, synthesis and evaluation. Thus, when being examined on the topics detailed in this syllabus candidates must demonstrate their capacity for :

- Comprehending and understanding the general rock engineering principles covered in this syllabus and applying these to solve real world mining problems
- Applying fundamental scientific knowledge, comprehension and understanding to predict the behaviour of rock materials in real world mining environments
- Performing creative procedural design and synthesis of mine layouts and support systems to control and influence rock behaviour and rock failure processes
- Using engineering methods and understanding of the uses of computer packages for the computation, modelling, simulation, and evaluation of mining layouts
- Communicating, explaining and discussing the reasoning, methodology, results and ramifications of all the above aspects in a professional manner at all levels.

3. PRIOR LEARNING

This portion of the syllabus assumes that candidates have prior learning and good understanding of :

- The field of fundamental mechanics appropriate to this part of the syllabus
- The application and manipulation of formulae appropriate to this part of the syllabus as outlined in the relevant sections of this document
- The terms, definitions and conventions appropriate to this part of the syllabus as outlined in the relevant sections of this document.

4. STUDY MATERIAL

This portion of the syllabus assumes that candidates have studied widely and have good knowledge and understanding of :

- The reference material appropriate to this part of the syllabus as outlined in the relevant sections of this document
- Other texts that are appropriate to this part of the syllabus but that may not be specifically referenced in this document

Information appropriate to this part of the syllabus published in journals, proceedings and documents of local mining, technical and research organisations.

SYLLABUS

1. STRESS AND STRAIN
1.1 STRESS AND STRAIN COMPONENTS

The candidate must be able to demonstrate knowledge and understanding of the above subject area by being able to:

- Describe, explain and illustrate the complete state of stress at a point

- Indicate the positive sense of normal and shear stresses
- Calculate the principal stresses from shear components in two dimensions
- Calculate the normal and shear components of stress on a plane at any orientation in a two-dimensional stress field given either the principal stresses or the stress components in the x-y directions
- Calculate the principal stresses from shear components in three dimensions
- Calculate the normal and shear components of stress on a plane at any orientation in a three-dimensional stress field given either the principal stresses or the stress components in the x-y and z directions
- Calculate the principal strains from strain components in two dimensions
- Calculate strain components in any orientation given either the principal strains or strain components in the x-y directions
- Determine the principal stresses making use of the Mohr-circle representation of stresses in two dimensions given the stress components on a plane
- Determine the stress components on a plane making use of the Mohr-circle representation of stresses in two dimensions given the principal stresses
- Describe, explain and illustrate the normal and shear stresses induced in the following members under their own weight :
  - A simply supported beam
  - A cantilever beam
  - A built-in beam
- Note the positions of maximum tension, maximum compression and maximum shear in each case.

2. CONSTITUTIVE BEHAVIOUR
2.1 CONSTITUTIVE RELATIONSHIPS

The candidate must be able to demonstrate knowledge and understanding of the above subject area by being able to:

- Identify, describe and explain the following constitutive relationships:
  - Elastic
  - Plastic
  - Elasto-plastic
  - Strain softening
  - Strain hardening
- Interpret these relationships citing material examples
- Calculate strain components from stress components for elastic materials in two dimensions using Hooke's law
- Calculate stress components from strain components for elastic materials in two dimensions using Hooke's law
- Calculate strain components from stress components for elastic materials in three dimensions using Hooke's law
- Calculate stress components from strain components for elastic materials in three dimensions using Hooke's law
- Describe and explain the concept of plane strain

- Calculate the elastic strain energy density in elastic materials
- Describe, explain and define the following terms:
  - Dilation Angle
  - Associated Flow
  - Non-associated Flow
- Describe, explain and indicate on a stress-strain graph for rock material the onset of inelastic behaviour, dilation and strain softening
- Identify, describe and explain time-dependent behaviour
- Interpret this behaviour citing material examples.

3. ROCK PROPERTIES

3.1 ROCK STRENGTH

The candidate must be able to demonstrate knowledge and understanding of the above subject area by being able to :

- Identify, describe and explain the possible modes of failure of rock in uniaxial compression
- Describe and explain the effect of orientation on strength results for anisotrophic rocks such as shales
- Sketch, describe and explain the complete axial stress-strain graph for rock in uniaxial compression
- Sketch, describe and explain the complete axial stress-strain graph for rock in triaxial compression
- Indicate the following for the above two cases :
  - the associated radial strain
  - the associated volumetric strain
  - the associated brittle-ductile transition
  - the associated residual strength
  - Hysteresis in the post peak portion of the curve
- Describe and explain brittle behaviour of rocks based upon Griffith's theory
- Explain the basis for the Griffith criterion
- Explain the basis for the Coulomb criterion
- Identify, describe and explain the assumptions of the Coulomb criterion
- Describe and explain the orientation of the failure surface predicted by the Coulomb criterion
- Describe and explain how the cohesion and friction angle required by the Coulomb criterion may be determined from rock tests
- Explain the basis for the Hoek and Brown criterion
- Describe and explain how to apply the Hoek and Brown criterion to predict rock failure
- Describe and explain how the 'm'-parameter for intact rock required for the Hoek and Brown criterion may be obtained
- Sketch, describe and explain :
  - The stress-strain behaviour of brittle rock in uniaxial compression
  - The effect of confinement on rock strength
  - The effect of confinement on the stress-strain behaviour of rock samples
  - What happens to rock after it has reached its peak strength

- Sketch, describe and explain :
- Rock fracture, Brittle failure
- Ductile deformation, Yield
- Peak strength, Residual strength
- Effective stress, Pore pressure
- The volume effect on the strength of intact rock.

3.2 JOINT STRENGTH

The candidate must be able to demonstrate knowledge and understanding of the abovesubject area by being able to :

- Represent and describe the occurrence of rock jointing, the number of jointsets and their orientations
- Describe and explain the effect of friction angle on joint strength
- Describe and explain the effect of infilling on joint strength
- Describe and explain the effect of water on joint strength
- Sketch, describe and explain the typical shear resistance-displacementbehaviour of smooth joints
- Sketch, describe and explain the typical shear resistance-displacementbehaviour of rough joints
- Sketch, describe and explain the effect of surface roughness on joint shearstrength
- Compare and contrast typical shear resistance versus shear displacementbehaviour of smooth and rough joints
- Describe and explain the effect of increasing the normal stress on thebehaviour of joints under shear loading
- Describe and explain the effect of surface roughness on joint shear strength
- Describe and explain the concept of roughness angle and its effect on initialshear resistance according to Patton's theory
- Describe and explain the effect of surface roughness on dilation during shearon a joint
- Describe and explain the effect of the above on rock stability in unconfinedsituations
- Describe and explain the effect of the above on rock stability in confined situations
- Apply Barton's equation to estimate the peak strength of joints
- Describe and explain the difference between joints, fractures and cracks.

4. STRESS IN ROCK AND ROCKMASSES

4.1 STRESS ESTIMATION

The candidate must be able to demonstrate knowledge and understanding of the abovesubject area by being able to :

- Explain Heim's Law
- Calculate the expected stress regime in rock based upon gravity and thePoisson effect only
- Explain and comment upon the reasons why stresses may vary from thosecalculated above
- Describe and explain virgin stresses encountered in the different mining areasof South Africa

- Describe and explain the principles of stress measurement using strain cells
- Explain and comment upon the reasons for the large variations in the results ofthe above stress measurements
- Describe and explain the principles of stress measurement based upon AEprinciples
- Explain and discuss the use of Kirsch equations in respect of stress determinations.

5. ROCKMASS PROPERTIES

The candidate must be able to demonstrate knowledge and understanding of the abovesubject area by being able to :

- Describe, explain and illustrate the volume effect on the strength of intact rock
- Describe, explain and illustrate the role of joint strength on the strength of arockmass
- Describe, explain and illustrate, the role of joint frequency on the strength of arockmass
- Describe, explain and illustrate the role of joint length on the strength of arockmass
- Describe, explain and illustrate the role of intact rock strength on the strengthof a rockmass
- Describe, explain and illustrate the role of groundwater on the strength of arockmass
- Describe and explain the general relationship between strength and seismicwave velocity in a rockmass.

6. ROCKMASS CLASSIFICATION

The candidate must be able to demonstrate knowledge and understanding of the abovesubject area by being able to :

- Apply Barton’s, Bieniawski’s and Loubscher’sgeomechanics classificationsystems to classify rockmasses
- Determine rockmass ‘m’ and ‘s’ parameters for the Hoek and Brown criterion,based upon rock classification
- Apply the Hoek and Brown criterion to estimate the strength of a rockmass
- Identify, describe and explain the limitations of the Hoek and Brown criterion
- Estimate rockmass deformability from joint stiffness and rockmass-classification results.

REFERENCES

1. STRESS AND STRAIN COMPONENTS

- Ryder JA & Jager AJ 2002 Rock Mechanics for Tabular Hard Rock Mines SIMRAC Jhb Chapter 3
- Brady BHG & Brown ET 1993 Rock Mechanics for Underground Mining Chapman & Hall New York Chapter 2
- Obert L & Duvall WI 1967 Rock Mechanics and the Design of Structures in Rock John Wiley & Sons New York Chapters 1, 2
- Jaeger JC & Cook NGW 1969 Fundamentals of Rock Mechanics Chapman & Hall London Chapter 2
- Budavari S (ed) 1986 Rock Mechanics in Mining Practice SAIMM Jhb Chapter 1, 3, 5

2. CONSTITUTIVE RELATIONSHIPS

- Ryder JA & Jager AJ 2002 Rock Mechanics for Tabular Hard Rock Mines SIMRAC Jhb Chapter 4
- Obert L & Duvall WI 1967 Rock Mechanics and the Design of Structures in Rock John Wiley & Sons New York Chapters 3, 6, 10
- Jaeger JC & Cook NGW 1969 Fundamentals of Rock Mechanics Chapman & Hall London Chapters 4, 5, 6, 7, 9, 10, 11, 12
- Budavari S (ed) 1986 Rock Mechanics in Mining Practice SAIMM Jhb Chapter 1

3. ROCK STRENGTH

- Ryder JA & Jager AJ 2002 Rock Mechanics for Tabular Hard Rock Mines SIMRAC Jhb Chapter 2
- Brady BHG & Brown ET 1993 Rock Mechanics for Underground MiningChapman & Hall New York Chapter 4
- Obert L & Duvall WI 1967 Rock Mechanics and the Design of Structures in RockJohn Wiley & Sons New York Chapters 10, 11
- Jaeger JC & Cook NGW 1969 Fundamentals of Rock Mechanics Chapman& Hall London Chapter 4, 6
- Budavari S (ed) 1986 Rock Mechanics in Mining Practice SAIMM Jhb Chapter 2

3.1 JOINT STRENGTH

- Ryder JA & Jager AJ 2002 Rock Mechanics for Tabular Hard Rock Mines SIMRAC Jhb Chapter 2
- Jaeger JC & Cook NGW 1969 Fundamentals of Rock Mechanics Chapman & Hall London Chapter 3

4. STRESS ESTIMATION

- Budavari S (ed) 1986 Rock Mechanics in Mining PracticeSAIMM Jhb Chapter 3
- Brady BHG & Brown ET 1993 Rock Mechanics for Underground Mining Chapman & Hall New York Chapter 5
- Obert L & Duvall WI 1967 Rock Mechanics and the Design of Structures in Rock John Wiley & Sons New York Chapters 14
- Jaeger JC & Cook NGW 1969 Fundamentals of Rock Mechanics Chapman & Hall London Chapter 14, 15

5. ROCKMASS PROPERTIES

- Ryder JA & Jager AJ 2002 Rock Mechanics for Tabular Hard Rock MinesSIMRAC Jhb Chapter 2
- Brady BHG & Brown ET 1993 Rock Mechanics for Underground Mining Chapman & Hall New York Chapter 3, 4
- Obert L & Duvall WI 1967 Rock Mechanics and the Design of Structures in RockJohn Wiley & Sons New York Chapters 15
- Jaeger JC & Cook NGW 1969 Fundamentals of Rock MechanicsChapman & Hall London Chapters 1, 17

6. ROCKMASS CLASSIFICATION

- Ryder JA & Jager AJ 2002 Rock Mechanics for Tabular Hard Rock MinesSIMRAC Jhb Chapter 7
- Stacey TR 2001 Best Practice Rock Engineering Handbook for ‘Other’ MinesSIMRAC Jhb Chapter 2
- Hoek E & Brown ET 1980 Underground Excavations in Rock IMM London Chapter 2

CHAPTER 2

2. STRESS AND STRAIN

INTRODUCTION

Chapter 2 deals with stress and strain as the most basic building blocks in rock mechanics theory. The stress acting at a point or on a surface initially induces deformation which may lead to failure. Since rock mechanics practitioners are interested in the stability of excavation walls, understanding of stress, strain and the result of these on the excavation walls is critical.

In this chapter the following is addressed:

- Stress and strain sign convention as used in rock engineering practice
- Stress at a point in two- and three dimensions
- Stress and strain transformations
- Mohr's circle in determining principal stress orientations and stresses acting on inclined planes from the X-axis or principal stress axes, and
- Normal and shear stresses acting in beams

NOTE

In this chapter reference is made to freely-available calculators on the Internet that may be of assistance to verify calculations.

LEARNING OUTCOMES

From the syllabus provided in Chapter 1, it follows that rock engineers must be able to demonstrate knowledge and understanding of stress and strain by being able to:

2.1 Describe, explain and illustrate the complete state of stress at a point
2.2 Indicate the positive sense of normal and shear stresses
2.3 Calculate the principal stresses from shear components in two dimensions
2.4 Calculate the normal and shear components of stress on a plane at any orientation in a two-dimensional stress field given either the principal stresses or the stress components in the x-y directions
2.5 Calculate the principal stresses from shear components in three dimensions
2.6 Calculate the normal and shear components of stress on a plane at any orientation in a three-dimensional stress field given either the principal stresses or the stress components in the x-y and z directions
2.7 Calculate the principal strains from strain components in two dimensions
2.8 Calculate strain components in any orientation given either the principal strains or strain components in the x-y directions
2.9 Determine the principal stresses making use of the Mohr-circle representation of stresses in two dimensions given the stress components on a plane

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2.10 Determine the stress components on a plane making use of the Mohr-circle representation of stresses in two dimensions given the principal stresses

2.11

- Describe, explain and illustrate the normal and shear stresses induced in the following members under their own weight :
- A simply supported beam
- A cantilever beam
- A built-in beam
- Note the positions of maximum tension, maximum compression and maximum shear in each case.

NOTE

The references used in each learning outcome are listed within the text where necessary as well as in a list of references at the end of each learning outcome.

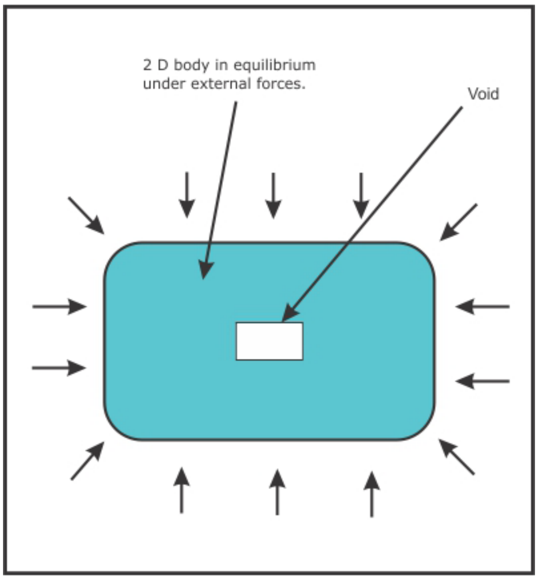
LEARNING OUTCOME 2.1

BRIDGING GAP 1

"Metric Prefixes" on page 222

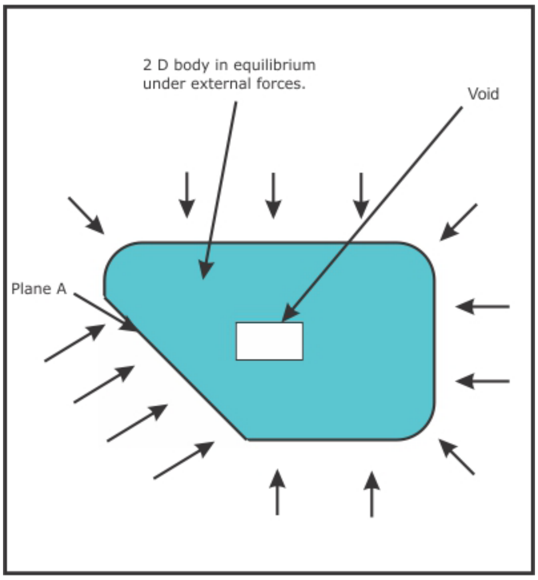
Describe, explain and illustrate the complete state of stress at a point

Stress can simply be defined as the ratio of the force acting on an area to the area being loaded. The SI unit (International System of Units) for stress is the Pascal (after Blaise Pascal) and can be defined as Newton / m^2 where Newton is the SI unit for Force. In rock engineering, the most common stress units used include kPa, MPa and GPa.



Before considering the stress at a point in a three-dimensional (3D) rock mass, understanding of stress in a two-dimensional (2D) body is important. A two-dimensional body in equilibrium (forces are in balance, body is stable or steady) is subjected to external forces as shown in Figure 1

Figure 1: A 2D body in equilibrium under external force.



To define the state of stress at a point in the body, the body is cut along an imaginary internal plane A (Figure 2) and the internal forces acting on the plane are analysed. It must be clear that if the body is in equilibrium, the sum of forces acting on plane A from the 'internal' and 'external' directions (only 'external' forces shown in Figure 2) should be equal.

Figure 2: Cutaway plane (plane A) in a 2D body.

The stress on a small elementary area ΔA can be given by the ratio

INTERESTING INFO

Blaise Pascal (19 June, 1623 to 19 August 1662), was a French mathematician, physicist, inventor, writer and Catholic philosopher. He was educated by his father, a tax collector in Rouen.

Pascal's earliest work was in the natural and applied sciences where he made important contributions to the study of fluids, and clarified the concepts of pressure and vacuum.

In 1642, while still a teenager, he started some pioneering work on calculating machines, and after three years of effort and 50 prototypes he invented the mechanical calculator. He built twenty of these machines (called the Pascaline) in the following ten years.

Following a mystical experience in late 1654, he had his "second conversion", abandoned his scientific work, and devoted himself to philosophy and theology. In this year, he also wrote an important treatise on the arithmetical triangle.

Between 1658 and 1659, he wrote on the cycloid and its use in calculating the volume of solids.

Pascal had poor health especially after his eighteenth year and his death came just two months after his 39th birthday.

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of the acting force ΔF and the area ΔA. At a point on plane A, the stress p can be defined as the limiting value of the average force per unit area where the area tends to 0 m^2(since the area of a point is very small and will approach zero).

$$p = \lim_{\triangle A \to 0} \frac{\triangle F}{\triangle A}$$

Along the cutting plane A, the stress p at a point has a direction and a magnitude, which means that it is a vector, called a traction vector.
The traction vectors may not lie perpendicular or parallel to the area on which it acts, and can be resolved into normal (perpendicular to plane A) and tangential (parallel to plane A) components, called the stress components for the plane A. A number of imaginary cutting planes can be selected and the stresses resolved into the stress components for each plane. It is clear that a large number of stress components can be derived for all the possible cutting planes. At the same time, the magnitude of these traction vectors are dependent on the orientation of the cutting plane to which it is related and can therefore not be used to express the complete state of stress at a point.

To solve this problem, the complete state of stress at a point is referred to as a 'tensor' of second order rather than a vector. The 'tensor of 2nd order' suggests that two (2) subscripts are required to define the stress component adequately. The stress tensor is represented by σ_{ij}, where the 1st subscript *i* indicates the orientation of the normal (perpendicular) to the cutting plane *i* and the 2nd subscript *j*, indicates the orientation of the components of the traction vector p_i associated with the cutting plane *i*.

IMPORTANT

The coordinate system used is a right-handed coordinate system, which means that the positive Y axis is 90° counter clockwise from the positive X axis, and the positive Z axis out of the paper. In 2D situations, any one of the axes can be ignored, but in most cases, the Z axis is excluded and the conditions sketched in the X-Y plane. Stress conditions where analyses in two dimensions are deemed appropriate, are called "Plane stress" conditions (see below).

It is important to note that the rock engineering sign convention utilised in these notes differs from general engineering sign convention. In rock engineering, compressive stresses are deemed positive and tensile stresses negative, whereas in engineering, compressive stress is indicated as negative and tensile stress as positive.

INTERESTING INFO

Cartesian co-ordinate system axes used in rock mechanics
The Cartesian coordinate system normally consists of X,Y axes in 2D and X,Y and Z axes in 3D. Different coordinate systems are often used in handbooks and should not be confused with each other.In **Figure 3,** the 2D XY, X'-Y' and the m-n axes notations often used in rock engineering are shown. In the case of stress transformations, the X',Y' and m,n axes are often applied at an angle to the normal X,Y axes.

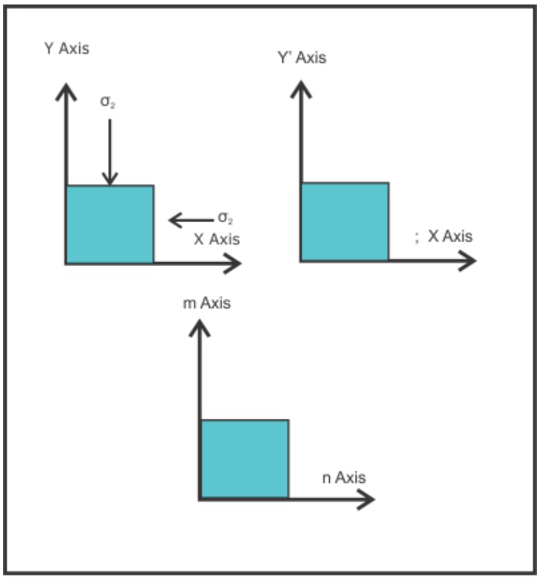

Figure 3: Co-ordinate systems commonly used in rock engineering practice

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Figure 4 below shows the positive (black) and negative (red) sides of a square 2D stress element.

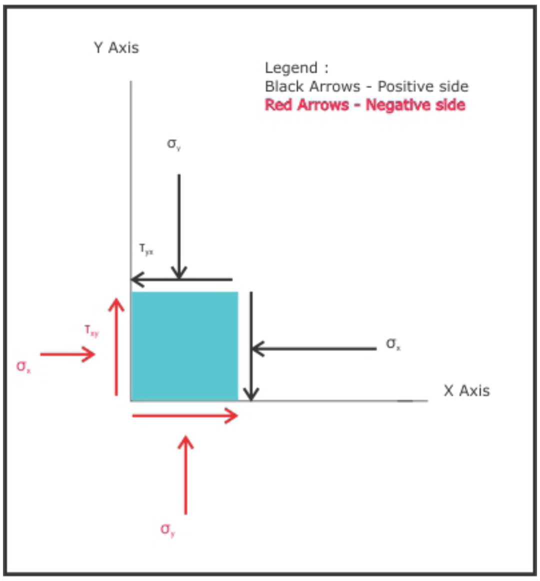
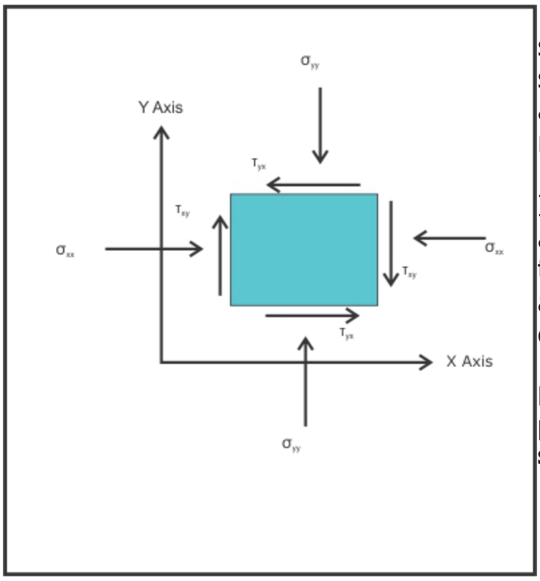


Figure 4: Sign convention used for positive stress components in rock engineering

It must be noted that the stress tensors are often denoted as follows:

- normal stress notations may be given as ‘σ_x’ or ‘σ_{xx}’ in the case of the normal stress acting on the X plane, or‘σ_y’or‘σ_{yy}’ in the case of the normal stress acting on the Y plane (the latter notation is preferred, but some handbooks do use the simpler single subscript notation for normal stresses, such as Rock Mechanics in Practice by Budavari)
- shear stresses are denoted as ‘τ_{xy}’and ’τ_{yx}’
- In both cases, the first subscript indicates the direction of the normal to the plane on which the stress acts and the second subscript indicates the axis direction in which the stress acts on the plane.

In 2D conditions, the stress components acting at a point are purposely limited to two coordinate axes only. For instance, the state of stress at a point may be presented in the X-Y plane only, ignoring the stress in the Z direction (or X-Z and Y-Z planes). This (ignoring the stress in the 3rd direction) can typically be allowed in underground situations, such as parallel sided longwalls or tunnels where the stress components in the 3rd direction are deemed not to significantly affect the analysis of stress around these excavations.



This condition is called “plane stress” and refers to the analyses of stress in a single 2D plane and the assumption that stress in the 3rd direction is zero.

In 2D conditions, the state of stress at a point can thus be described in terms of two (2) normal (σ_{xx}, σ_{yy}) and two (2) shear stress ($\tau_{xy}=\tau_{yx}$) components as indicated in Figure 5.

Figure 5: Sign convention for the positive stresses acting on the square 2D element

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The state of stress in three dimensions is expressed using the same conventions described for the 2D conditions, but includes stress components on all three orthogonal planes X-Y, X-Z and Y-Z. The state of stress at a point can therefore be described in terms of three(3) normal (σ_{xx}, σ_{yy}, σ_{zz}) and six (6) shear stress ($\tau_{xy}=\tau_{yx}$, $\tau_{xz}=\tau_{zx}$, $\tau_{zy}=\tau_{yz}$) components.

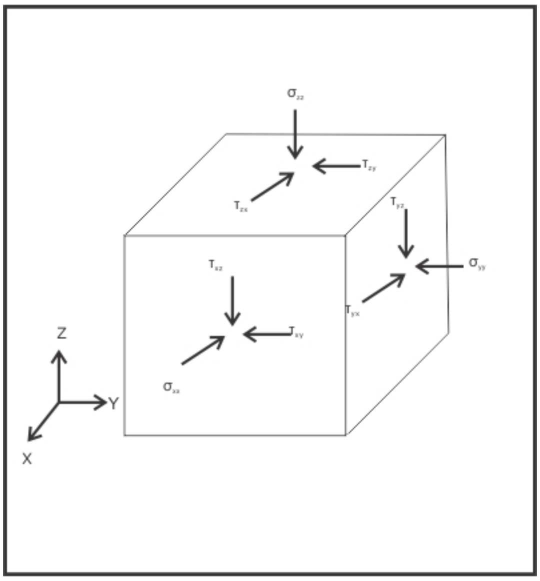

Figure 6: Stress on a 3 D elemental cube

REFERENCE

- Page 114,115 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 9, 10 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 162 - Appendix A, Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
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- Page166 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 2.2

INDICATE THE POSITIVE SENSE OF NORMAL AND SHEAR STRESSES

CONNECTION 1

Refer to "LEARNING OUTCOME 2.1" on page 11 where stress sign notations and orientations were discussed. Critical issues are repeated in this outcome.

BRIDGING GAP 2

"Co-ordinate Systems" on page 227

Rock mechanics sign conventions for:

- **Co-ordinate system**

  In 2D the positive Y-axis is vertically upwards and the positive X-axis is horizontally to the right.In 3D, the right-hand coordinate system means that the positive Z-axis is upwards in the direction of the right-hand thumb if the fingers indicate the direction from the positive X-axis to the positive Y-axis.

- **Normal stress**

  Positive normal stresses and strains are compressive. The positive normal stress is orientated towards the negative coordinate axis.

- **Shear stress**

  Positive shear stress rotates a body towards the origin of positive stress faces and away from the origin of negative stress faces or Shear stress on a positive face is positive when it acts 'towards' the origin and on a negative face 'away' from the origin.

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Although strain is discussed in later sections within this chapter, it is considered necessary to mention the sense of positive strain in this section (Figure 7).

- ***Normal strain***

  Positive normal strain results in decreasing sample dimensions (shortening)

- ***Shear strain***

  Positive shear strain results in increasing the right angle during the change in body shape.

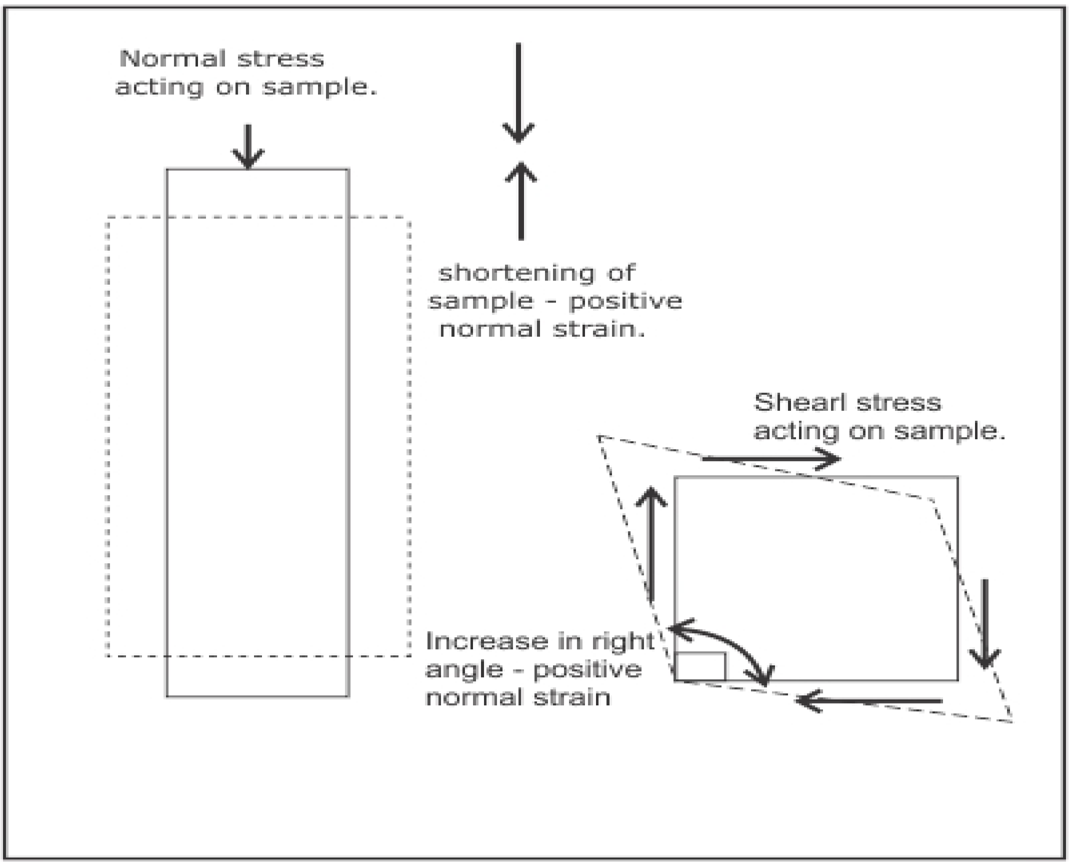

Figure 7: Positive normal and shear strains

- Page 118, 121 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 4, 12 - Rock mechanics in mining practice, 1983, S Budavari et al.
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- Page 11 - Handbook on hard rock strata control, 1995, AJS Spearing
- Page 10 - Rock mechanics in coal mining, 1976, MDG Salamon and KI Oravecz
- Page 8,41 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

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LEARNING OUTCOME 2.3

Calculate the principal stresses from normal and shear components in two dimensions

IMPORTANT

The state of stress at any point in a body can be described in several ways, making use of stress tensors with reference to specific coordinate axes. The same state of stress can be described using normal and shear stresses related to any selected set of Cartesian coordinate axes (X-Y, rotated X'-Y' or m-n axes) or principal stresses, which by definition are the normal stresses when shear stresses are equal to zero (using the principal stress axes) or even in tangential and radial normal and shear stresses (using the polar coordinate system).

The determination of the state of stress in terms of one axis system from a known state of stress in another axis system is called stress transformation or rotation of stresses. This section explains how the state of stress can be transformed from a normal axis system to a principal stress axis system. In the normal axis system, the state of stress is given in terms of normal and shear stresses (σ_x, σ_y and τ_{xy}) and in the principal stress axis system, the state of stress is required in terms of principal stresses (σ_1 and σ_2). For complete state of stress description, the directions of the rotated stress components are essential as only the orientations of the original stress components are known.
In order to calculate the principal stresses from normal and shear components in two dimensions, we therefore need the normal stress (σ_x, σ_y) and shear stress (τ_{xy}) magnitudes.

IMPORTANT

Rotation is not completed unless the magnitude and orientation (direction) of the newly-rotated stress components are known. Just knowing the magnitude of the major principal stress is not enough if the impact on an underground excavation is to be estimated. The direction from which the stress acts on the excavation is critical to be able to indicate whether hangingwall or sidewall concerns will be created.

BRIDGING GAP 3

"Cartesian coordinate system" on page 227

The following equations can be used to calculate the principal stresses from normal and shear stresses, or rather to transform or rotate normal and shear stresses into / to principal stresses.

$$\sigma_1 = \tfrac{1}{2} (\sigma_x + \sigma_y) + \tfrac{1}{2} \sqrt{[(\sigma_x - \sigma_y)2 + 4\,\tau_{xy}^2]}$$

$$\sigma_2 = \tfrac{1}{2} (\sigma_x + \sigma_y) - \tfrac{1}{2} \sqrt{[(\sigma_x - \sigma_y)2 + 4\,\tau_{xy}^2]}$$

NOTE

The only difference in the equations is the positive / negative sign in front of the square roots. Calculation of the principal stresses can be simplified by executing the 1st term and 2nd term once only and then adding them to get the major and subtracting them to get the minor principal stresses.

EXAMPLE

If the state of stress at a point can be described by the normal and shear stresses of magnitude σ_x = 48 MPa, σ_y= 17 MPa and τ_{xy}= τ_{yx}= 14 MPa, determine the magnitude and orientation of the principal stresses that would represent this same state of stress.

The orientation of the principal stresses can be determined using $\tan 2\theta = (2\tau_{xy})/((\sigma_x-\sigma_y))$. In determining the orientations of the principal stresses, knowledge of basic trigonometry is required.

A general graphical illustration of the stress state referred to in this example is shown above in Figure 8 and includes a 2D body diagram and a Mohr circle. The body diagram shows an infinitely small square area subjected to normal stresses σ_x ,σ_y and $\tau_{xy}= \tau_{YX}$ within the X-Y axis system and also shows the probable principal stresses acting in the X_p-X_p axis system, which was rotated anticlockwise through an angle Θ from the

back

IMPORTANT

NOTE

X-Y axes. The Mohr circle represents the same state of stress in what is called 'shear stress (τ on the vertical axis) – normal stress (σ_n on the horizontal axis) space'.

Any state of stress for which rotations are required should always be presented in the body diagram format as it makes the transformation and evaluation easier to understand. This means that a stress state provided in words, such as in this example should immediately be presented on a body diagram before starting with the analysis.
In some rock engineering handbooks, the positive τ axis is orientated vertically downward, but in these notes it will be maintained vertically upward.
The same example will be used in section 2.9 in the Mohr's circle representation.

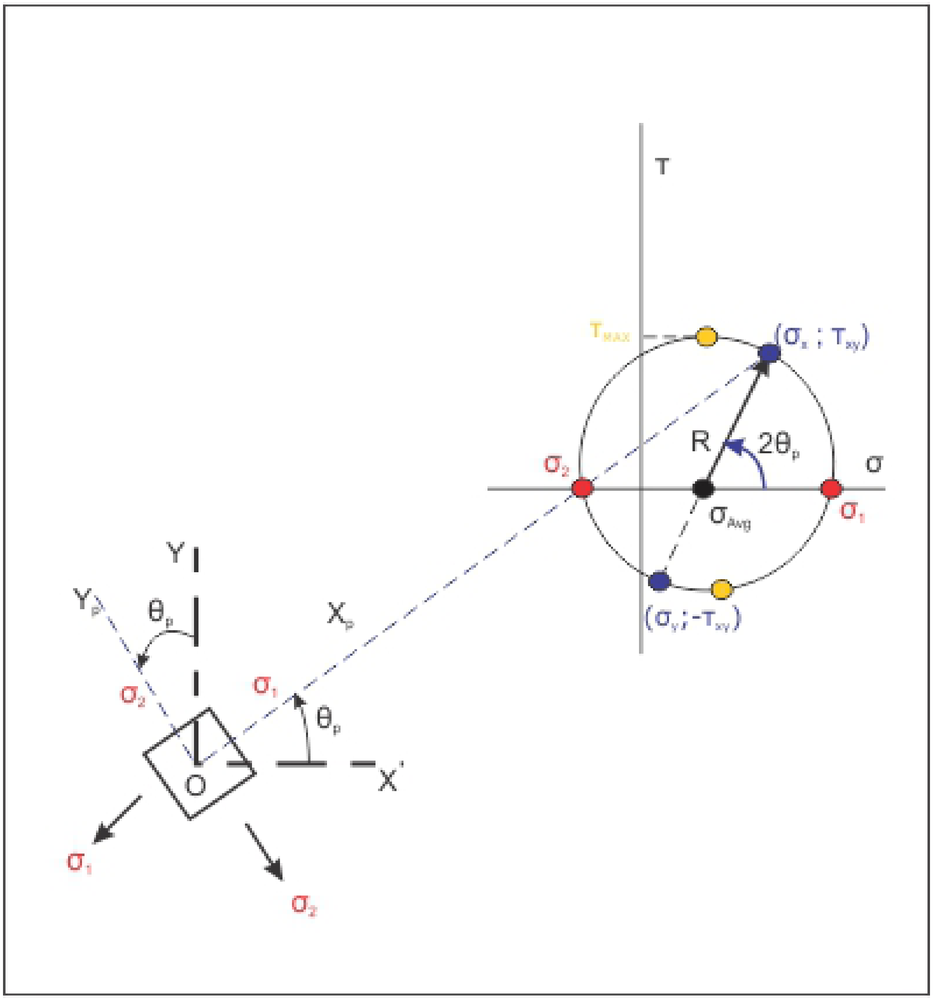

Figure 8: Graphical illustration of the state of stress

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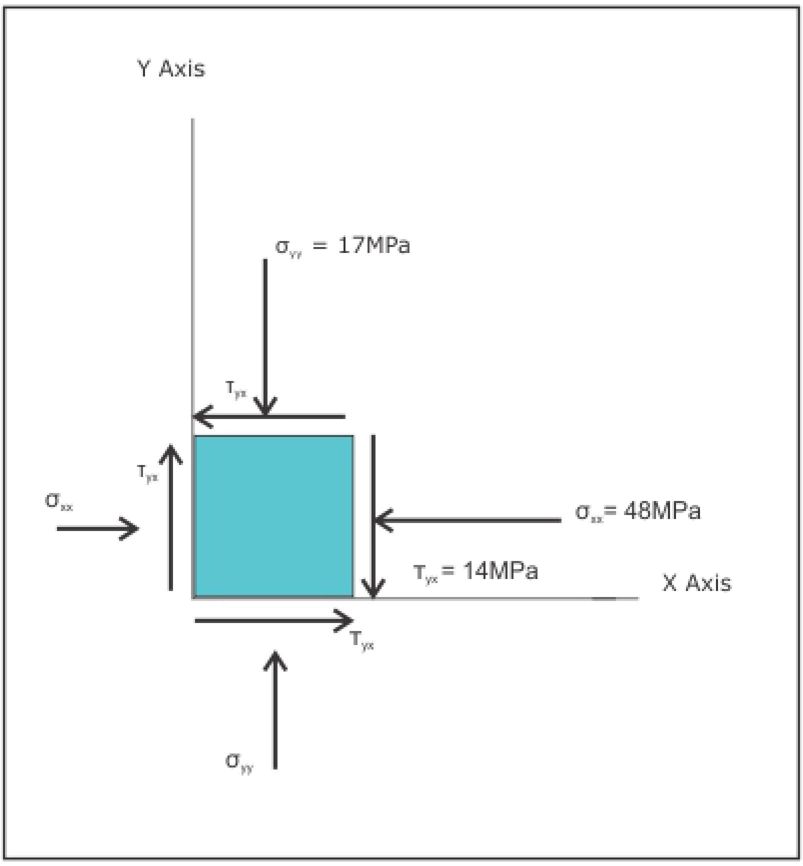
CONNECTION 2

The Mohr circle representation of the state of stress is discussed in "LEARNING OUTCOME 2.9" on page 47

To determine the principal stress magnitudes and orientations required in the example, the following process is suggested:



1. The state of stress given in the example is presented on the body diagram above. If the body diagram is provided, this step has already been completed.

2. Use the information given to determine the magnitudes of the principal stresses.

$\sigma_1 = ½ (\sigma_x + \sigma_y) + ½ \sqrt{} [(\sigma_x - \sigma_y)^2 + 4\ \tau_{xy}{}^2]$

$= ½ (48x10^6 + 17x10^6) + ½\sqrt{} [(48\ x10^6$

$- 17\ x10^6)2 + 4. (14\ x10^6)^2]$Pa

$=32.5\ x10^6 + 20.887\ x10^6$Pa

= $\underline{53.387\ x10^6 Pa}$

= $\underline{53.387 MPa}$

$\sigma_2 = ½ (\sigma_x + \sigma_y) - ½\sqrt{} [(\sigma_x - \sigma_y)^2 + 4\ \tau_{xy}{}^2]$

$= ½ (48x10^6 + 17x10^6) - ½\sqrt{} [(48\ x10^6$

$- 17\ x10^6)^2 + 4. (14\ x10^6)^2]$Pa

$=32.5\ x10^6 - 20.887\ x10^6$Pa

$= 11.613\ x10^6$Pa

= $\underline{11.613 MPa}$

3. Test the answers: An easy test of the answers is to calculate the sum of the normal stresses and the sum of the principal stresses. They should be equal, i.e.

$\sigma_x + \sigma_y = \sigma_1 + \sigma_2$

48 MPa + 17 MPa = 53.387 MPa + 11.613 MPa

65 MPa = 65 MPa

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4. The orientations of the principal stresses (σ_1 and σ_2) are normally indicated as θ_1 and θ_2 respectively and are also called the 'principal stress directions'. They are determined from the normal and shear stresses given in the example using the equation below.

Tan2θ=(2τxy)/((σx-σy))

Solve for θ using the equation above.

$Tan\ 2\theta = 2\ \tau_{xy} / (\sigma_x - \sigma_y)$

$=2(14x10^6) / (48\ x10^6 - 17\ x10^6)$

$=28\ x10^6 / 31\ x10^6$

$=0.9032$

NOTE

Solving for 2θ (by typing 0.9032 into the calculator and pressing the ATAN button). According to basic trigonometry, there exists two angles that are 180° apart, for which the
Tan2θ = 0.9032
2θ = 42.089°
OR
2θ = 42.089° + 180° = 222.089°

IMPORTANT

One of the two orientations must be allocated to the major and the other to the minor principal stresses before the stress transformation is completed.

NOTE

By applying basic trigonometry, the selection of the appropriate angles can be made. All angles between 0° and 360° can be plotted on a full circle by applying the standard convention of measuring the angle anti-clockwise from the 'positive X-axis'.

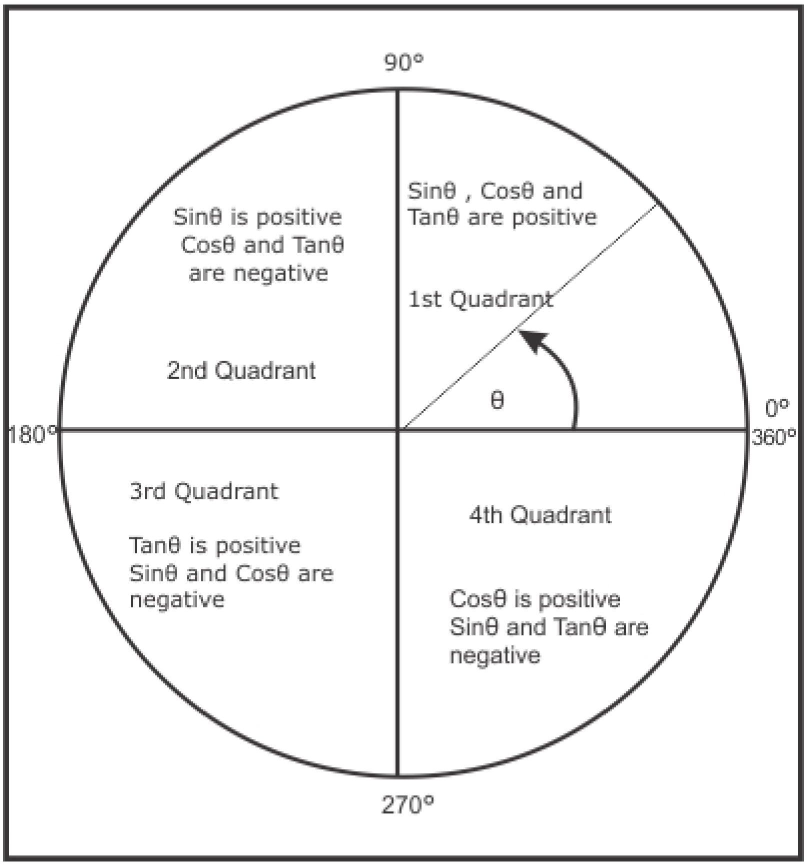
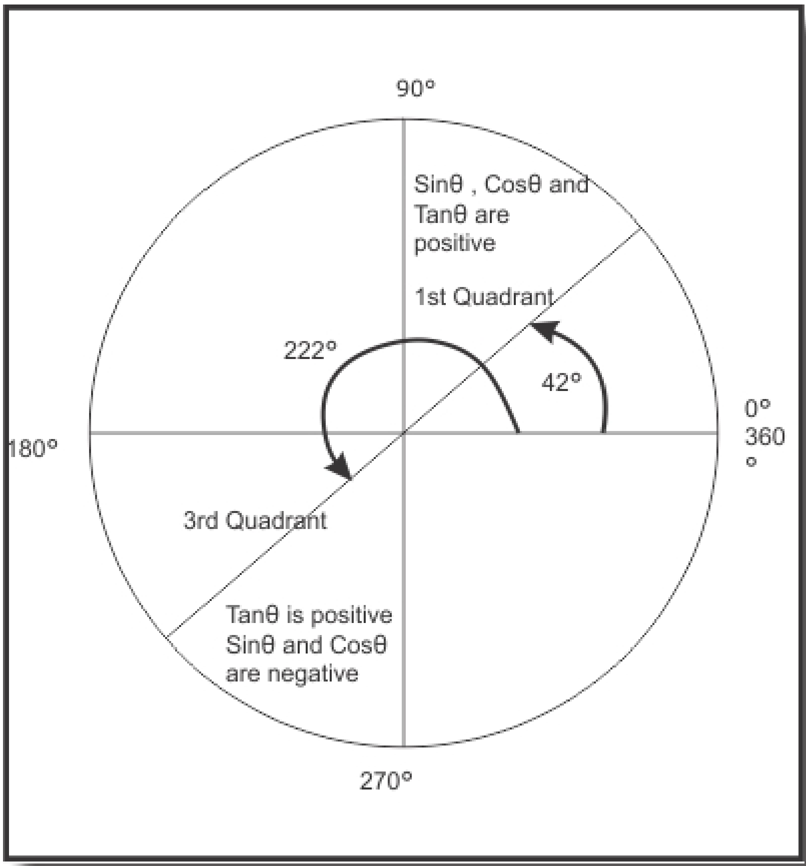


Figure 9: Quadrants and positive trigonometry notations

Plot the two angles calculated on the diagram, as shown below.
Since it is known that 2θ=Sin2θ/Cos2θ , the signs of the individual quotients (Sin2θ and Cos2θ) will determine the sign of the Tan2θ result. By revisiting the Tan2θ calculation, the signs of the quotients can be selected from step 3. In this example both the top and bottom quotients are positive. This means that the major principal stress plots in quadrant 1 where both Sinθ and Cosθ are positive.

Tan $2\theta = 2\,\tau_{xy} / (\sigma_x - \sigma_y)$:Step 1
$= 2(14\times10^6) / (48\times10^6 - 17\times10^6)$:Step 2
$= (28\times10^6)/(31\times10^6)$ or Sin2θ/Cos2θ :Step 3



This selection allows the allocation of orientations to the individual principal stresses:

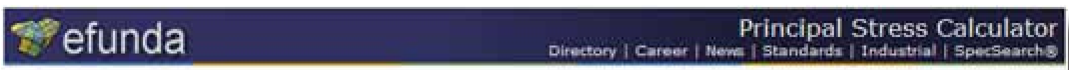
$2\theta_1 = 42.089°$ OR $2\theta_2 = 42.089° + 180° = 222.089°$

$\theta_1 = 21.045°$ OR $\theta_2 = 111.045°$

NOTE

The difference between the angles calculated for θ_1 and θ_2 is 90°. This is in agreement with the definition of principal stresses as perpendicular to each other.

The calculator shown below can be used to check the calculated answers. However, since some of the calculators freely available on the Internet are designed for engineering applications, care must be taken with their assumed stress notations.



REFERENCE

- Page 120, 123 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 17,19,21 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman

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- Page 162 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 7 - Rock mechanics in mining practice, 1983, S Budavari et al
- Page 92 - Underground excavations in rock, 1980, E Hoek and ET Brown
- Page27 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 2.4

CALCULATE THE NORMAL AND SHEAR STRESS COMPONENTS OF STRESS ON A PLANE AT ANY ORIENTATION IN A TWO-DIMENSIONAL STRESS FIELD GIVEN EITHER THE PRINCIPAL STRESSES OR THE STRESS COMPONENTS IN THE X-Y DIRECTIONS

NOTE

The rotation of principal stresses or stress components in the X-Y direction into normal and shear stresses on an arbitrary plane allows the user to determine the normal or 'clamping stress' and shear or 'disturbing stress' on the plane. This allows the user to evaluate potential for slip (according to the Mohr-Coulomb failure criterion) along the particular plane if the plane's properties (cohesion and friction angle) are known.

To calculate the normal and shear components of stress on a plane at any orientation in a two dimensional stress field, the following information is required:

1. The principal stress magnitudes and the angle between one of the principal stress axes and the normal to the plane.

   The normal (σ_n) and shear (τ_n) stresses acting on a plane whose normal is oriented at an angle θ from the major principal axis (σ_1), can be calculated by using the following equations:

   $$\sigma_n = \sigma_1 \text{Cos}^2\theta + \sigma_2 \text{Sin}^2\theta$$

   and

   $$\tau_{nm} = -\tfrac{1}{2} (\sigma_1 - \sigma_2) \text{Sin } 2\theta$$

   If the principal stresses, their orientations and the orientation of the arbitrary plane are provided, the magnitudes and the angle between σ_1 and σ_n are placed within the above equations and the normal and shear stress on the plane calculated.

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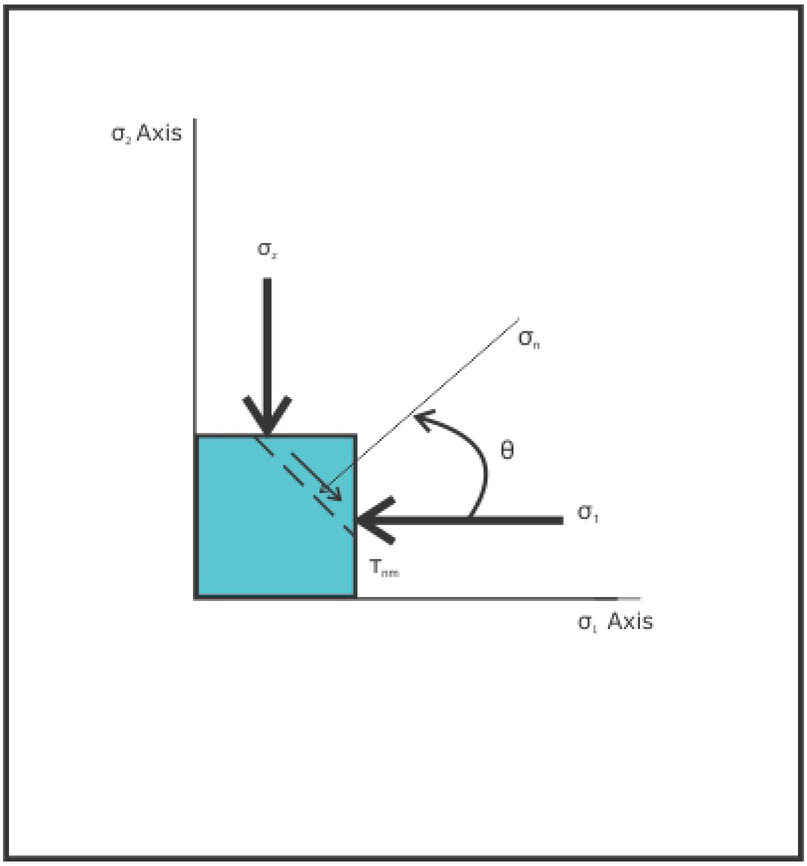

Figure 10: Body diagram for stress rotation

This calculation is similar to rotating the principal stress axes through an angle to an m-n axes system.

2. The normal and shear stresses in a specified axis system. The normal (σ_n) and shear (τ_n) stresses acting on a plane whose normal is oriented at an angle θ from the given normal stress axis(σ_x), can be calculated by using the following equations

$$\sigma_x' = \sigma_x \, Cos^2\theta + 2\tau_{xy} Sin\theta \, Cos\theta + \sigma_y \, Sin^2\theta$$
$$\sigma_y' = \sigma_x Sin^2 \theta \; - 2\tau_{xy} Sin\theta \, Cos\theta + \sigma_y \, Cos^2\theta$$
$$\tau_{xy}' = (\sigma_y - \sigma_x) \, Sin\theta \, Cos\theta + \tau_{xy} \, (Cos^2\theta - Sin^2\theta)$$

In this case it is necessary to rotate the given normal and shear stresses to an axis system where one of the new axes coincides with the normal to the arbitrary plane and the other is orientated parallel to the plane. This rotation is shown below and indicates that shear stress is now present on the side of the rotated body that represents the arbitrary plane, while the normal stresses are acting normal to and parallel to the plane. The solution of the equations above therefore result in two normal stresses (σ_x' and σ_y') and a shear stress (τ_{xy}'), and requires some engineering judgement to select the correct normal stress that is acting NORMAL to the arbitrary plane.

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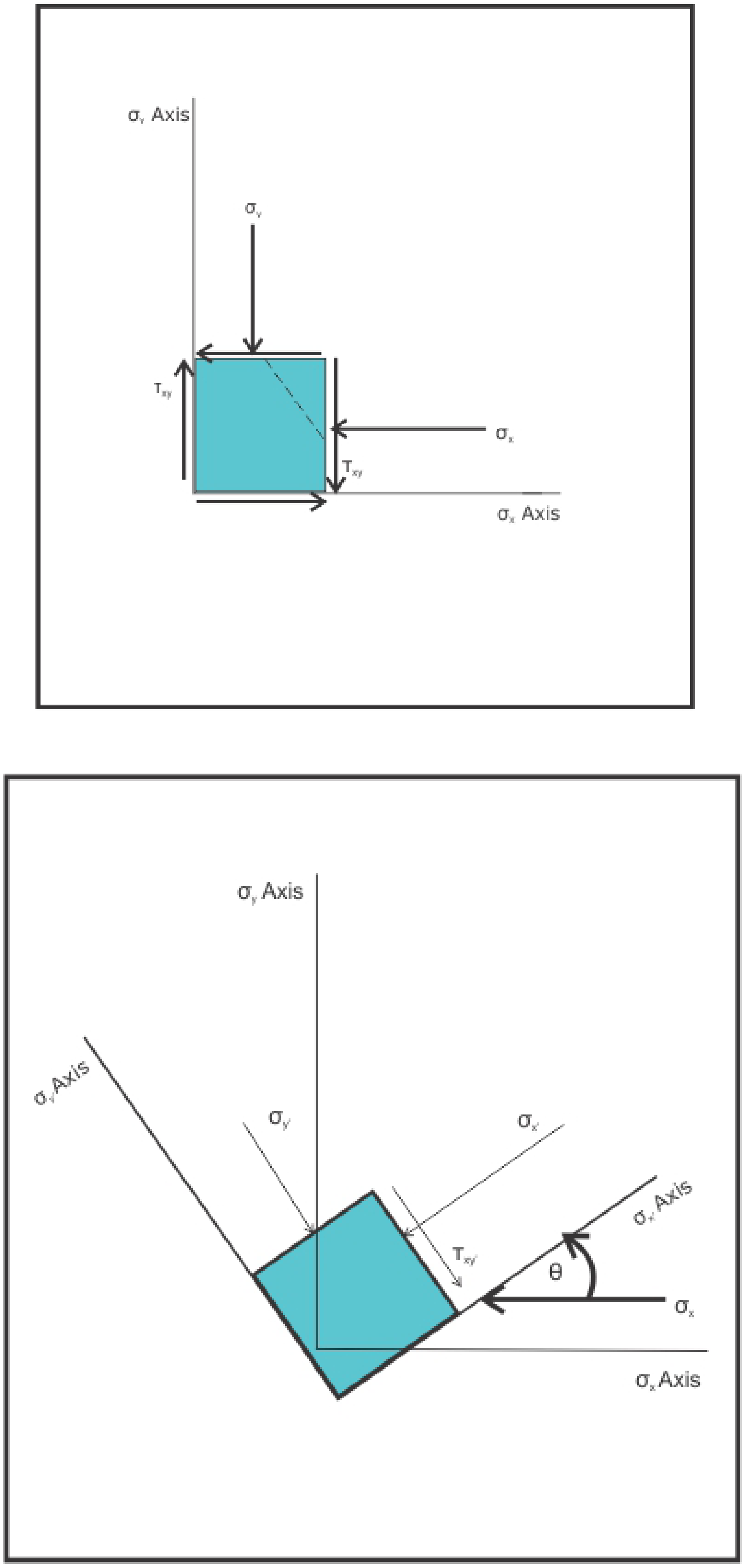

Figure 11: Rotation of the axes

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EXAMPLE

As an introduction to the stress transformations onto an arbitrary plane within a body mentioned above, a simple axes rotation is shown in this example. To calculate the magnitude of the normal and shear stress components in the rotated X'-Y' axes system from the original X-Y axes system, the equations listed in the section above are used. In the example shown in Figure 12 on the right, σx = 60 MPa, σy = 12 MPa, τxy= 8 MPa and the angle through which the axes are rotated is 40° (anticlockwise from the X axis).

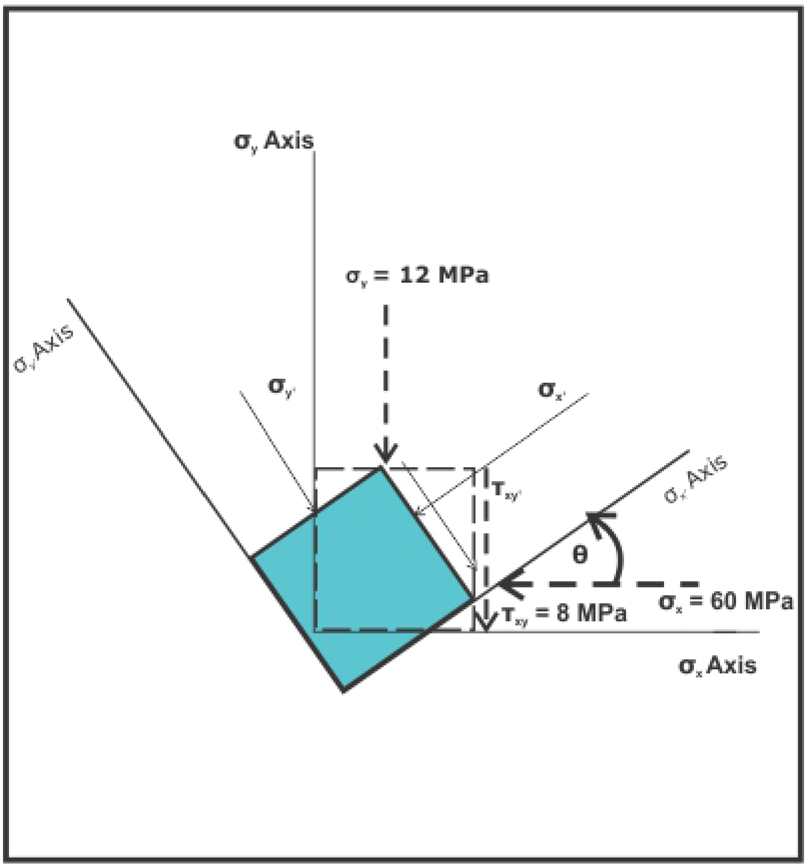

Figure 12: Stress transformation

$\sigma_x' = \sigma_x \; Cos^2\theta + 2\tau_{xy} Sin\theta \; Cos\theta + \sigma_y \; Sin^2\theta$

$= 60x10^6.Cos^2(4)0+2.(8x10^6).Sin40.Cos40+12x10^6.Sin^2(40)$

$=60x10^6.(0.587)+2.(8x10^6).(0.492).+12x10^6.(0.413)$

$= 48.048x10^6Pa.$

Or $\sigma_x' \underline{= 48.048MPa}$.

$\sigma_y' = \sigma_x \; Sin^2 \; \theta \; - 2\tau_{xy} Sin\theta Cos\theta + \sigma_y \; Cos^2\theta$

$= 60x10^6.Sin^2(40)-2.(8x10^6).Sin40.Cos40 + 12x10^6.Cos^2(40)$

$=60x10^6.(0.413)- 2.(8x10^6).(0.492)+ 12x10^6.(0.586)$

$= 23.94x10^6Pa$

Or $\sigma_y' \underline{= 23.94MPa}$

The shear stress τ_{xy}' is calculated as follows.

$\tau_{xy}' = (\sigma_y - \sigma_x) \; Sin\theta Cos\theta + \tau_{xy} \; (cos^2\theta - sin^2\theta)$

$= (12 \; x10^6 - 60 \; x10^6).Sin40Cos40+ 8x10^6.(Cos^2(40) - Sin^2(40))$

$= -48 \; x10^6.(0.492) + 8x10^6.(0.174)$

$= -22.227x10^6Pa$

Or $\tau_{xy}' \underline{= -22.227MPa}$

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EXAMPLE

To calculate the magnitude of the normal and shear stress on an arbitrary plane in a body from principal stresses and the angle between the σ_1 and the normal to the plane, the equations listed below are used.

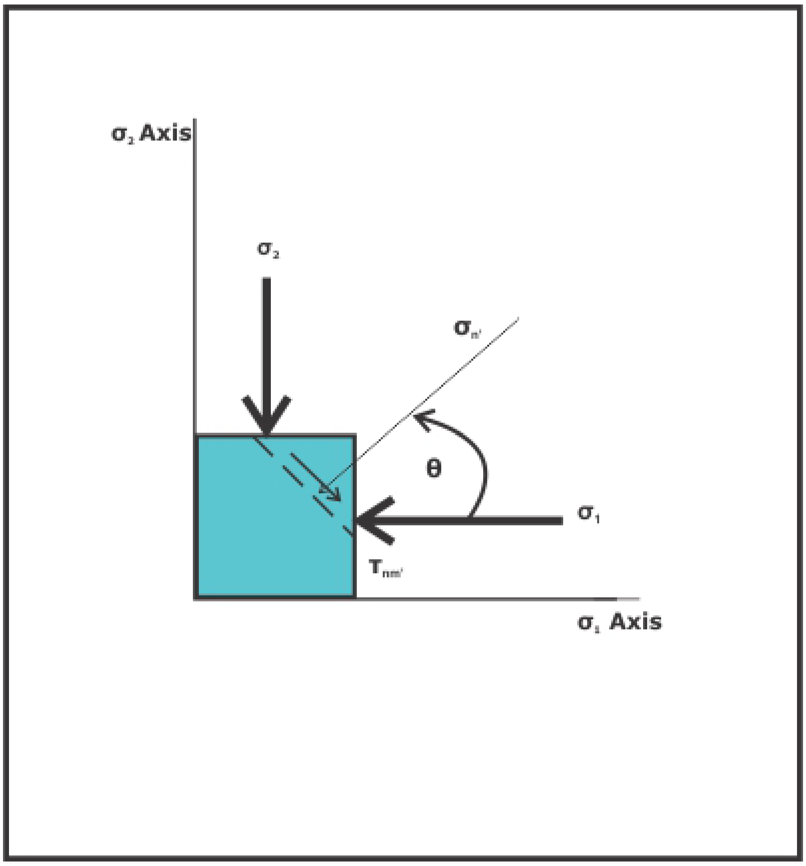

Figure 13: Stress components on an arbitrary plane from principal stresses

$\sigma_n = \sigma_1 \, Cos^2\theta + \sigma_2 \, Sin^2\theta$

and

$\tau_{nm} = -\ ½ \, (\sigma_1 - \sigma_2) \, Sin\ 2\theta$

In the example shown in Figure 13 above, $\sigma_1 = 60$ MPa, $\sigma_2 = 12$ MPa and the angle between σ_1 and σ_n is 40°.

$\sigma_n = \sigma_1 \, Cos^2\theta + \sigma_2 \, Sin^2\theta$
$= 60x10^6.Cos^2(40) + 12x10^6.Sin^2(40)$
$= 60x10^6.(0.587) + 12x10^6.(0.413)$
$= 40.176\ x10^6$ Pa

Or σ_n = <u>40.176 MPa</u>

$\tau_{nm} = -\ ½ \, (\sigma_1 - \sigma_2). \, Sin\ 2\theta$
$= -\ ½ \, (60x10^6 - 12\ x10^6). \, Sin\ 2(40)$
$= -\ ½ \, (48x10^6). \, Sin\ (80)$
$= -\ ½ \, (48x10^6). \, (0.985)$
$= -23.64\ x10^6$

Or τ_{nm} <u>= -23.64 MPa</u>

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EXAMPLE

To calculate the magnitude of the normal and shear stress on an arbitrary plane in a body from normal and shear stresses in the X-Y axes system. In this example, the X-Y axes are rotated anticlockwise towards the X'-Y' axes system through an angle Θ so that one of the new rotated axes falls along the normal to the arbitrary plane. Two normal stresses σ_x' and σ_y' and the shear stress τ_{xy}' are determined using the equations given below. A selection must then be made to select the normal stress that forms the normal to the arbitrary plane.

In the example shown in Figure 14 below, σ_x= 60 MPa, σ_y = 12 Mpa, τxy= 8MPa and the rotational angle = 40 degrees.

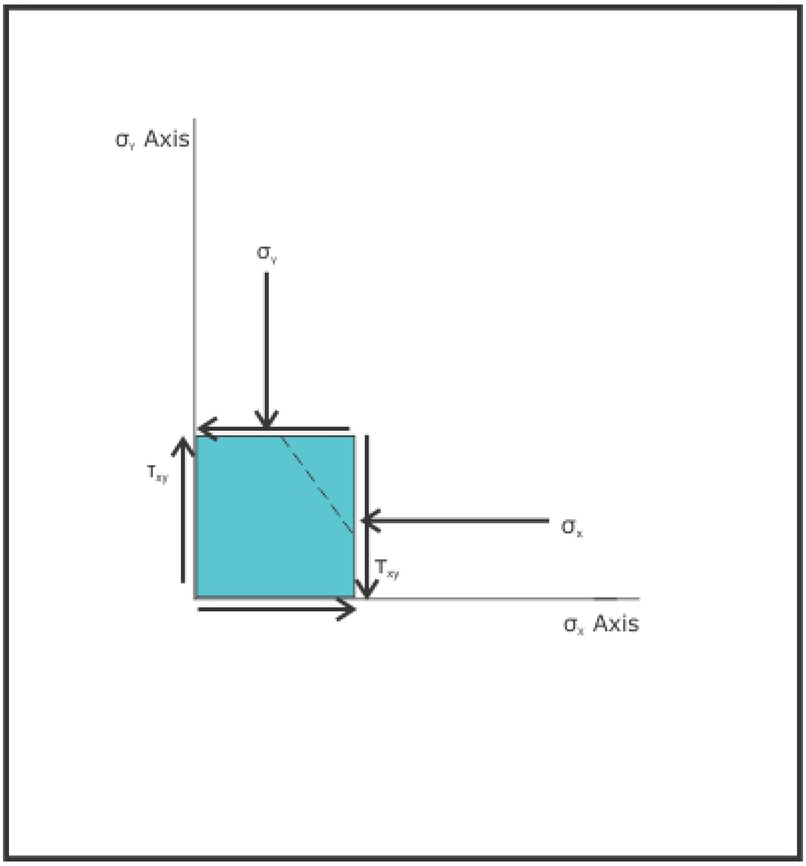

Figure 14 Stress components on a plane from normal and shear stress components

NOTE

The dashed line represents the arbitrary plane on which the stress solution is requested. Rotate the X-Y axis anticlockwise through the required 40° until the X' axis falls on the normal arbitrary plane. The determination of the normal stresses σ_x' and σ_y' and the shear stress τ_{xy}' provides the solution of the normal stress on the plane and the shear stress along the plane.

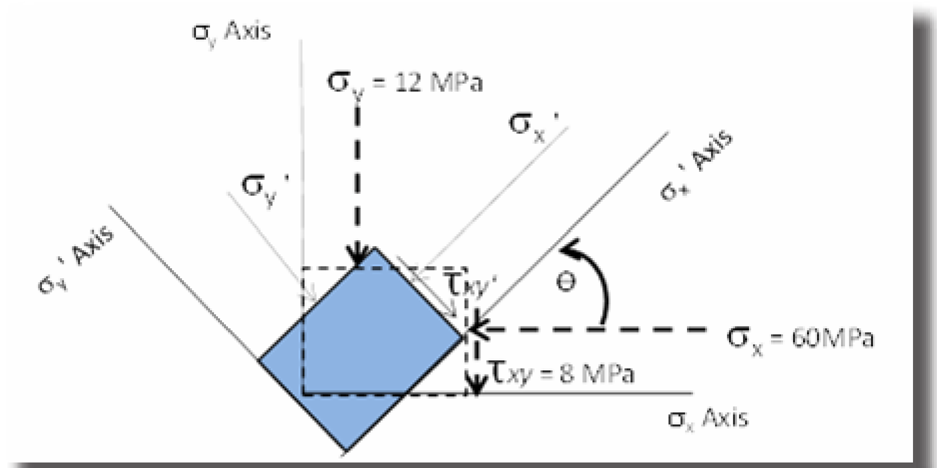

Figure 15 Stress transformation

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$\sigma_x' = \sigma_x \,Cos^2\theta + 2\tau_{xy} Sin\theta\, Cos\theta + \sigma_y \,Sin^2\theta$
$=60x10^6.Cos2(40)+2.(8x10^6).Sin40.Cos40+12x10^6.Sin^2(40)$
$=60x10^6.(0.587)+2.(8x10^6).(0.492)+ 12x10^6.(0.413)$
$= 48.048x10^6Pa.$

Or σ_x' $\underline{= 48.048MPa.}$

$\sigma_y' = \sigma_x \sin^2\theta - 2\tau_{xy}\sin\theta\cos\theta + \sigma_y \cos^2\theta$
$=60x10^6.Sin^2(40)-2.(8x10^6).Sin40.Cos40 + 12x10^6.Cos^2(40)$
$=60x10^6.(0.413)- 2.(8x10^6).(0.492)+ 12x10^6.(0.5860)$
$= 23.94x10^6Pa$

Or σ_y' $\underline{= 23.94MPa}$

The shear stress τ_{xy}'is calculated as follows.

$\tau_{xy}' = (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$
$=(12x10^6 - 60x10^6).Sin40.Cos40 + 8x10^6.(Cos^2(40) - Sin^2(40))$
$= -48\ x10^6.(0.492) + 8x10^6.(0.174)$
$= -22.227x10^6Pa$

Or τ_{xy}' $\underline{= -22.227MPa}$

The answers are the same as in the first example in this section since what was done here is a simple axes rotation. The selection of the appropriate normal stress to the plane depends on an analysis of the body diagram. Referring to the body diagram in Figure 15 it is clear that that the axis that falls along the normal to the plane is X' and therefore σ_x' forms the normal stress on the arbitrary plane.

- Page 5,7 - Rock mechanics in mining practice, 1983, S Budavari et al.

LEARNING OUTCOME 2.5

Calculate the principal stresses from normal-and shear stress components in three dimensions

The word normal was added as it was incorrectly recorded in the syllabus. The normal stresses are required to calculate the principal stress magnitudes and direction cosines in three dimensions.

The state of stress at any point in a three dimensional environment can be described using stress components on three orthogonal planes in an X, Y and Z axes system, and consists of the following nine components.

- • σ_x ; τ_{xy} and τ_{xz}
- • σ_y ; τ_{yx} and τ_{yz}
- • σ_z ; τ_{zx} and τ_{zy}

Where $\tau_{xy} = \tau_{yx}$; $\tau_{xz} = \tau_{zx}$ and $\tau_{yz} = \tau_{zy}$(body is in rotational equilibrium). Since these shear components with interchangeable subscripts are equal, only six (6) components need to be known to be able to fully describe the state of stress, namely (Figure 16):

- • σ_x ; τ_{xy} ;
- • σ_y ; τ_{yx} ;
- • σ_z ; τ_{zx}

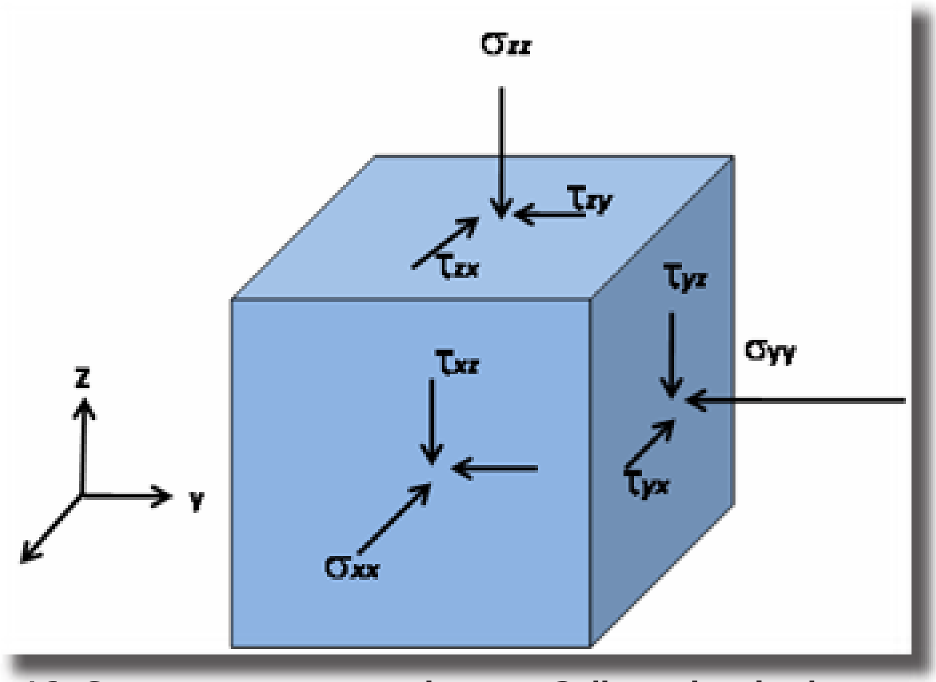

Figure 16: Stress components acting on a 3 dimensional cube

The normal stresses are indicated in various literature as either σ_x, σ_y *and* σ_z or σ_{xx}, σ_{yy} and σ_{zz}.

The stress components in 3d cases are also sometimes written in matrix format as shown below.

Stress Matrix in 3D

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

In matrix format the normal stress components (σ_{xx}, σ_{yy}, σ_{zz}*) are written diagonally from the top left to bottom right positions in the 3D matrix. The shear stress components* τ_{xy}, *and* τ_{xz} *acting parallel to the plane to which* σ_{xx} *is the normal stress, are positioned along the first line in the matrix.The shear stress components* τ_{yx}, *and* τ_{yz} *acting parallel to the plane to which* σ_{yy} *is the normal stress, are positioned along the second line in the matrix and the shear stress components,* τ_{zx}, *and* σ_{zz}, *acting parallel to the plane to which* σ_{zz} *is the normal stress, are positioned along the third line in the matrix. The 1st column in the matrix holds the components that work parallel to the X axis, the 2*nd *column the components that work parallel to the Y axis and the 3*rd*, the components that work parallel to the Z axis.*

Some definitions must be understood prior to explaining how to calculate principal stresses from normal and shear stress components.

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BRIDGING GAP 3

"Cubic Equation" on page 230 in mathematics, a cubic equation is an equation that has three roots (possible solutions for x) and a typical cubic equation is shown below.

$f(x)=ax^3+bx^2+cx+d$

The graph on the right shows that a cubic equation has three possible x-values as solution (intersecting the x axis in three places).

Stress invariants:

Stress invariants refers to three constants (I1, I2 and I3) that are determined from the stress components (either normal and shear or principal stress components), but which are independent of the coordinate system used (xyz or principal).

Important the three stress invariants I1, I2 and I3 can be calculated from the equations shown below.

$$I1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = \sigma_1 + \sigma_2 + \sigma_3$$

$$I2 = -(\sigma_{yy}\sigma_{zz}+\sigma_{zz}\sigma_{xx}+\sigma_{xx}\sigma_{yy})+\tau_{yz}^2+\tau_{zx}^2+\tau_{xy}^2 = -(\sigma_2\sigma_3+\sigma_3\sigma_1+\sigma_1\sigma_2)$$

$$I3 = \sigma_{xx}\sigma_{yy}\sigma_{zz}+2\tau_{yz}\tau_{zx}\tau_{xy}-\sigma_{xx}\tau_{yz}^2-\sigma_{yy}\tau_{zx}^2-\sigma_{zz}\tau_{xy}^2 = \sigma_1\sigma_2\sigma_3$$

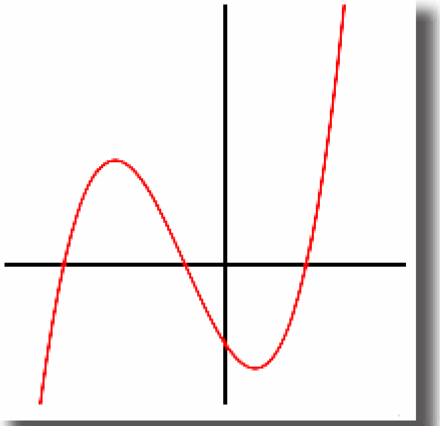

Figure 17: cubic equation with three roots (three x-axis intersections)

Use is made of a cubic equation that includes the stress invariants to solve for the three (cubic equation has three solutions) orthogonal principal stresses. The roots of this cubic equation provide the major principal stress σ_1 the intermediate principal stress σ_2 and the minor principal stress σ_3 as solutions to the cubic equation below.

$$\sigma_p^3 - I_1\sigma_p^2 + I_2\sigma_p - I_3 = 0$$

Resolving such a cubic equation is best done by means of computer programmes, but can be done using basic calculations as discussed below.

$$\sigma_p^3 - I_1\sigma_p^2 + I_2\sigma_p - I_3 = 0$$

In order to find the magnitudes of the principal stresses we first need to calculate the values of the constant q (from the invariants) and angle ø (from invariants and angle ø) , where :

$$Q = (3I_2+I_1^2)^{0.5}$$

And

$$\cos ø = (9I_1I_2+27I_3+2I_1^3)/2Q^3$$

The principal stresses can now be calculated using the following equation:

$$\sigma_p = {}^1/_3(2Q\cos(ø/3+k.120°)+I_1)$$

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EXAMPLE

Calculate the magnitude of the principal stresses in 3D from the normal and shear stress components given below (same values as used on page 142 of Ryder and Jager).

CONNECTION 3

Refer to the "Stress and Strain" on page 230

Where the constant Q, angle ø have been determined previously and the variable 'k' takes on values of 0, 1 and 2 to determine the three solutions to the quadratic equation. The three solutions found for the three values of the variable k must be sorted in descending order of magnitude to give σ_1 (major principal stress = largest result), σ_2 (intermediate principal stress = intermediate result) and σ_3 (minor principal stress = smallest result).

σ_x = 30 MPa;
σ_y = 20MPa;
σ_z = 10 MPa;
τ_x = 6 MPa;
τ_{yz} = -4 MPa and
τ_{zx} = 2 MPa

Calculate the stress invariants

$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$
$= 30 + 20 + 10$
$= \underline{60MPa}$

$I_2 = -(\sigma_{yy}\sigma_{zz}+\sigma_{zz}\sigma_{xx}+\sigma_{xx}\sigma_{yy})+\tau_{yz}^2+\tau_{zx}^2+\tau_{xy}^2$
$= -(20.10 + 10.30 + 30.20) + (-4)^2 + (2)^2 + (6)^2$
$= -(1100) + 16 + 4 + 36$
$= \underline{-1044\ MPa}$

$I_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\tau_{yz}\tau_{zx}\tau_{xy} - \sigma_{xx}\tau_{yz}^2 - \sigma_{yy}\tau_{zx}^2 - \sigma_{zz}\tau_{xy}^2$
$= (30.20.10) + 2(-4.2.6) – (30.-42) – (20.22) – (10.62)$
$= (6000) – 96 – 480 – 80 – 360$
$= \underline{4984\ MPa}$

The value of the constant q can now be calculated:

$Q = (3I_2 + I_1^2)^{0.5}$
$= (3.(-1044) + (60)2)^{0.5}$
$= (-3132 + 3600)^{0.5}$
$= (468)^{0.5}$
$= \underline{21.633MPa}$

Now also determine the angle ø:

$\text{Cos } Ø = (9I_1I_2 + 27I_3 + 2I_1^3) / 2Q^3$
$= [(9.60.-1044) + (27.4984) + (2.603)] / [2.(21.633)^3]$
$= [-563760 + 134568 + 432000] / [20247.912]$
$= [2808] / [20247.912]$
$= \underline{0.1387}$

Therefore:

$Ø$ = ARC COS (0.1387)
= $\underline{82.03°}$

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Place the values for constant Q and angle ø into the equation to solve for the principal stresses and execute with k values of 0, 1 and 2.

FOR k= 0

$$
\begin{aligned}
\sigma_P &= {}^1/_3\ [2QCOS\ (Ø/3 + k.120°) + I1] \\
&= {}^1/_3\ [2(21.633).\ COS(82.03°/3 + 0.120°) + 60] \\
&= {}^1/_3\ [43.266\ COS\ (27.34° + 0°) + 60] \\
&= {}^1/_3\ [43.266\ COS\ (27.34°) + 60] \\
&= {}^1/_3\ [43.266(0.888) + 60] \\
&= {}^1/_3\ [38.431 + 60] \\
&= {}^1/_3\ [98.431] \\
&= \underline{32.810\ MPa}
\end{aligned}
$$

FOR k= 1

$$
\begin{aligned}
\sigma_P &= {}^1/_3\ [2QCOS\ (Ø/3 + k.120°) + I1] \\
&= {}^1/_3\ [2(21.633).\ COS(82.03°/3 + 1.120°) + 60] \\
&= {}^1/_3\ [43.266\ COS\ (27.34° + 120°) + 60] \\
&= {}^1/_3\ [43.266\ COS\ (147.34°) + 60] \\
&= {}^1/_3\ [43.266(-0.841) + 60] \\
&= {}^1/_3\ [-36.425 + 60] \\
&= {}^1/_3\ [23.575] \\
&= \underline{7.858\ MPa}
\end{aligned}
$$

FOR k= 2

$$
\begin{aligned}
\sigma_P &= {}^1/_3\ [2QCOS\ (Ø/3 + k.120°) + I1] \\
&= {}^1/_3\ [2(21.633).\ COS(82.03°/3 + 2.120°) + 60] \\
&= {}^1/_3\ [43.266\ COS\ (27.34° + 240°) + 60] \\
&= {}^1/_3\ [43.266\ COS\ (267.34°) + 60] \\
&= {}^1/_3\ [43.266(-0.046) + 60] \\
&= {}^1/_3\ [-2.007 + 60] \\
&= {}^1/_3\ [57.992] \\
&= \underline{19.331\ MPa}
\end{aligned}
$$

IMPORTANT

The results calculated now need to be sorted in descending order.

Therefore:

$$
\begin{aligned}
\sigma_1 &= 32.81MPa, \\
\sigma_2 &= 19.33\ MPa\ \text{and} \\
\sigma_3 &= 7.86MPa
\end{aligned}
$$

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INTERESTING INFO

To the right is a printout of a 3D stress calculator from the Internet used to verfy the answers.

CONNECTION 4

http://www.efun-da.com/formulae/solid_mechanics/mat_mechanics/plane_stress.cfm

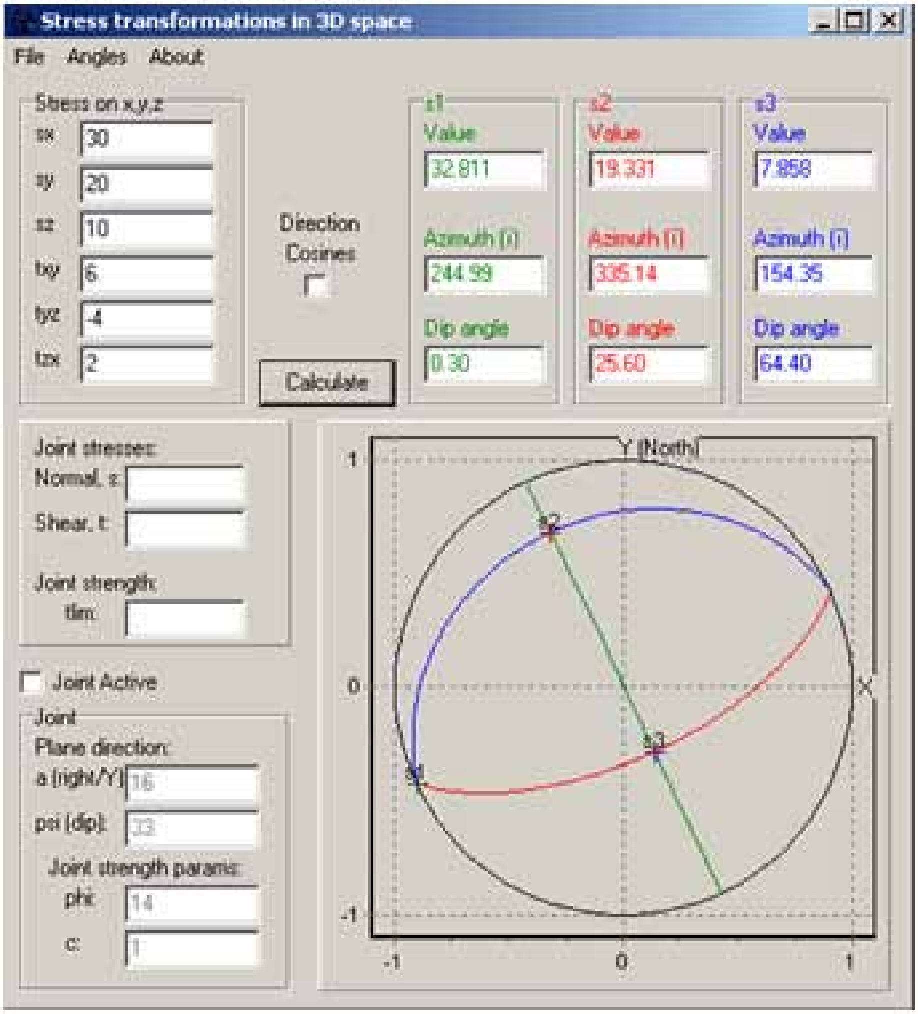

Figure 18: 3 D stress calculator

EXAMPLE

Determine the magnitudes of the principal stresses from the following normal and shear stress components.

σ_x = 48 MPa;
σ_y = 32MPa;
σ_z = 14 MPa;
τ_{xy} = 4 MPa;
τ_{yz} = 2 MPa and
τ_{zx} = 5 MPa.

Calculate the stress invariants

$I_1 = \sigma_x + \sigma_y + \sigma_z$
$= 48 + 32 + 14$
$= \underline{94MPa}$

$I_2 = -(\sigma_{yy}\sigma_{zz}+\sigma_{zz}\sigma_{xx}+\sigma_{xx}\sigma_{yy})+\tau_{yz}^2+\tau_{zx}^2+\tau_{xy}^2$
$= -(32.14 + 14.48 + 48.32) + (2)^2 + (5)^2 + (4)^2$
$= -(2656) + 4 + 25 + 16$
$= \underline{-2611 \text{ MPa}}$

$I_3 = \sigma_{xx}\sigma_{yy}\sigma_{zz} + 2\tau_{yz}\tau_{zx}\tau_{xy} - \sigma_{xx}\tau_{yz}^2 - \sigma_{yy}\tau_{zx}^2 - \sigma_{zz}\tau_{xy}^2$
$= (48.32.14) + 2(2.5.4) - (48.22) - (32.52) - (14.42)$
$= (21504) + 80 - 192 - 800 - 224$
$= \underline{20368MPa}$

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The value of the constant q can now be calculated:

$$\begin{aligned} Q &= (3I_2 + I_1^{2})^{0.5} \\ &= (3.-2611 + (94)^2)^{0.5} \\ &= (-7833 + 8836)^{0.5} \\ &= (1003)^{0.5} \\ &= \underline{31.670\text{MPa}} \end{aligned}$$

Now also determine the angle ø:

$$\begin{aligned} \text{Cos } \varnothing &= (9I_1I_2 + 27I_3 + 2I_1^{3}) / 2Q^3 \\ &= [(9.94.-2611) + (27.20368) + (2.943)] / [2.(31.670)^3] \\ &= [-2208906 + 549936 + 1661168] / [63529.316] \\ &= [21980] / [63529.316] \\ &= \underline{0.0346} \end{aligned}$$

Therefore:

$$\begin{aligned} \varnothing &= \text{ARC Cos } (0.0346) \\ &= \underline{88.02°} \end{aligned}$$

Place the values for constant Q and angle ø into the equation to solve for the principal stresses and execute with k values of 0, 1 and 2.

For k= 0

$$\begin{aligned} \sigma_P &= {}^1/_3\ [2Q\cos (ø/3 + k.120°) + I_1] \\ &= {}^1/_3\ [2(31.670).\ \cos(88.02°/3 + 0.120°) + 94] \\ &= {}^1/_3\ [63.340 \cos (29.33° + 0°) + 94] \\ &= {}^1/_3\ [63.340 \cos (29.33°) + 94] \\ &= {}^1/_3\ [63.340 (0.872) + 94] \\ &= {}^1/_3\ [55.216 + 94] \\ &= \underline{49.738 \text{ MPa}} \end{aligned}$$

For k= 1

$$\begin{aligned} \sigma_P &= {}^1/_3\ [2Q\cos (\varnothing/3 + k.120°) + I_1] \\ &= {}^1/_3\ [2(31.670).\ \text{COS}(88.02°/3 + 1.120°) + 94] \\ &= {}^1/_3\ [63.340 \text{ COS } (29.33° + 120°) + 94] \\ &= {}^1/_3\ [63.340 \text{ COS } (149.33°) + 94] \\ &= {}^1/_3\ [63.340 (-0.860) + 94] \\ &= {}^1/_3\ [-54.479 + 94] \\ &= {}^1/_3\ [39.520] \\ &= \underline{13.173 \text{ MPa}} \end{aligned}$$

for k=2

$$\begin{aligned} \sigma_P &= {}^1/_3\ [2\text{QCOS } (\varnothing/3 + k.120°) + I_1] \\ &= {}^1/_3\ [2(31.670).\ \text{COS}(88.02°/3 + 2.120°) + 94] \\ &= {}^1/_3\ [63.340 \text{ COS } (29.33° + 240°) + 94] \\ &= {}^1/_3\ [63.340 \text{ COS } (269.330°) + 94] \\ &= {}^1/_3\ [63.340 (-0.012) + 94] \\ &= {}^1/_3\ [-0.741 + 94] \\ &= {}^1/_3\ [93.259] \\ &= \underline{31.086 \text{ MPa}} \end{aligned}$$

The results calculated now need to be sorted in descending order.

Therefore:

$$\sigma_1 = 49.7\text{MPa};$$
$$\sigma_2 = 31.1\text{MPa};$$
$$\sigma_3 = 13.2\text{MPa}$$

INTERESTING INFO

On the right is a printout of a 3D stress calculator from the internet used to verify the answers.

CONNECTION 5

http://www.efunda.com/formulae/solid_mechanics/mat_mechanics/plane_stress.cfm

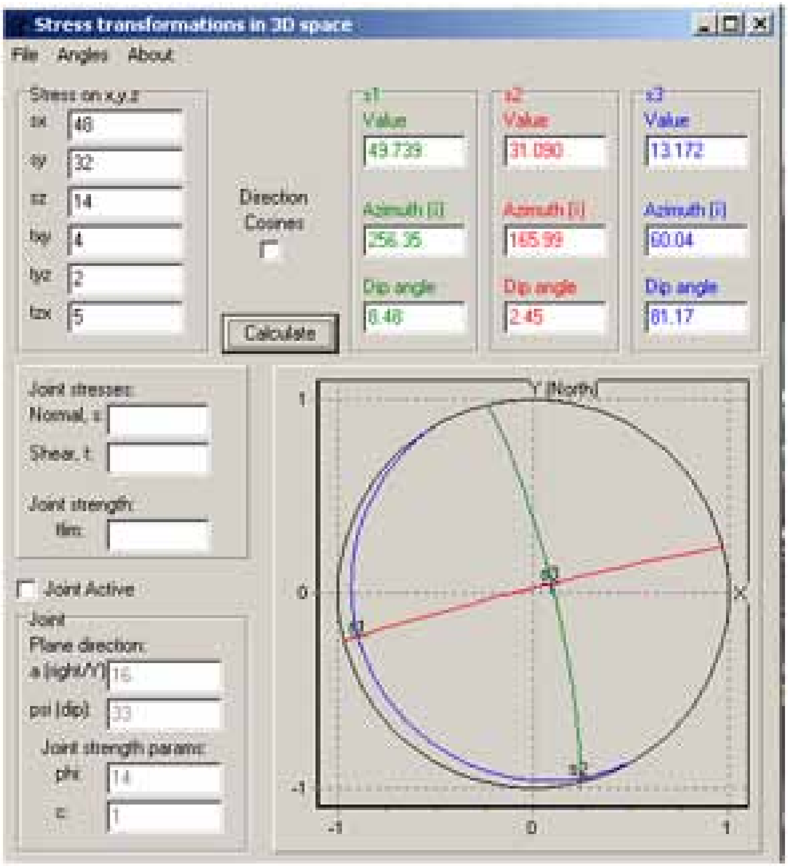

Figure 19: 3 D stress calculator

Matrix: a matrix is a collection of numbers arranged into a fixed number of rows and columns. Below, in figure 20, figure 21 and figure 22, typical 3x3 matrices are shown, each with three rows and three columns.Persons familiar with matrix multiplications can use it to calculate the magnitudes of the principal stresses and for stress transformation purposes.

$$\begin{vmatrix} 1 & -2 & 3 \\ 0 & 8 & 4.6 \\ 4 & -1 & 0 \end{vmatrix}$$

Figure 20: A typical 3 x 3 matrix

$$\begin{vmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{vmatrix}$$

Figure 21: 3D normal and shear stress matrix

$$\begin{vmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{vmatrix}$$

Figure 22: 3D principal stress matrix

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- Page 127, 139 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 165, 139 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 17,27,32,38 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 15 - Handbook on hard rock strata control, 1995, AJS Spearing
- Page 18,23 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 2.6

CALCULATE THE NORMAL AND SHEAR COMPONENTS OF STRESS ON A PLANE AT ANY ORIENTATION IN A THREE-DIMENSIONAL STRESS FIELD GIVEN EITHER THE PRINCIPAL STRESSES OR THE STRESS COMPONENTS IN THE X-Y AND Z DIRECTIONS

Some definitions must be understood prior to explaining how to calculate normal and shear stress components on a plane at any orientation in three dimensions.

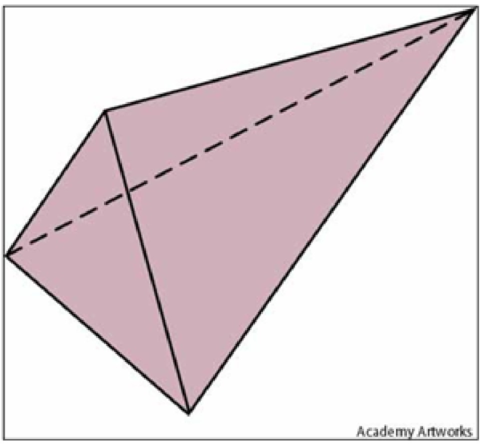

Figure 23: Example of a tetrahedron

Tetrahedron: *A tetrahedron is a solid figure that has four planar faces, and a regular tetrahedron has equilateral triangles. Figure 23 shows a tetrahedron.*

Direction cosines: *In practice, the state of stress expressed in normal stress components may be more conveniently expressed in the plane of a fault. The stress components would then have to be re expressed in terms of a new coordinate system, whose axes can be considered to be rotated in relation to the original coordinate system. Direction cosines provide a mathematical tool to express the new system in terms of the old system using cosines of the angles each new co-ordinate axis makes with the corresponding old axis.*

In two dimensions, consider a set of axes rotated in relation to an original set. The rotated system is referred to as the "dashed" system, and named X'_j, while the original system is called X_j. Figure 24 below shows the dashed system of axes rotated through an angle θradians in relation to the un-dashed (original) system.

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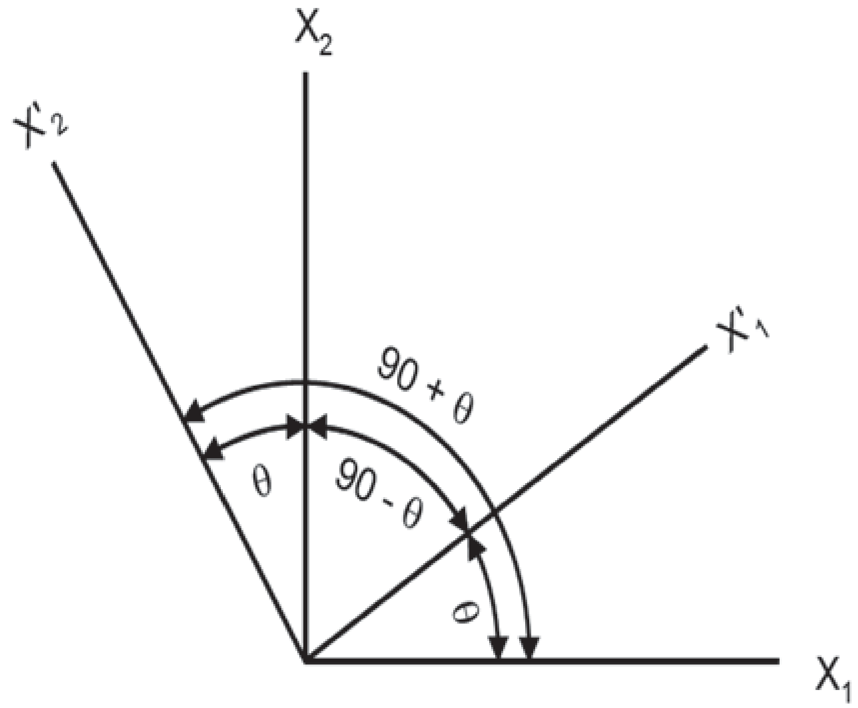

Figure 24: Dashed axes rotated through angle θ with respect to un-dashed axes

There are four angles, defined by directional cosines λ_{ij}, relating the axes to each other. A directional cosine is defined as the cosine of the angle between a dashed axis and an undashed axis, in the plane defined by the two axes.

In three dimensions, the definition holds and the naming of the directional cosines is then based on which axes define the angle; here the subscript i in λ_{ij} is defined by X'_i, and j after X_j (i.e. 1st subscript refers to the dash axis and the 2nd subscript to the un-dashed axis).

$$\lambda_{11} = \text{Cos}\ (X'_1 - X_1) = \text{Cos}\ \theta$$
$$\lambda_{12} = \text{Cos}\ (X'_1 - X_2) = \text{Cos}\ (90^{\circ} - \theta) = \text{Sin}\ \theta$$
$$\lambda_{21} = \text{Cos}\ (X'_2 - X_1) = \text{Cos}\ (90^{\circ} + \theta) = -\text{Sin}\ \theta$$
$$\lambda_{22} = \text{Cos}\ (X'_2 - X_2) = \text{Cos}\ \theta$$

The directional cosines λ_{ij} can be arranged in matrix form as follows:

$$\lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} Cos\theta & Sin\theta \\ -Sin\theta & Cos\theta \end{bmatrix}$$

In three dimensions, the direction cosine of a line are the cosines of the direction angles, where the direction angles, Beta, Psi and Theta are the positive directions of respectively the X, Y and Z axes of a rectangular coordinate system (Figure 25).

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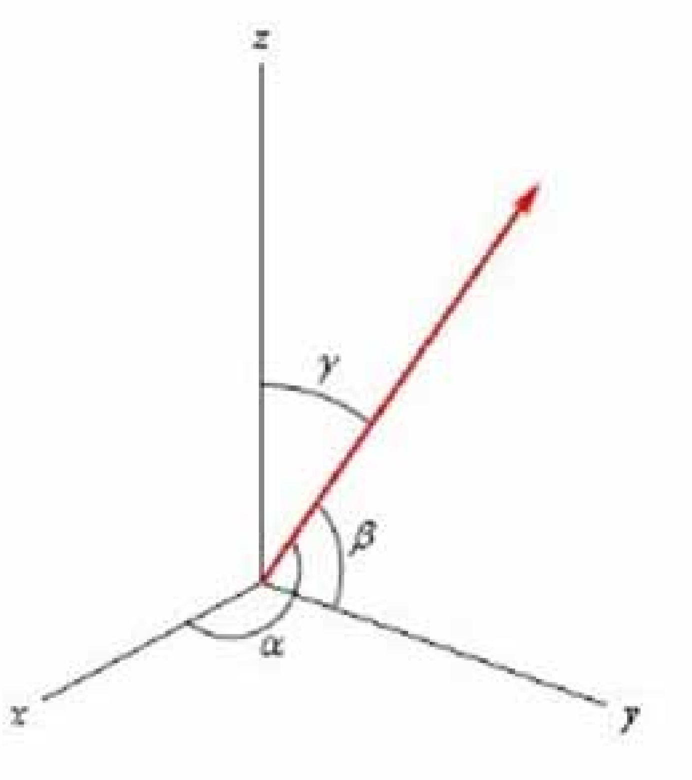

Figure 25 Betha, Psi and Thetha angles

Stress rotational formulae: Stress rotational formulae are formulae that can be used to calculate the state of stress in an arbitrarily coordinated system eg. (X', Y' and Z') in terms of the direction cosines and the six normal stress components.

In order to fully understand 3D stress transformation pages 139 -141 of A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager , should be worked through.

Figure 26 shows a tetrahedron drawn on the XYZ coordinate system. The normal to the arbitrary ABC plane is given by X'and represents the orientation of the plane ABC. In other words, if it is possible to describe the normal to the plane X' , then it is possible to know the orientation of the plane ABC. The direction cosines (cosines of the angles that X' makes with the X,Y and Z axes), assist in determining this orientation.

If the tetrahedron shaped element is in equilibrium, the stress components (σ_x ;σ_y ; σ_z ;τ_{xy};τ_{yz} and τ_{zx}) on the orthogonal planes must balance out the traction vector P on the plane ABC.

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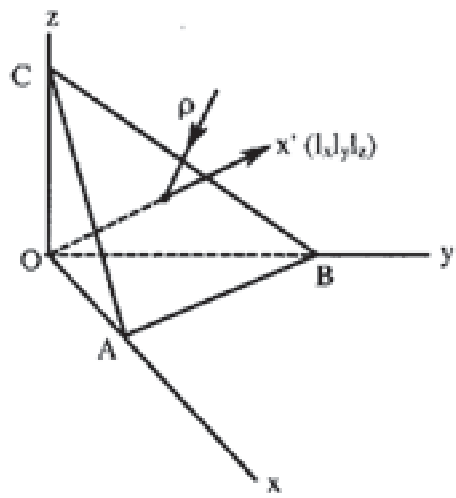

Figure 26: Tetrahedron (Ryder and Jager)

- Pages 139-141 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

The components of the traction vectors P_x, P_y and P_z along the x, y and z directions are given by the equations shown below.

$$p_x = \sigma_{xx}\lambda_{xx} + \tau_{yx}\lambda_{xy} + \tau_{zx}\lambda_{xz}$$
$$p_y = \tau_{xy}\lambda_{xx} + \sigma_{yy}\lambda_{xy} + \tau_{zy}\lambda_{xz}$$
$$p_z = \tau_{xz}\lambda_{xx} + \tau_{yz}\lambda_{xy} + \sigma_{zz}\lambda_{xz}.$$

If one now considers plane ABC the normal stress σ_{xx}' that acts normal to the x'plane can now be determined by adding all the traction vectors P_x ,P_y and P_z along the x' direction.

The formulae shown below were derived from the projection of P_x ,P_y and P_z in the x' direction and adding these projections.

$$\sigma_{xx}' = \sigma_{xx}\lambda_{xx}^2 + \sigma_{yy}\lambda_{xy}^2 + \sigma_{zz}\lambda_{xz}^2 + 2\tau_{xy}\lambda_{xx}\lambda_{xy} + 2\tau_{yz}\lambda_{xy}\lambda_{xz} + 2\tau_{zx}\lambda_{xz}\lambda_{xx}$$

In order to determinethe shear stresses acting on plane x' the formulae shown below are used.

$$\tau_{x'y'} = \sigma_{xx}\lambda_{xx}\lambda_{yx} + \sigma_{yy}\lambda_{xy}\lambda_{yy} + \sigma_{zz}\lambda_{xz}\lambda_{yz} + \tau_{xy}(\lambda_{xx}\lambda_{yy} + \lambda_{xy}\lambda_{yx}) +$$
$$\tau_{yz}(\lambda_{xy}\lambda_{yz} + \lambda_{xz}\lambda_{yy}) + \tau_{zx}(\lambda_{xz}\lambda_{yx} + \lambda_{xx}\lambda_{yz})$$

with a similar result for $\tau_{x'z'}$ by projecting in the z' direction.

By means of re-aligning the formulae used to calculate $\sigma_{xx'}$ the other normal and shear stress components acting on the planes normal to y' or z' can be found . The formulae derived to calculate $\sigma_{yy'}$ is shown below.

$$\sigma_{yy'} = \sigma_{xx}\lambda_{yx}^2 + \sigma_{yy}\lambda_{yy}^2 + \sigma_{zz}\lambda_{yz}^2 + 2\tau_{xy}\lambda_{yx}\lambda_{yy} + 2\tau_{yz}\lambda_{yy}\lambda_{yz} + 2\tau_{zx}\lambda_{yz}\lambda_{yx}$$

To determine the state of stress on plane ABC, it is more convenient to set up a new coordinate system called X’Y’Z’ where the X’ axis is normal to plane ABC (Figure 25). This new coordinate system can be defined by using the original XYZ axes and the orientation of the new aces system as given by the direction cosines below.

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	X′	Y′	Z′
X	λ_{xx}	λ_{yx}	λ_{zx}
Y	λ_{xy}	λ_{yy}	λ_{zy}
Z	λ_{xz}	λ_{yz}	λ_{zz}

Applying the definition of the direction cosines λ_{xx} is the cosine of the angle between the X′ and X axes, λ_{xy} is the cosine of the angle between the X′ and Y axes, etc.

Stress rotation formulae were then derived by solving the components of the traction vector P (P_x,P_y and P_z) and projecting them onto the new coordinate axes. The results of this process provided:

$$\sigma_{xx'} = \sigma_{xx}\lambda_{xx}^2 + \sigma_{yy}\lambda_{xy}^2 + \sigma_{zz}\lambda_{xz}^2 + 2\tau_{xy}\lambda_{xx}\lambda_{xy} + 2\tau_{yz}\lambda_{xy}\lambda_{xz} + 2\tau_{zx}\lambda_{xz}\lambda_{xx}$$

$$\sigma_{yy'} = \sigma_{xx}\lambda_{yx}^2 + \sigma_{yy}\lambda_{yy}^2 + \sigma_{zz}\lambda_{yz}^2 + 2\tau_{xy}\lambda_{yx}\lambda_{yy} + 2\tau_{yz}\lambda_{yy}\lambda_{yz} + 2\tau_{zx}\lambda_{yz}\lambda_{yx}$$

$$\sigma_{zz'} = \sigma_{xx}\lambda_{zx}^2 + \sigma_{yy}\lambda_{zy}^2 + \sigma_{zz}\lambda_{zz}^2 + 2\tau_{xy}\lambda_{zx}\lambda_{zy} + 2\tau_{yz}\lambda_{zy}\lambda_{zz} + 2\tau_{zx}\lambda_{zz}\lambda_{zx}$$

$$\tau_{x'y'} = \sigma_{xx}\lambda_{xx}\lambda_{yx} + \sigma_{yy}\lambda_{xy}\lambda_{y}y + \sigma_{zz}\lambda_{xz}\lambda_{yz} + \tau_{xy}(\lambda_{xx}\lambda_{yy} + \lambda_{xy}\lambda_{yx}) + \tau_{yz}(\lambda_{xy}\lambda_{yz} + \lambda_{xz}\lambda_{yy}) + \tau_{zx}(\lambda_{xz}\lambda_{yx} + \lambda_{xx}\lambda_{yz})$$

IMPORTANT

When doing a 3D stress transformation calculation it is important that the correct value of the direction cosine be used in the formulae. It is therefore a good practice to have a copy of the stress transformation matrix shown below next to the matrix indicating the actual values of the direction cosines (prevent confusion).

Below in Figure 27 the stress transformation matrix is shown and in Figure 28 the values of the direction cosines are shown in matrix form.

	X′	Y′	Z′
X	λ_{xx}	λ_{yx}	λ_{zx}
Y	λ_{xy}	λ_{yy}	λ_{zy}
Z	λ_{xz}	λ_{yz}	λ_{zz}

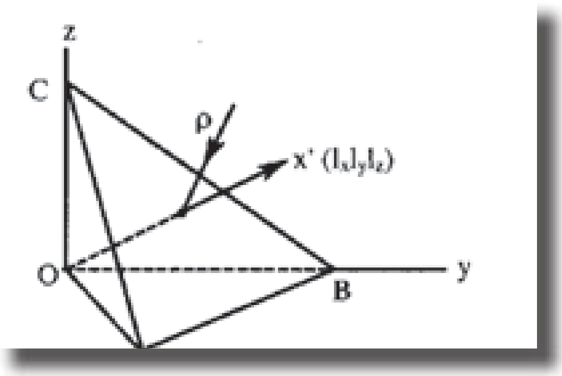

Figure 27: Example of a stress transformation matrix

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EXAMPLE

Given the following state of stress: σ_{xx} = 12MPa; σ_{yy} = 8 MPa; σ_{zz} = 5MPa, τ_{xy} = 1MPa; τ_{yz} = -1MPa and τ_{zx} = 2MPa, and the following direction cosines in matrix format, perform the stress rotation.

$$\begin{bmatrix} 0.960261 & -0.07399 & -0.269116 \\ 0.157097 & 0.940262 & 0.30204 \\ 0.230692 & -0.332314 & 0.914521 \end{bmatrix}$$

Figure 28: Values of the direction cosines used in the example

σ_{xx}'can be calculated from the formulae shown below.

$$\begin{aligned}\sigma_{xx}' = {} & \sigma_{xx}\lambda_{xx}^2 + \sigma_{yy}\lambda_{xy}^2 + \sigma_{zz}\lambda_{xz}^2 + 2\tau_{xy}\lambda_{xx}\lambda_{xy} + 2\tau_{yz}\lambda_{xy}\lambda_{xz} + 2\tau_{zx}\lambda_{xz}\lambda_{xx} \\ = {} & 12(0.960261)^2+8(0.157097)^2+5(0.230692)^2+2(1)(0.960261)(0.157097)+2(-1)(0.157097)(0.230692)+2(2)(0.230692)(0.960261)\text{ MPa} \\ = {} & \underline{12.644\text{ MPa}}\end{aligned}$$

$$\begin{aligned}\sigma_{yy'} = {} & \sigma_{xx}\lambda_{yx}^2 + \sigma_{yy}\lambda_{yy}^2 + \sigma_{zz}\lambda_{yz}^2 + 2\tau_{xy}\lambda_{yx}\lambda_{yy} + 2\tau_{yz}\lambda_{yy}\lambda_{yz} + 2\tau_{zx}\lambda_{yz}\lambda_{yx} \\ = {} & 12(-0.07399)^2+8(0.940262)^2+5(-0.332314)^2+2(1)(-0.07399)(0.940262)+2(-1)(0.940262)(-0.332314)+2(2)(-0.332314)(-0.07399) \\ = {} & \underline{8.275\text{ MPa}}\end{aligned}$$

$$\begin{aligned}\sigma_{zz'} = {} & \sigma_{xx}\lambda_{zx}^2 + \sigma_{yy}\lambda_{zy}^2 + \sigma_{zz}\lambda_{zz}^2 + 2\tau_{xy}\lambda_{zx}\lambda_{zy} + 2\tau_{yz}\lambda_{zy}\lambda_{zz} + 2\tau_{zx}\lambda_{zz}\lambda_{zx} \\ = {} & 12(-0.269116)^2+8(0.30204)^2+5(0.914521)^2+2(1)(-0.269116)(0.30204)+2(-1)(0.30204)(0.914521)+2(2)(0.914521)(-0.269116) \\ = {} & \underline{4.0811\text{MPa}}\end{aligned}$$

$$\begin{aligned}\tau_{x'y'} = {} & \sigma_{xx}\lambda_{xx}\lambda_{yx}+\sigma_{yy}\lambda_{xy}\lambda_{y}y+\sigma_{zz}\lambda_{xz}\lambda_{yz}+\tau_{xy}(\lambda_{xx}\lambda_{yy}+\lambda_{xy}\lambda_{yx})+\tau_{yz}(\lambda_{xy}\lambda_{yz}+\lambda_{xz}\lambda_{yy})+\tau_{zx}(\lambda_{xz}\lambda_{yx}+\lambda_{xx}\lambda_{yz}) \\ = {} & 12(0.960261)(-0.07399)+8(0.157097)(0.940262)+5(0.914521)(-0.332314)+1(0.960261)(0.940262)+0.940262)+(-1)(0.157097)(-0.07399)+(0.230692)(0.940262)+2(0.230692)(-0.07399)+(0.960261)(-0.332314) \\ = {} & \underline{0\text{ MPa}}\end{aligned}$$

A similar result for $\tau_{y'z'}$ and $\tau_{x'z'}$ by projecting in the y' and z' directions will be calculated as the direction cosines chosen in this example coincides with the direction cosines of the principal stresses.

REFERENCE

- Also see the format of these equations used in Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman - Page 26 Equations 11-13

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When calculating from principal stresses to normal and shear stress components relative to the x, y and z planes, the same formulae as in the previous example will be used. The only difference will be that Sigma 1, 2 and 3 values will replace the normal stress values in the formulae, and as principal stresses have no shear components a value of 0 will be used in the formulae for shear stresses

- Chapter 3 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

EXAMPLE

Calculate the normal and shear components on planes x, y and z given the principal stress magnitudes.Input data σ_1=15.832MPa σ_2=7.4MPa and σ3=0.768MPa

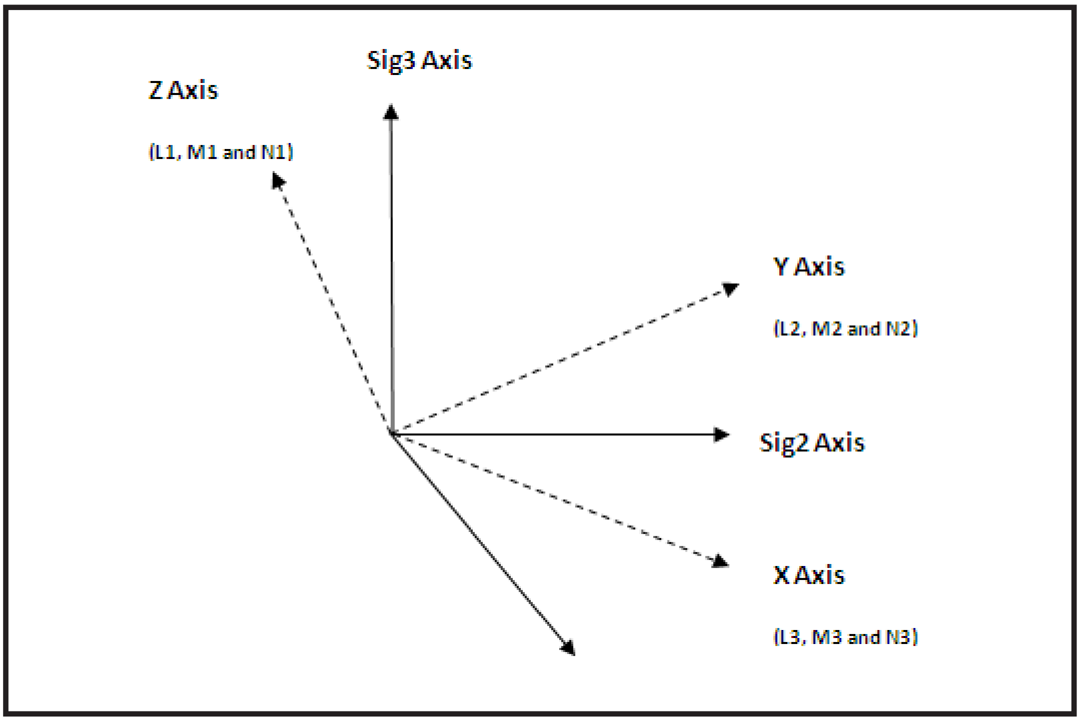

Figure 29: Stress transformation example

Shear stresses = 0 as the state of stress is given in principal stresses therefore the formulae can be simplified as shown below as the parts containing the shear stresses reduce to a value of zero.

With this method it is important to firstly fix the rotational angles from the principal stress to the normal axes (principal stress to XYZ coordinate system). Remember that it is a rotation from a principal stress coordinate system to an XYZ coordinate system.

- In this example the method explained in Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman, pages 25 – 26 were used.

If we assume the rotational angles θ and λ to be 20° and 42° respectively, the direction cosines can be calculated from:

$l_1 = \cos\theta$, $m_1 = \sin\theta$ and $n_1 = 0$
$l_2 = -\sin\theta$, $m_2 = \cos\theta$ and $n_2 = 0$
$l_3 = \sin\theta . \cos\lambda$, $m_3 = \sin\theta . \sin\lambda$ and $n_3 = \cos\theta$

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Therefore the calculated direction cosines are (in matrix format):

$$\begin{bmatrix} 0.93969262 & -0.34202014 & 0.2541705 \\ 0.3420201 & 0.93969262 & 0.22885615 \\ 0.0 & 0.0 & 0.93969262 \end{bmatrix}$$

NOTE

These direction cosines can also be written as:

	1	2	3
I	0.93869262	-0.34202014	0.2541705
m	0.3420201	0.03969262	0.2288561
n	0.0	0.0	0.93969262

σ_{xx} $= \sigma_1\lambda_{xx}^2 + \sigma_2\lambda_{xy}^2 + \sigma_3\lambda_{xz}^2$ or $\sigma_{xx} = \sigma_1 l_1^2 + \sigma_2 m_1^2 + \sigma_3 n_1^2$
$= 15.832(0.93869262)^2 + 7.4(0.3420201)^2 + 0.768(0)^2$
$=$ <u>14.816 MPa</u>.

σ_{yy} $= \sigma_1\lambda_{yx}^2 + \sigma_2\lambda_{yy}^2 + \sigma_3\lambda_{yz}^2$ or $\sigma_{yy} = \sigma_1 l_2^2 + \sigma_2 m_2^2 + \sigma_3 n_2^2$
$= 15.832(-0.34202014)^2 + 7.4(0.0396262)^2 + 0.768(0)^2$
$=$ <u>1.864MPa</u>.

σ_{zz} $= \sigma_1\lambda_{zx}^2 + \sigma_2\lambda_{zy}^2 + \sigma_3\lambda_{zz}^2$ or $\sigma_{yy} = \sigma_1 l_3^2 + \sigma_2 m_3^2 + \sigma_3 n_3^2$
$= 15.832(0.2541705)^2 + 7.4(0.22885615)^2 + 0.768(0.93969262)^2$
$=$ <u>2.089 MPa</u>

Calculate τ_{xy}; τ_{yz} and τ_{zx} using the equations shown below.

REFERENCE

- Page 26, Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman

NOTE

The formulae described by Ryder and Jager in Chapter 3 in the Textbook on rock engineering for tabular hard rock mines can also be used.

τ_{xy} $= l_1 l_2 \sigma_1 + m_1 m_2 \sigma_2 + n_1 n_2 \sigma_3$
$= (0.93869262)(-0.34202014)(15.832) + (0.3420201)(0.03969262)(7.4) + (0)(0)(0.768)$
$=$ <u>- 4.982 MPa</u>

τ_{yz} $= l_2 l_3 \sigma_1 + m_2 m_3 \sigma_2 + n_2 n_3 \sigma_3$
$= (-0.34202014)(0.2541705)(15.832) + (0.0396262)(0.22885615)(7.4) + (0)(0.93969262)(0.768)$
$=$ <u>-1.314 MPa</u>

τ_{zx} $= l_3 l_1 \sigma_1 + m_3 m_1 \sigma_2 + n_3 n_1 \sigma_3$
$= (0.2541705)(0.93869262)(15.832) + (0.22885615)(0.3420201)(7.4) + (0.93969262)(0)(0.768)$
$=$ <u>4.357 MPa</u>

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REFERENCE

- Page 139 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 165 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 17 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman

LEARNING OUTCOME 2.7

Calculate the principal strains from strain components in two dimensions

Strain describes the deformation of a body as a result of stress. Normal strain is defined as ε(epsilon) and the rotational strain called the shear strain is represented by the γ(gamma). Normal strain is the result of applied normal stresses acting on a body resulting in positive strain (changes size of the body) when decreasing and negative strain result when increasing sample dimensions.

Shear strain resulting from an applied shear stress produces a distortion (changes shape of the body) which is defined as an angular change in a right angle of the sample. A positive shear strain (Figure 30) represents a decrease in the right angle and a negative shear strain represents an increase in the right angle. Shear strain can also be defined as one half of the change in the original right angle measured in radians.

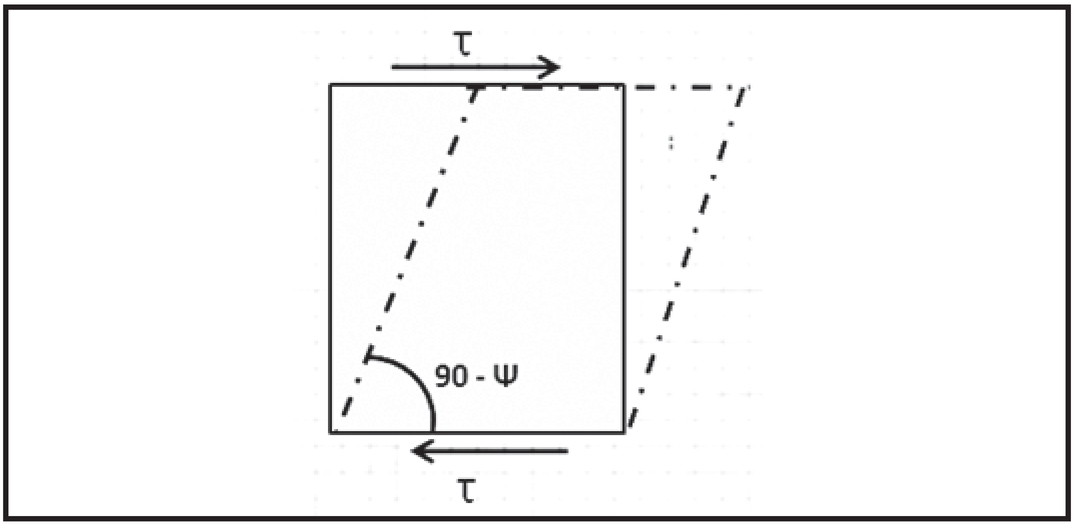

Figure 30: Positive shear strain

In rock engineering practice strain transformations are commonly used in performing in-situ stress measurements, where strains are measured with equiangular or rectangular strain rosettes. The measured strains are then converted to principal strains and then to principal stresses. With this method both the principal stress magnitudes and their orientations can ultimately be determined.

Simultaneous solution of the normal strain equations gives the shear strain component for the 0°-45°-90° rosette as

$$\Gamma_{090} = \varepsilon_{45} - \tfrac{1}{2}(\varepsilon_0 + \varepsilon_{90})$$

The normal and shear strain component for the 0°-60°-120° rosette as

$$\Gamma_{090} = (1/\sqrt{3}).(\varepsilon_{60} - \varepsilon_{120})$$
$$\varepsilon_{90} = (2/3).(-\tfrac{1}{2}\varepsilon_0 + \varepsilon_{60} + \varepsilon_{120})$$

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and principal strains and their orientations as

$$\varepsilon_{1,2} = \tfrac{1}{2}(\varepsilon_0 + \varepsilon_{90}) \pm \tfrac{1}{2}\sqrt{[(\varepsilon_0 - \varepsilon_{90})^2 + (2\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90})^2]}$$

$$\text{Tan}2\theta = (2\,\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90}) / (\varepsilon_0 - \varepsilon_{90})$$

for a 0°-45°-90° rosette and

$$\varepsilon_{1,2} = (\varepsilon_0+\varepsilon_{60}+\varepsilon_{120})/3 \pm (2/3)\sqrt{[\varepsilon_0{}^2+\varepsilon_{60}{}^2+\varepsilon_{120}{}^2-(\varepsilon_0\varepsilon_{60}+\varepsilon_{60}\varepsilon_{120}+\varepsilon_{120}\varepsilon_0)]}$$

$$\text{Tan}2\theta = (\sqrt{3})\,(\varepsilon_{60} - \varepsilon_{120}) / (2\varepsilon_0 - \varepsilon_{60} - \varepsilon_{120})$$

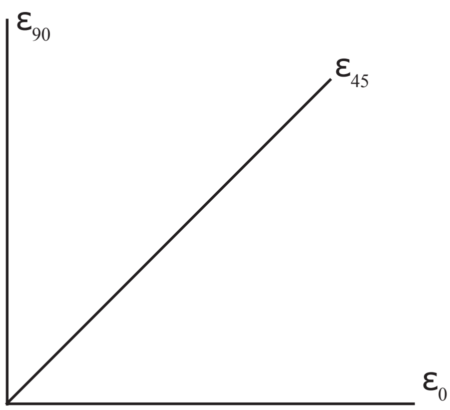
for a 0°-60°-120° rosette.

REFERENCE

- Page 138 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

EXAMPLE

A rectangular (0°-45°-90° degree) rosette gives the following strain readings: ε_0 = 3 x10-4 ε_{45} = 1.5 x 10-4 ε_{90} = 0. Calculate the magnitude and orientations of the principal strains on the surface.



NOTE

$\varepsilon_0 = \varepsilon_x$ and $\varepsilon_{90} = \varepsilon_y$

$$\begin{aligned}\varepsilon_1 &= \tfrac{1}{2}(\varepsilon_0 + \varepsilon_{90}) + \tfrac{1}{2}\sqrt{[(\varepsilon_0 - \varepsilon_{90})^2 + (2\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90})^2]}\\
&= \tfrac{1}{2}(3\text{ x}10^{-4} + 1.5\text{ x }10^{-4}) + \tfrac{1}{2}\sqrt{[(3\text{ x}10^{-4} - 0)^2 + (2(1.5\text{ x }10^{-4}) - 3\text{ x}10^{-4} - 0)^2]}\\
&= 0.00003\\
&= \underline{3.75\text{ x }10^{-4}}\end{aligned}$$

$$\begin{aligned}\varepsilon_2 &= \tfrac{1}{2}(\varepsilon_0 + \varepsilon_{90}) + \tfrac{1}{2}\sqrt{[(\varepsilon_0 - \varepsilon_{90})^2 + (2\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90})^2]}\\
&= \tfrac{1}{2}(3\text{ x}10^{-4} + 1.5\text{ x }10^{-4}) - \tfrac{1}{2}\sqrt{[(3\text{ x}10^{-4} - 0)2 + (2\text{x}1.5\text{ x }10^{-4} - 3\text{ x}10^{-4} - 0)2]}\\
&= \underline{0.000075}\end{aligned}$$

Calculate the principal strain directions .

$$\begin{aligned}\text{Tan}2\theta &= (2\,\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90}) / (\varepsilon_0 - \varepsilon_{90})\\
&= (2(1.5\text{ x }10^{-4}) - 1.5\text{ x }10^{-4} - 0) / (1.5\text{ x }10\text{ -4-0})\\
&= 1.5\text{ x }10^{-4}/1.5\text{ x }10^{-4}\\
&= 1\\
\text{Tan}2\theta &= 1\end{aligned}$$

CONNECTION 5

Refer to "LEARNING OUTCOME 2.3" on page 16 for a method to solve principal strain orientations

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Therefore

$2\theta = 45°$ or

$2\theta = 45° + 180°$

$\underline{= 225°}$

Therefore

$2\theta_1 = 45°$ and $2\theta_2 = 225°$ giving

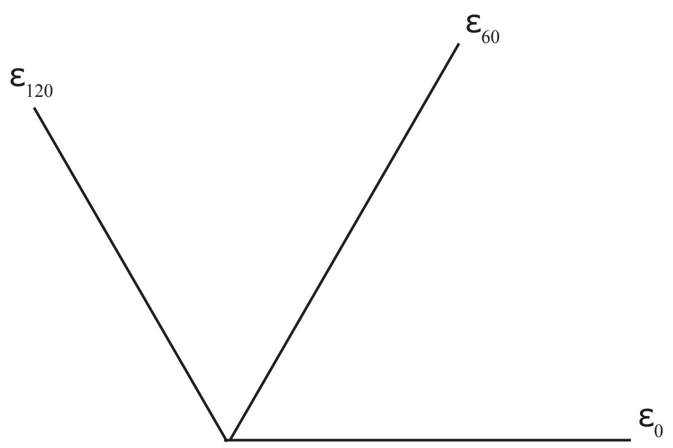
$\theta_1 = 22.5°$ and $\theta_2 = 112.5°$

EXAMPLE

A 0°-60°-120° degree strain rosette gives the following strain readings:

$\varepsilon_0 = 3.2 \times 10^{-4}$ $\varepsilon_{60} = 1.5 \times 10^{-4}$

$\varepsilon_{120} = 1.2 \times 10^{-4}$



REFERENCE

- Page 138 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

$$\varepsilon_1 = (\varepsilon_0+\varepsilon_{60}+\varepsilon_{120})/3+(2/3)\sqrt{[\varepsilon_0^{\,2}+\varepsilon_{60}^{\,2}+\varepsilon_{120}^{\,2}-(\varepsilon_0\varepsilon_{60}+\varepsilon_{60}\varepsilon_{120}+\varepsilon_{120}\varepsilon_0)]}$$

$$= [(3.2\text{x}10^{-4}+1.5\text{x}10^{-4}+1.2\text{x}10^{-4})/3]+(2/3)\sqrt{[(3.2\text{x}10^{-4})^2+(1.5\text{x}10^{-4})^2+(1.2 \times 10^{-4})^2-((3.2 \text{ x}10^{-4})(1.5 \times 10^{-4})+(1.5 \times 10^{-4})(1.2 \times 10^{-4}) + (1.2 \times 10^{-4})(3.2 \text{ x}10^{-4}))]}$$

$$= \underline{4.4583 \times 10}^{-4}$$

$$\varepsilon_2 = (\varepsilon_0+\varepsilon_{60}+\varepsilon_{120})/3+(2/3)\sqrt{[\varepsilon_0^{\,2}+\varepsilon_{60}^{\,2}+\varepsilon_{120}^{\,2}-(\varepsilon_0\varepsilon_{60}+\varepsilon_{60}\varepsilon_{120}+\varepsilon_{120}\varepsilon_0)]}$$

$$= [(3.2\text{x}10^{-4}+1.5\text{x}10^{-4}+1.2\text{x}10^{-4})/3]-(2/3)\sqrt{[(3.2\text{x}10^{-4})^2+(1.5\text{x}10^{-4})^2+(1.2 \times 10^{-4})^2-((3.2 \text{ x}10^{-4})(1.5 \times 10^{-4}) + (1.5 \times 10^{-4})(1.2 \times 10^{-4}) + (1.2 \times 10^{-4})(3.2 \text{ x}10^{-4}))]}$$

$$= \underline{-5.205 \times 10}^{-5}$$

CONNECTION 5

Refer to "LEARNING OUTCOME 2.3" on page 16 for method to solve for principal strain orientations

Calculate the principal strain directions.

$$\text{Tan } 2\theta = (\sqrt{3})(\varepsilon_{60}-\varepsilon_{120})/\ (2\varepsilon_0-\varepsilon_{60}-\varepsilon_{120})$$

$$= (\sqrt{3})(1.5\text{x}10^{-4}-1.2\text{x}10^{-4})/2(3.2\text{x}10^{-4})-1.5\text{x}10^{-4}-1.2\text{x}10^{-4})$$

$$= 5.205\text{x}10^{-5}/3.7\text{x}10^{-4}$$

$$= \underline{0.1407}$$

$$\text{Tan}2\theta_1 = -5.205 \times 10^{-5}/3.7 \text{ x}10^{-4}$$

$$= \underline{0.1407}$$

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Therefore

$2\theta = 8°$ or

$2\theta = 8°+180° = 188°$

Therefore

$2\theta_1 = 8°$ and $2\theta_2 = 188°$

giving

$\theta 1 = 8°$ and $\theta 2 = 188°$

Therefore ε_1 direction $=4°$

Therefore ε_2 direction $= 94°$

REFERENCE

- Page 12 - Rock mechanics in mining practice, 1983, S Budavari et al.
- Page 47 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 133 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 163 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden

LEARNING OUTCOME 2.8

CALCULATE STRAIN COMPONENTS IN ANY ORIENTATION GIVEN EITHER THE PRINCIPAL STRAINS OR STRAIN COMPONENTS IN THE X-Y DIRECTIONS

The normal and shear stain components can be calculated from the principal strains. In such a case the rotational angle θ from the principal stain direction to the ε_x position must be known.

EXAMPLE

For principal strains of $1.5x10^{-3}$ and $-2.2x10^{-3}$, and an angle of 25.791° between the normal and principal strain axes, calculate the normal strains.

$$\varepsilon_x = \varepsilon_1\cos^2\theta + \varepsilon_2\sin^2\theta$$
$$= (1.5X10^{-3})\cos^2(25.791°) + (-2.20X10^{-3})\sin^2(25.791°)$$
$$= 0.00080$$
$$= \underline{8x10^{-4}}$$

$$\varepsilon_y = \varepsilon_1\sin^2\theta + \varepsilon_2\cos^2\theta$$
$$= (1.5X10^{-3})\sin^2(25.791°)+(-2.20X10^{-3})\cos^2(25.791°)$$
$$= -0.0015$$
$$= \underline{-15x10^{-4}}$$

Strain can also be calculated at any orientation relative to the XY axis by means of rotating it to an X' and Y' axis system. To do this the angle θ between the x' and x axes must be known.

NOTE

The equations shown below are used to do these strain transformations.

$$\varepsilon_{x'} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) + \tfrac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \gamma_{xy}\sin 2\theta$$
$$\varepsilon_{y'} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) + \tfrac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta - \gamma_{xy}\sin 2\theta$$
$$\gamma_{x'y'} = \tfrac{1}{2}(\varepsilon_y - \varepsilon_x)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

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EXAMPLE

If ε_x = 0.00015, ε_y = 0.00025 and γ_{xy} = 0.00012, calculate the normal and shear strain on a plane orientated at 42° to the XY axis.

$$\begin{aligned}
\varepsilon_{x'} &= \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) + \tfrac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \gamma_{xy}\sin 2\theta \\
&= \tfrac{1}{2}(0.00015 + 0.00025) + \tfrac{1}{2}(0.00015 - 0.00025) \\
&\quad \cos 2(42°) + (0.00012)\sin 2(42°) \\
&= 0.000314 \\
&= \underline{3.14 \times 10^{-4}}
\end{aligned}$$

$$\begin{aligned}
\varepsilon_{y'} &= \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) + \tfrac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta - \gamma_{xy}\sin 2\theta \\
&= \tfrac{1}{2}(0.00015 + 0.00025) - \tfrac{1}{2}(0.00015 + 0.00025) \\
&\quad \cos 2(42°) - \gamma_{xy}\sin 2(42°) \\
&= 0.0000859 \\
&= \underline{8.59\ X10^{-5}}
\end{aligned}$$

$$\begin{aligned}
\gamma_{x'y'} &= \tfrac{1}{2}(\varepsilon_y - \varepsilon_x)\sin 2\theta + \gamma_{xy}\cos 2\theta \\
&= \tfrac{1}{2}(0.00015 - 0.00025)\sin 2(42°) + \gamma xy \cos 2(42°) \\
&= 0.0000623 \\
&= 6.23\ x10\text{-}5
\end{aligned}$$

REFERENCE

- • Page 14 - Rock mechanics in mining practice, 1983, S Budavari et al.

LEARNING OUTCOME 2.9

DETERMINE THE PRINCIPAL STRESSES MAKING USE OF THE MOHR-CIRCLE REPRESENTATION OF STRESSES IN TWO DIMENSIONS GIVEN THE STRESS COMPONENTS ON A PLANE

INTERESTING INFO

The Mohr circle was introduced by Otto Mohr in 1882 and is still extensively used to graphically visualize relationships between normal and shear stress. It is also used to determine the principal stresses (σ_1 and σ_2) magnitudes and their orientations from normal and shear stress components (σ_1, σ_2 and τ_{xy}). As the stress element is rotated away from the principal directions, the normal and shear stress components will always lie on the Mohr circle. In Figure 31 an example of a Mohr circle representing a specific state of stress is shown

.

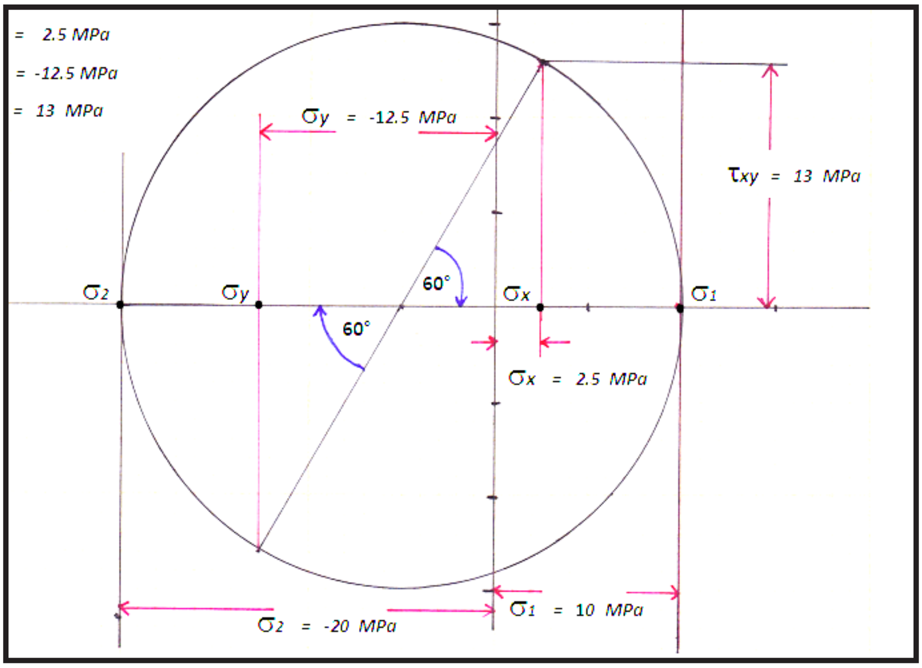

Figure 31: Mohr circle representation of a state of stress
In the Mohr circle, points on the circle represent a combination of normal and shear stresses at various orientations.

IMPORTANT

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The horizontal axis is the normal stress (σ_n) axis and the vertical axis is the shear stress (τ)axis.

The centre of the circle C is determined from (σ_x +σ_y)/2) (Figure 31) while the radius of the Mohr circle R can be determined from the formulae shown below.

$$R = \sqrt{\frac{(\sigma_x - \sigma_y)^2}{2} + \tau_{xy}^2} \text{ or } R = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

The value of the maximum and minimum principal stresses can be found on the Mohr circle where the circle crosses the horizontal axis (normal stress axis).

The maximum principal stress (σ_1)is the maximum of the two values value where the circle intersects the normal stress axis and can be determined from σ_1 = C + R (Figure 31).

The minor principal stress (σ_2) is the minimum of the 2 values where the circle intersects the normal stress axis and can be determined from σ_2 = C - R (Figure 31).

The first part of the equation used to calculate σ_1 and σ_2 (see below) is equal to C (centre point of the Mohr circle) and the last part is equal to R (radius in the Mohr circle).

$$\sigma_1 = ½ (\sigma_x + \sigma_y) + ½ \sqrt{[(\sigma_x - \sigma_y)^2 + 4\,\tau_{xy}{}^2]}$$

$$\sigma_2 = ½ (\sigma_x + \sigma_y) - ½ \sqrt{[(\sigma_x - \sigma_y)2 + 4\,\tau_{xy}{}^2]}$$

The orientations of the principal stresses can be measured from the X axis in a counter clockwise direction toward σ_1 and σ_2 and positions in the Mohr circle and are called θ_1 and θ_2.

NOTE

Check the correctness of the magnitudes of principal stresses determined from the Mohr circle by using the following equation $\sigma_x + \sigma_y = \sigma_1 + \sigma_2$.

When the magnitudes of the principal stresses are known the magnitude of σ_x, σ_y and τ_{xy} at any orientation from the principal stresses can be graphically determined.

In the Mohr circle, the direction of rotation around the centre of the circle is clockwise by 2θ. In a body diagram, the rotation is counter clockwise and is plotted as θ only.

In the Mohr circle the maximum shear stress (τ_{max}) as well as the normal stress (σ_n) magnitude at maximum shear can be determined. The orientation of maximum shear can also be read off from the Mohr circle. The τmax = R and σ_n = C and the orientation angle is the angle measured from the normal stress axis to the τ_{max} line in the Mohrcircle, measured in a clockwise direction.

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EXAMPLE

If σ_x = 48 MPa, σ_y = 17 MPa and τ_{xy} = 14 MPa, draw the body diagram representing this stress state.

ANIMATION

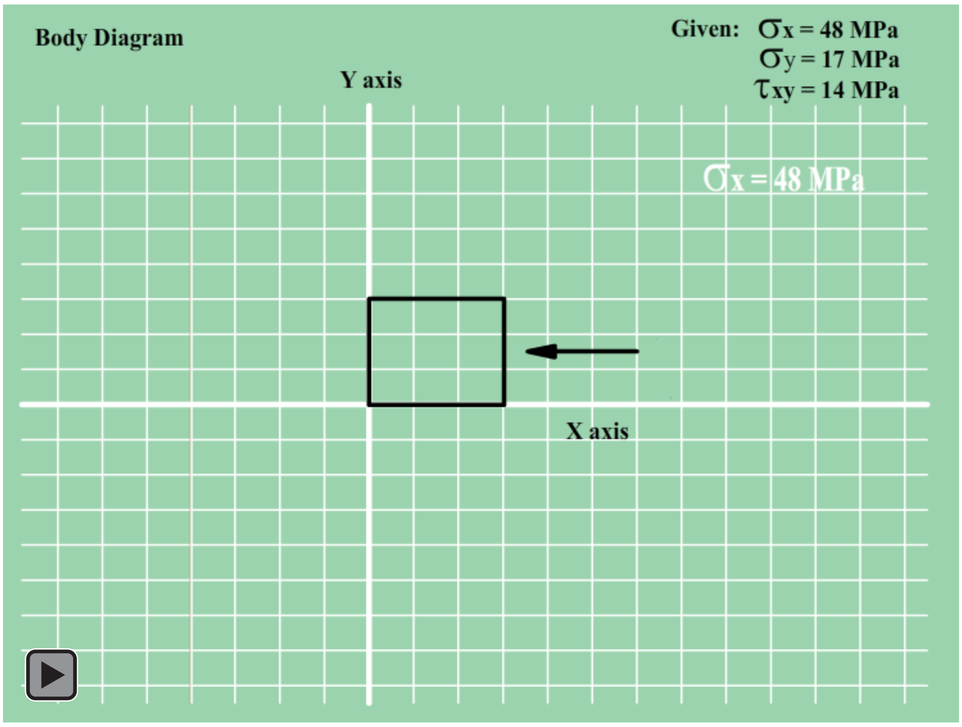

Figure 32: Animated Body diagram

IMPORTANT

The direction of the shear stress is pointing downwards. This will cause a clockwise rotation in the body diagram. Take note when drawing τ_{xy} that it is perpendicular to the X-axis (first subscript) and parallel to the Y-axis (second subscript). See body diagram below.

NOTE

A positive shear stress causes a clockwise rotation in body diagram. A negative shear stress causes a counter-clockwise rotation in body diagram (Right-hand side perpendicular to the X-axis.) See body diagram below.In a case as shown in Figure 31, the shear stress must be plotted above the normal stress axis (σ_n) in the Mohr circle.In the body diagram the angles are measured counter clockwise. In the Mohr circle the angles are measured clockwise with 2Θ

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EXAMPLE

Draw a Mohr circle from the information given below: σ_x = 48 MPa, σ_y = 17 MPa and τ_{xy} = 14 MPa.

The centre of the circle C is determined from

$(\sigma_x + \sigma_y)/2) = (48MPa + 17MPa)/2 = 30.5$ MPa and while

$\sigma_1 = C + R$ and

$\sigma_2 = C - R$

$\sigma_1 = 32.5 + 20.887 = 53.387$ MPa and

$\sigma_2 = 32.5 - 20.887 = 11.613$ MPa.

Principal stress magnitudes measured on the Mohr circle can be checked against the normal stresses as shown below.

Check:

$$\sigma_1+\sigma_2=\sigma_x+\sigma_y$$
$$53.387+11.613=48+17 \text{ MPa}$$
$$65\text{MPa}=65\text{MPa}$$

ANIMATION

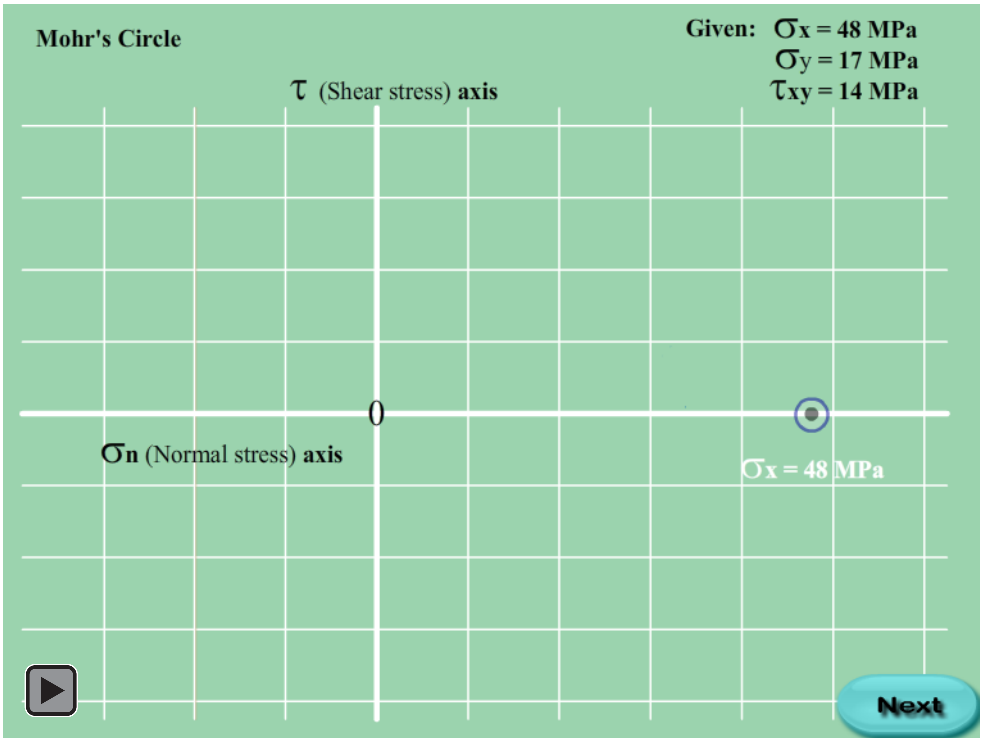

Figure 33: Animated Mohr circle

In the example τ_{max} can be read off as 20.887 MPa or calculated as τ_{max} = R= 20.887MPa as calculated in the animation of the Mohrcircle above, using the formula shown below.

$$R = \sqrt{\frac{(\sigma_x - \sigma_y)^2}{2} + \tau_{xy}^2}$$

NOTE

When you attempt the answers in the animated self test it is advised to physically plot the Mohr circle on graph paper and also make use of basic calculations on a scientific calculator and check these with the test results given.

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ANIMATION

Sample Video, on how to plot X and Y

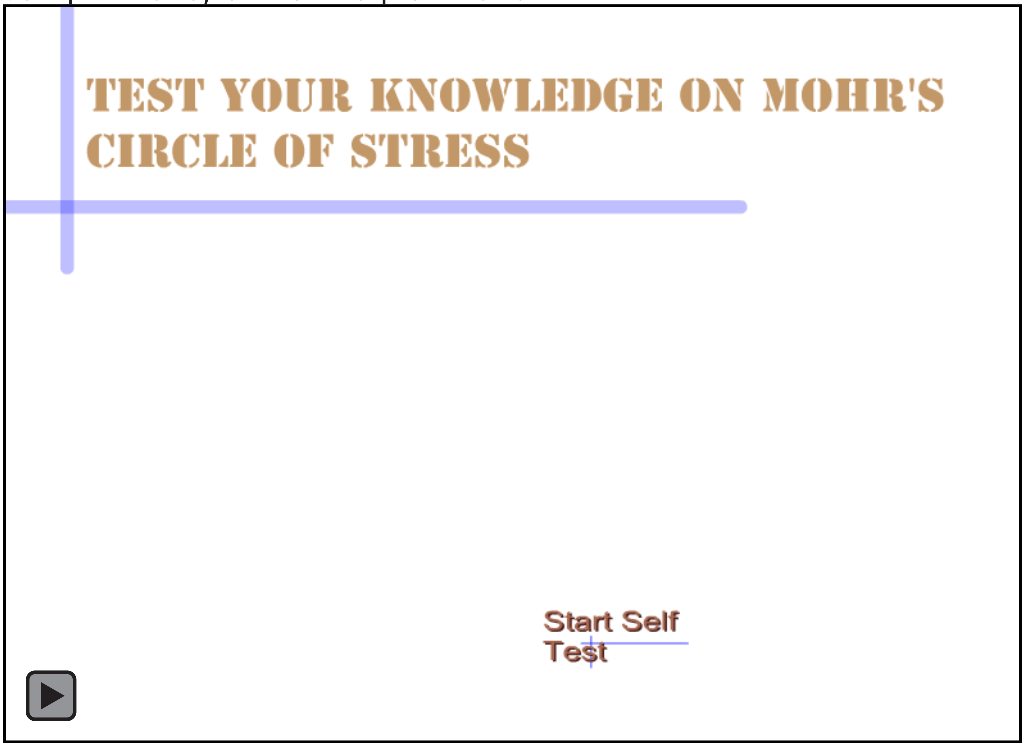

Figure 34: Animated self test

EXAMPLE

Using the Mohr circle, find the magnitude and directions of the principal stresses for the state of stress specified: σ_x = 2.5MPa, σ_y = -12.5 MPa and τ_{xy} = 13 MPa

Draw a body diagram representing the state of stress (Figure 35).

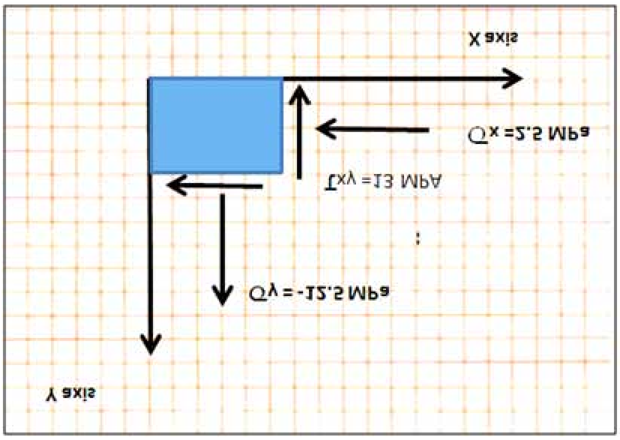

Figure 35: Body diagram

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NOTE

The shear stress along the positive side of the body diagram is pointing downwards indicating a positive shear stress. In such a case the shear stress must be plotted above the normal stress line in the Mohr circle (Figure 36).

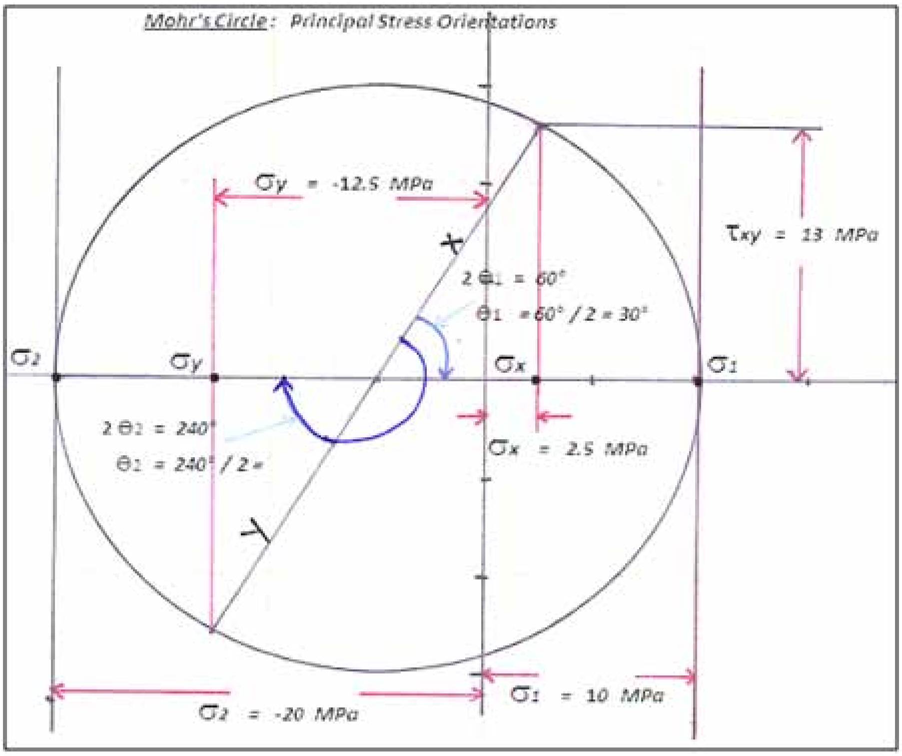

Figure 36: Magnitude and orientation of the principal stresses

From the Mohr circle plotted above that was graphically constructed to scale, the results are as follows:

The magnitudes of the major and minor principal stresses are:
σ_1=10 MPa and σ_2= -20 MPa

The orientation of the major principal stressmeasured from the x axis is:
2 x θ_1= 60°, therefore θ_1= 30°

The orientation of the minor principal stress measured from the x axis is:
2 x θ_2= 240°, therefore θ_2= 120°

NOTE

A 2nd method to determine the magnitudes of the principal stresses is to calculate it using the C (centre of Mohr circle) and R (radius of the Mohr circle).

C = $(\sigma_x + \sigma_y)/2)$ = (2.5 +-12.5)/2 = -5MPa.

R can be determined from the equation

$$R = \sqrt{\frac{(\sigma_x - \sigma_y)^2}{2} + \tau_{xy}^2}$$

R = 15 MPa.

σ_1 = C+R = -5 + 15 MPa =10 MPa and
σ_2 = C − R = -5 − 15 MPa = -20 MPa.

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NOTE

A 3rd method is to calculate it as described under "LEARNING OUTCOME 2.3" on page 16

$$\sigma_1 = ½ (\sigma_x + \sigma_y) + ½ \sqrt{[(\sigma_x - \sigma_y)^2 + 4 \tau_{xy}{}^2]}$$
$$\sigma_2 = ½ (\sigma_x + \sigma_y) - ½ \sqrt{[(\sigma_x - \sigma_y)2 + 4 \tau_{xy}{}^2]}$$

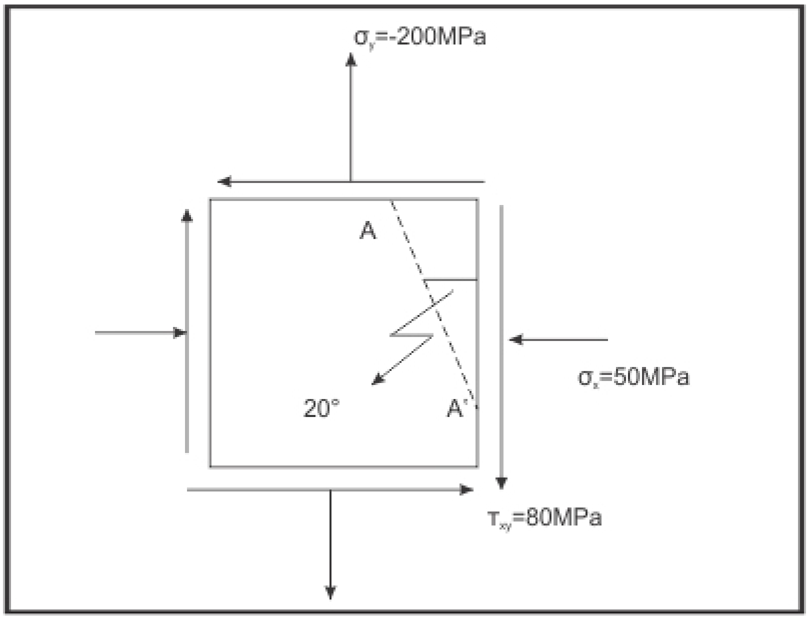
$$\sigma_1 = ½ (\sigma_x + \sigma_y) + ½ \sqrt{[(\sigma_x - \sigma_y)^2 + 4 \tau_{xy}{}^2]}$$
$$= ½ (2.5 + -12.5)+ ½ \sqrt{[(2.5 - -12.5)^2 + 4 \times 132]}$$
$$= \underline{10\ MPa}$$

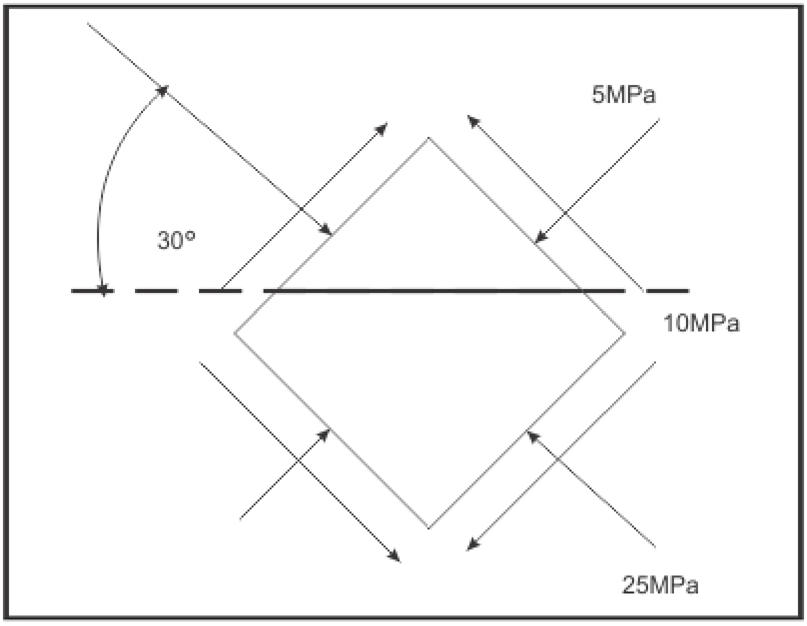
$$\sigma_2 = ½ (\sigma_x + \sigma_y) - ½ \sqrt{[(\sigma_x - \sigma_y)2 + 4 \tau_{xy}{}^2]}$$
$$= ½ (2.5 + -12.5) -½ \sqrt{[(2.5 - -12.5)^2 + 4 \times 132]}$$
$$= \underline{-20\ MPa}$$

PRACTICAL APLICATION

Typical test/ examination questions (Professor Budavari).

For the stress state shown below determine the maximum normal and shear stresses and the orientations of the planes on which these are acting





Given the stress state shown below determine the magnitude and orientation of the principal stresses.

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Answer sheet

σ_{max} = 73.41 MPa ;

orientation of plane onto which $\sigma_{max} = \sigma_1$ works = -73.69° from x;

σ_{max} = 148.41 MPa;

orientation of plane on which maximum shear stress works= 28.69° from x.

Place the X-axis along 25 MPa stress component

θ_1= 29.142 MPa;

θ_2= 0.858 MPa;

θ_1= 22.50;

θ_2= 112.5o from x.

REFERENCE

- Page 9 - Rock mechanics in mining practice, 1983, S Budavari et al.
- Page 118, 124 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 23,35,94 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 172 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 94 - Underground excavations in rock, 1980, E Hoek and ET Brown

LEARNING OUTCOME 2.10

Determine the stress components on a plane making use of the Mohr-circle representation of stresses in two dimensions given the principal stresses

IMPORTANT

In the Mohr circle, points on the circle represent a combination of normal and shear stresses at various orientations. When the magnitudes of the principal stresses are known it is easy to determine the magnitude of σ_x, σ_y and τ_{xy} at any orientation to the principal stresses.

EXAMPLE

Calculate the normal and shear stresses acting on a plane rotated through 25° measured from the principal stress axes (which coincide with the standard XY axes), when the major principal stress is 65 MPa and the minor principal stress 5 MPa.

Given: σ_1 = 65MPa and σ_2= 5 MPa and θ= 25°.
First draw the body diagram representing this state of stress (Figure 37) and then the Mohr circle (Figure 38) keeping in mind that the angle in the Mohr circle is 2 x 25° = 50°.

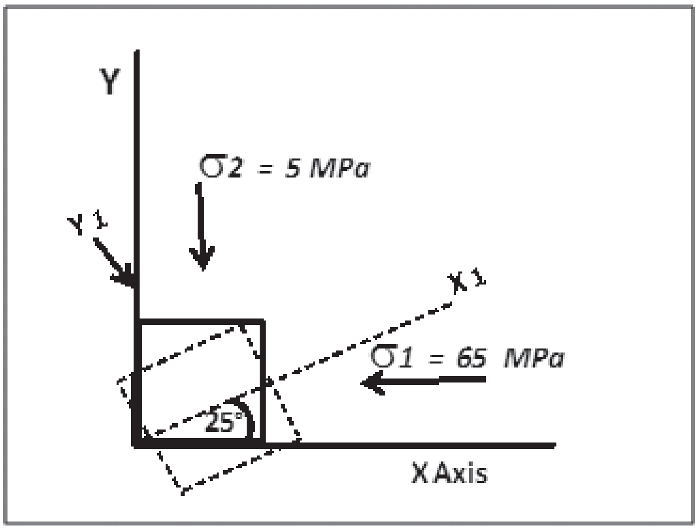

Figure 37: The body diagram

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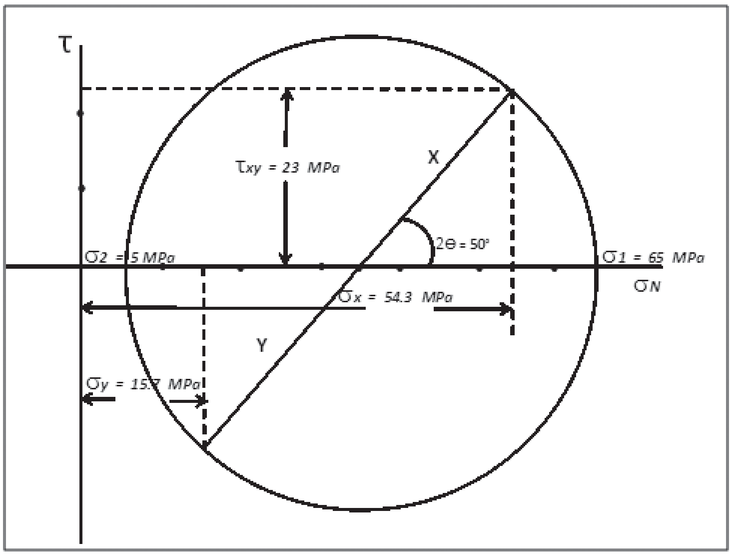
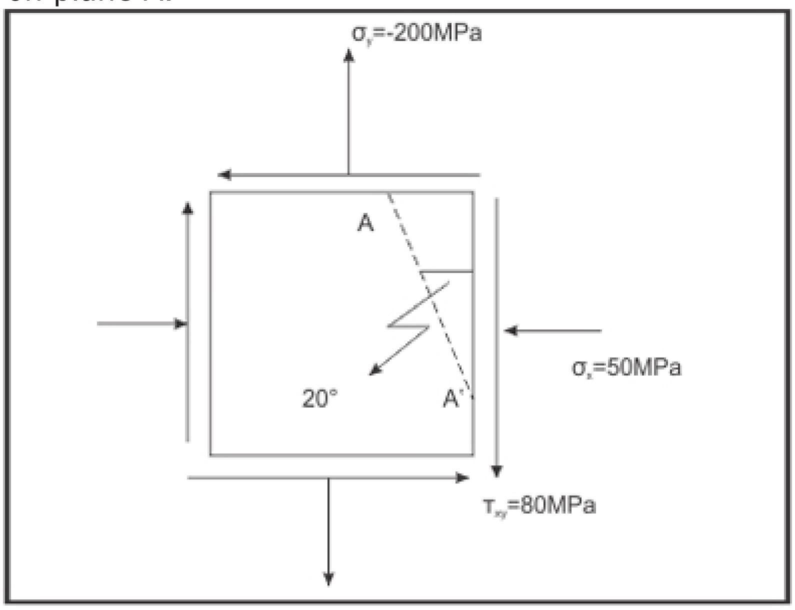


Figure 38: Mohr circle

PRACTICAL APLICATION

Typical test/ examination questions (Professor Budavari).

For the stress state shown below, determine the normal and shear stresses acting on plane A.



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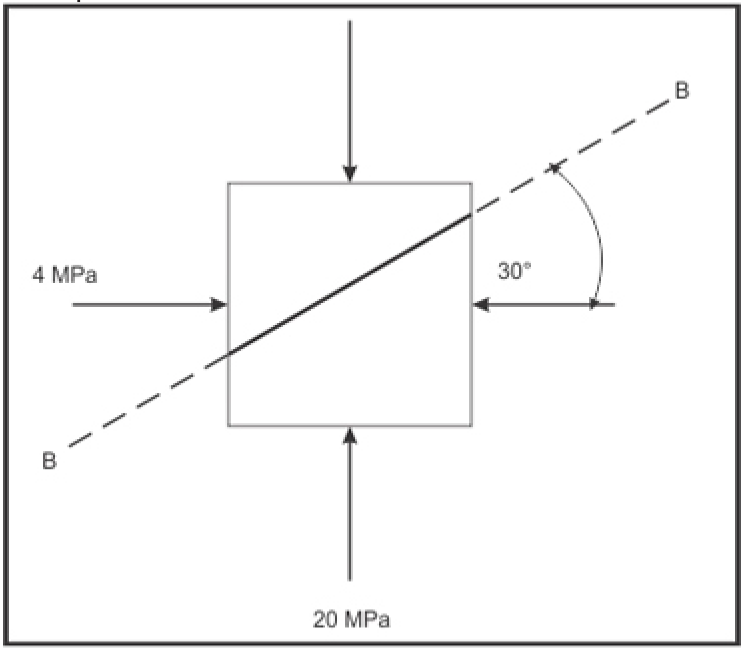
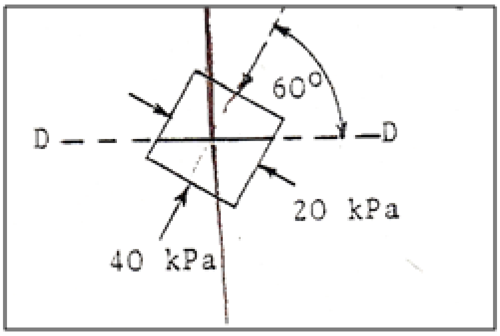
Given the following stress state, compute the magnitude of the stress components on plane B-B.



Given the stress state shown below, find the magnitude of the stress components on plane D-D graphically.



Answer sheet

$\sigma_n = 72.18$ MPa ; $\tau_{nm} = -19.06$MPa
$\sigma_n = 16.000$ MPa ; $\tau_{nm} = -6.928$ MPa
$\sigma_n = 35$ kPa ; $\tau_{nm} = -8.75$ kPa

REFERENCE

- Page 118, 125 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 35 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 172 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 15 - Rock mechanics in mining practice, 1983, S Budavari et al.

LEARNING OUTCOME 2.11

Describe, explain and illustrate the normal and shear stresses induced in the following members under their own weight :A simply (freely) supported beam, cantilever beam

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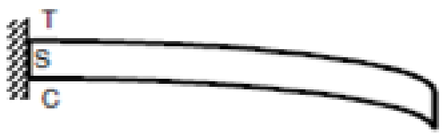
AND BUILT-IN BEAM. NOTE THE POSITIONS OF MAXIMUM TENSION, MAXIMUM COMPRESSION AND MAXIMUM SHEAR IN EACH CASE.

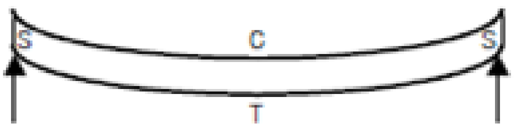
In normal underground mining situations, especially mining in sedimentary (layered) strata, the bedded or layered host rock acts as a beam. The beams can act as:

- a 'built-in' beam: The sides of the beam are 'built' into the surrounding material, such as a hangingwall layer that protrudes into the tunnel sidewalls,

- a 'cantilever' beam: The beam is only supported on one side and allows the beam to 'cantilever' towards the unsupported side, such as a brow that is not supported,



- a 'freely supported' or 'simply supported' beam: The beam is supported at two points along the beam only, such as the beam between two support units in the underground environment.



2.11.1 A built-in beam

The stresses and deflection of the beam can be calculated:

- The magnitude of the maximum tensile stress is calculated using

$$\sigma_T = -\frac{1\gamma L^2}{2t} + \sigma_n$$

Note the minus sign

- The magnitude of the maximum compressive stress is calculated using

$$\sigma_C = \frac{1\gamma L^2}{2t} + \sigma_n$$

- The magnitude of the maximum deflection is calculated using

$$\delta = \frac{1}{32}\frac{\gamma L^4}{Et^2}$$

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- The horizontal stress required for buckling can be calculated from

$$\sigma_h = \frac{1}{3} E \frac{t^2 \pi^2}{L^2}$$

Where γ= unit weight of beam (γ=ρg), t = thickness of beam, L = length of unsupported span and E = lateral Young's modulus.

2.11.2 A cantilever beam

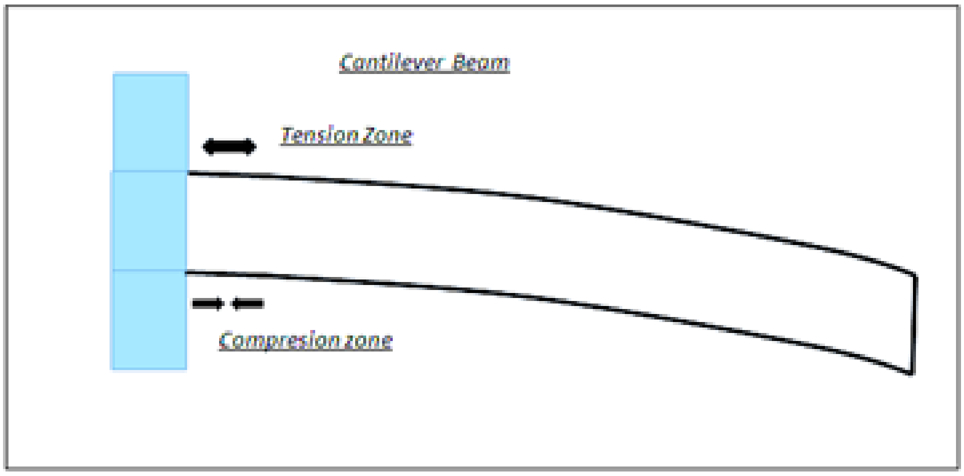

Figure 40 Cantilever beam

The stresses and deflection of the beam can be calculated:

- The magnitude of the maximum tensile stress is calculated using

$$\sigma_T = -\frac{3}{1} \frac{\gamma L^2}{t}$$

- The magnitude of the maximum compressive stress is calculated using

$$\sigma_C = \frac{3}{1} \frac{\gamma L^2}{t}$$

- The magnitude of the maximum deflection is calculated using

$$\delta = \frac{3}{2} \frac{\gamma L^4}{E t^2}$$

Where = unit weight of beam (=ρg), t = thickness of beam, L = length of unsupported span and E = lateral Young's modulus.

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2.11.3 A freely or simply supported beam

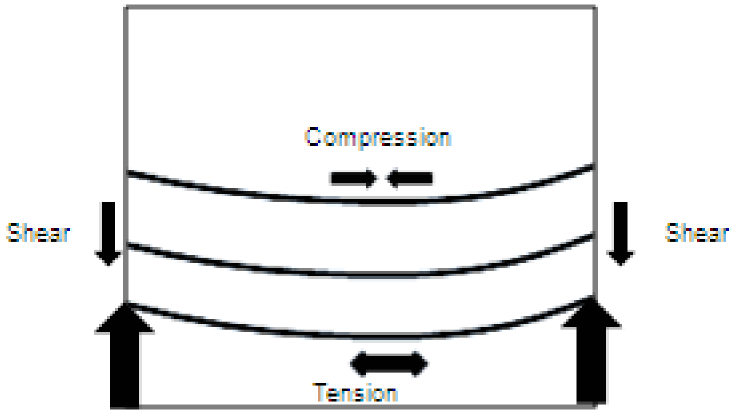

Figure 41 Simply supported beam

The stresses and deflection of the beam can be calculated:

- The magnitude of the maximum tensile stress is calculated using

$$\sigma_T = -\frac{3}{4}\frac{\gamma L^2}{t} + \sigma_h$$

- The magnitude of the maximum compressive stress is calculated using

$$\sigma_C = \frac{3}{4}\frac{\gamma L^2}{t} + \sigma_h$$

- The magnitude of the maximum deflection is calculated using

$$\delta = \frac{5}{32}\frac{\gamma L^4}{E t^2}$$

- The horizontal stress required for buckling can be calculated from

$$\sigma_h = \frac{1}{2} E \frac{t^2 \pi^2}{L^2}$$

Where = unit weight of beam (=ρg), t = thickness of beam, L = length of unsupported span and E = lateral Young's modulus.

CONNECTION 7

A calculator for beam calculations can be found on the webpage shown to the right.

REFERENCE

- Page 174 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 162 - "Fundamentals of Rock Mechanics, J.C. Jaeger and N.G.W. Cook"
- Page 5,174 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 21 - Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page

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CHAPTER 3

3.CONSTITUTIVE RELATIONSHIPS

INTRODUCTION

With understanding of how stresses and strains can be calculated (Chapter 2), the following questions arise:

- How do rock types strain?
- Do all rock types strain in a similar manner?
- Are there portions of the strains that appear similar?
- Etc.

Knowledge of constitutive relationships, i.e. how rocks strain when stressed, is critical in the evaluation of the behaviour of rocks. Even though this chapter will only evaluate a few of the possible types of strain behaviour, it is essential that these stress-strain relationships are understood well enough to allow the use of the concepts in future rock engineering work.

LEARNING OUTCOMES

The candidate must be able to demonstrate knowledge and understanding of the above subject area by being able to:

3.1 Identify, describe and explain the following constitutive relationships:

- Elastic
- Plastic
- Elasto-plastic
- Strain softening
- Strain hardening

3.2 Interpret these relationships citing material examples
3.3 Calculate strain components from stress components for elastic materials intwo dimensions using Hooke's law
3.4 Calculate stress components from strain components for elastic materials in two dimensions using Hooke's law
3.5 Calculate strain components from stress components for elastic materials in three dimensions using Hooke's law
3.6 Calculate stress components from strain components for elastic materials in three dimensions using Hooke's law
3.7 Describe and explain the concept of plane strain
3.8 Calculate the elastic strain energy density in elastic materials
3.9 Describe, explain and define the following terms:

- Dilation angle
- Associated flow
- Non-associated flow

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3.10 Describe, explain and indicate on a stress-strain graph for rock material the onset of inelastic behaviour, dilation and strain softening
3.11 Identify, describe and explain time-dependent behaviour

- Interpret this behaviour citing material examples.

- *Page 114, A textbook on rock mechanics for tabular hard rock mines, 2002,* JA Ryder and AJ Jager

LEARNING OUTCOME 3.1

IDENTIFY, DESCRIBE AND EXPLAIN THE FOLLOWING CONSTITUTIVE RELATIONSHIPS: ELASTIC, PLASTIC, ELASTO-PLASTIC, STRAIN HARDENING AND STRAIN SOFTENING

Material under stress will deform. The deformation is governed by the constitutive behaviour of the material. This is especially evident in sample rock testing, as rock types such as quartzite and shale behave totally different when stressed.

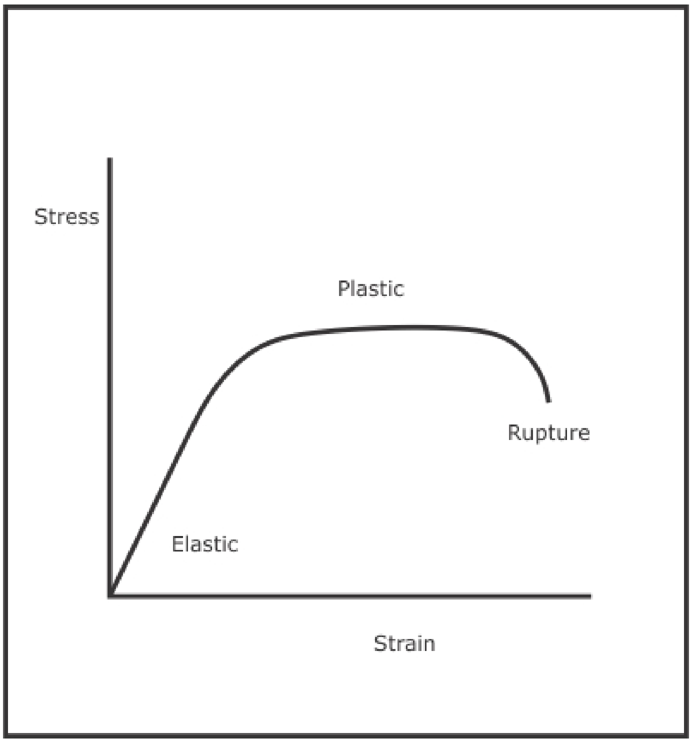

Figure 1: Generalised stress-strain curve for rocks

The following describe the behaviour of rocks during deformation:

- *Ductile behaviour : Rocks experience large amounts of plastic deformation before rupturing;*
- *Brittle behaviour: Rocks that exhibit elastic behaviour followed by rupture (Figure 2);*
- *Rupture is the loss of cohesion in the material and occurs prior to significant amounts of plastic deformation (Figure 3);*
- *Elastic limit : Ductile rocks deform elastically to a yield point (stress value of which is the yield strength), beyond this point, plastic deformation occurs with increasing stress;*
- *Rupture point: (rupture strength) Brittle rocks experience elastic deformation until a rupture point is reached, where the rock deforms by brittle rupture;*

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- *Failure point is reached when a brittle rock loses all resistance to stress and crumbles, or experiences increased deformation;*
- *Failure is difficult to discern in plastic deformation;*
- *Ultimate strength: Maximum stress that a rock can support before failure.*

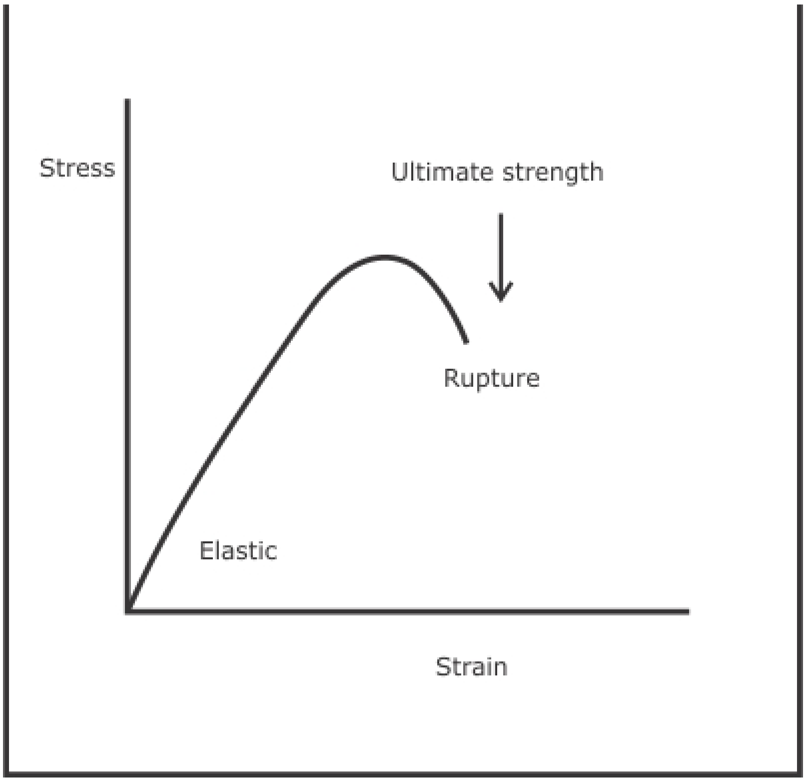

Figure 2: Brittle rock deformation

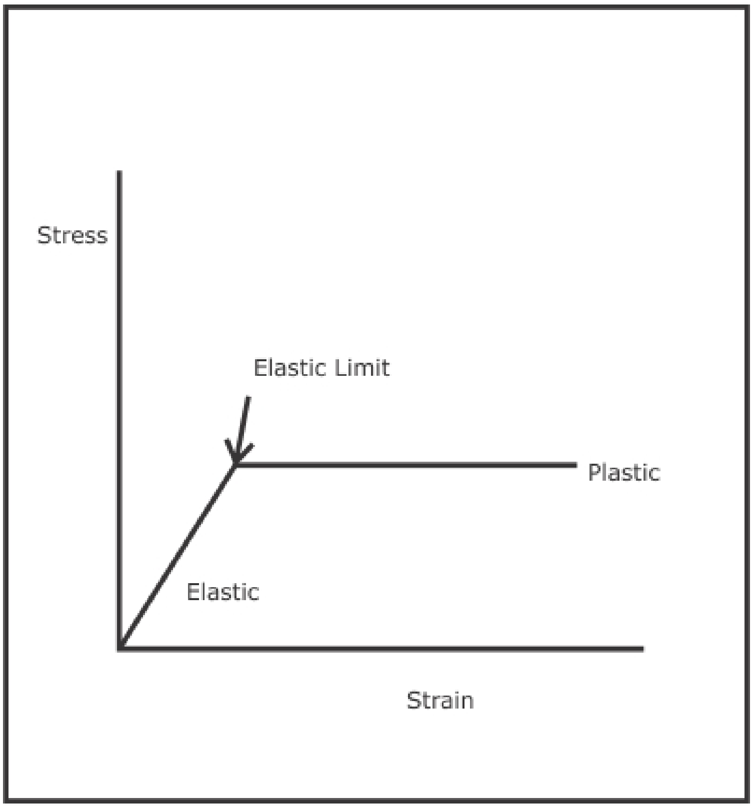

Figure 3: Ductile rock deformation

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Illustrated below, in Figure 4 are examples of idealised constitutive material behaviour

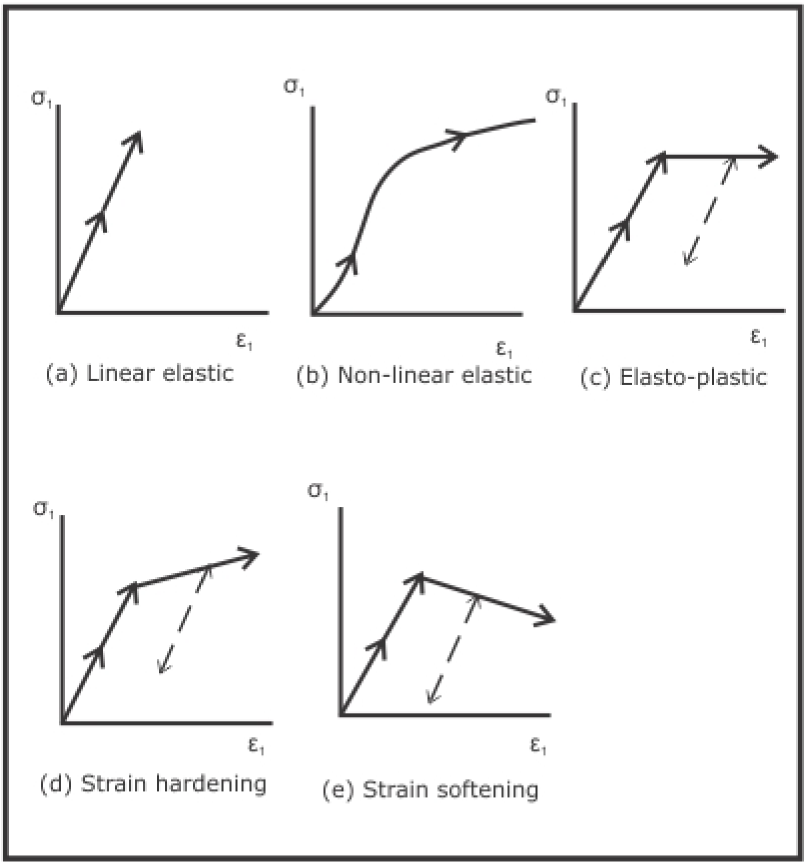

Figure 4: Idealised constitutive behaviour (Ryder and Jager)

NOTE

There are three types of deformation, namely elastic, plastic and rupture deformation. The visible effects of strain in rocks are usually of plastic or rupture variety as elastic strain produces little long term features.

Elastic deformation occurs when a body is deformed in response to a stress, but returns to its original shape when stress is removed. Stress is totally reversible or recoverable. Viscoelastic (anelastic) deformation is totally recoverablebut it is not instantaneous recovery. Recovery is time dependent and is described in terms of strain rate. Most rocks have elastic and anelastic properties at small stress magnitudes.

DEFINITION

In the elastic analysis it is assumed that elastic strain is a function of stress only. This is not strictly true since there is time dependence to the elasticity. The general term used for this time dependence is 'anelasticity'. Strain rate is the change in strain with respect to time and is denoted: $\varepsilon = d\varepsilon/dt$.

IMPORTANT

Elasticity implies that the strain (deformation) will recover if the stress is removed. In the case of a linear elastic material the increase in strain is directly proportional to the increase in stress – see linear line segment from the origin to point A in Figure 6. Most rock types exhibits linear elastic behaviour during the first part of the load deformation graph.

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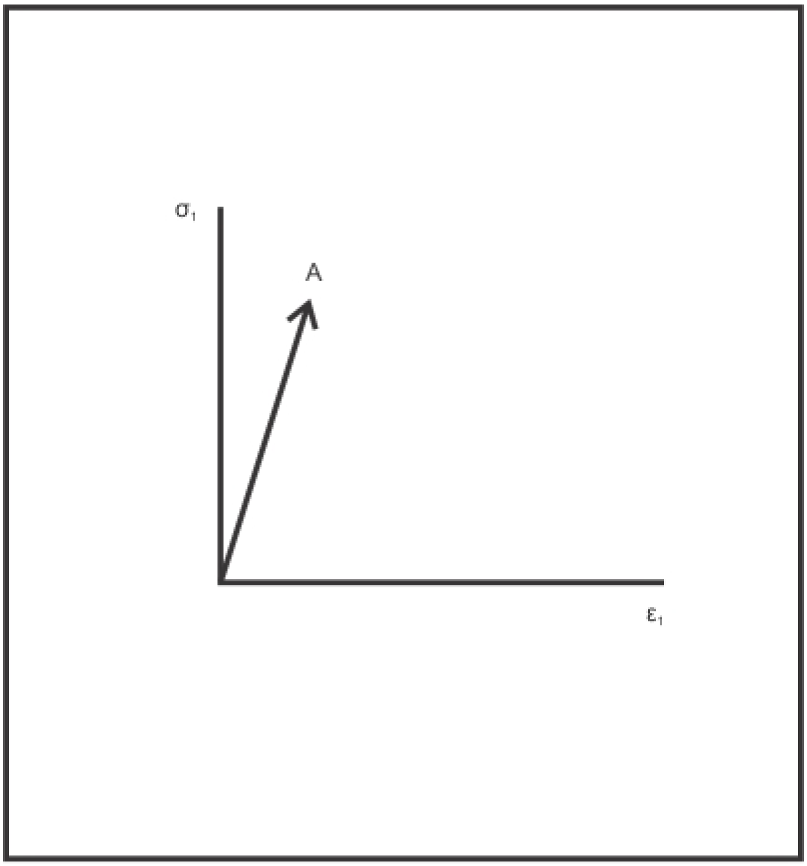

Figure 5: Linear elastic behaviour

When stress is applied, strain is instantaneous; i.e. not time dependent. Furthermore, instantaneous recovery ensues upon removal of stress. Some rocks at shallow depths and for short periods of time, approach ideal elastic behaviour during small magnitudes of deformation. E= Young's Modulus of Elasticity and is the constant that describes the slope of the line in the stress vs strain relationship (Figure 6) and is a measure of resistance to elastic distortion. As E increases, the slope becomes steeper, i.e. the rock stiffness increases.

Elastic behaviour is also known as Hookean behaviour in which stress is proportional to strain.

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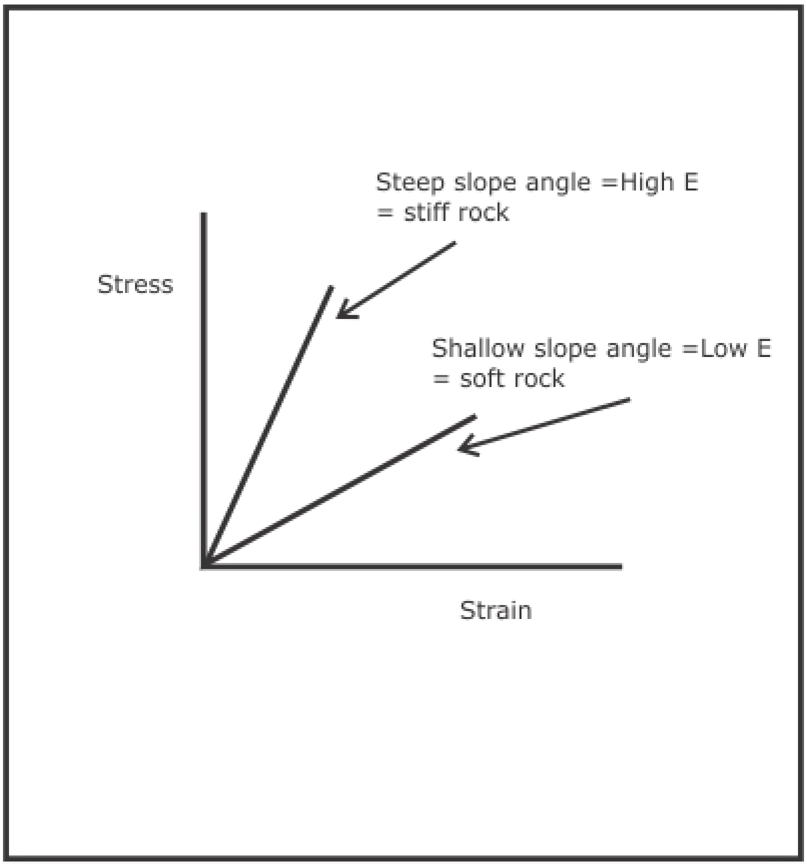

Figure 6: Rock stiffness

Elastic Behaviour may also be described for shear strain and volume change:

- Shear strain ($\gamma=\tau/G$) where G is the modulus of rigidity (or resistance to change in shape of stressed material);
- Volume change (dilation) where $\sigma = K(\frac{v - v_Q}{v_Q})$ where K is the bulk modulus (or the ratio of the pressure change to the resultant dilation $K(\frac{\Delta_p}{\Delta_v})$ and is a measure of the material incompressibility or of resistance to change in shape);
- Poisson's Ratio where $v=\varepsilon 1/\varepsilon 3$ (or the ratio of the strain normal to compressive stress to the strain parallel to compressive stress). Poisson's ratio is another means to describe the relationship between volume change and stress where v may range from 0 (compressible) to 0.5 (incompressible). Incompressible materials maintain constant volume regardless of the stress applied. Most rocks have a Poisson's ratio of approximately 0.25-0.35. Poisson's ratio describes the ability of a material to shorten parallel to $\sigma 1$ without corresponding elongation in the $\sigma 3$ direction.

IMPORTANT

In Figure 7 the stress strain curve illustrates a non-linear elastic behaviour, as the increase in strain is not directly proportional to the increase in stress. This normally occurs as the body is stressed beyond its yield point.

Plastic deformation is irreversible strain without visible fractures. Stress is applied to a rock body and deformation occurs. When stresses are removed, a portion of the strain remains. That portion of the rock that is deformed has experienced plastic strain. Permanent plastic deformation precludes visible fractures. The material deforms but does not

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break and produce visible fractures. Microscopic fracturing may however occur. Plastic strain is not recoverable or reversible.

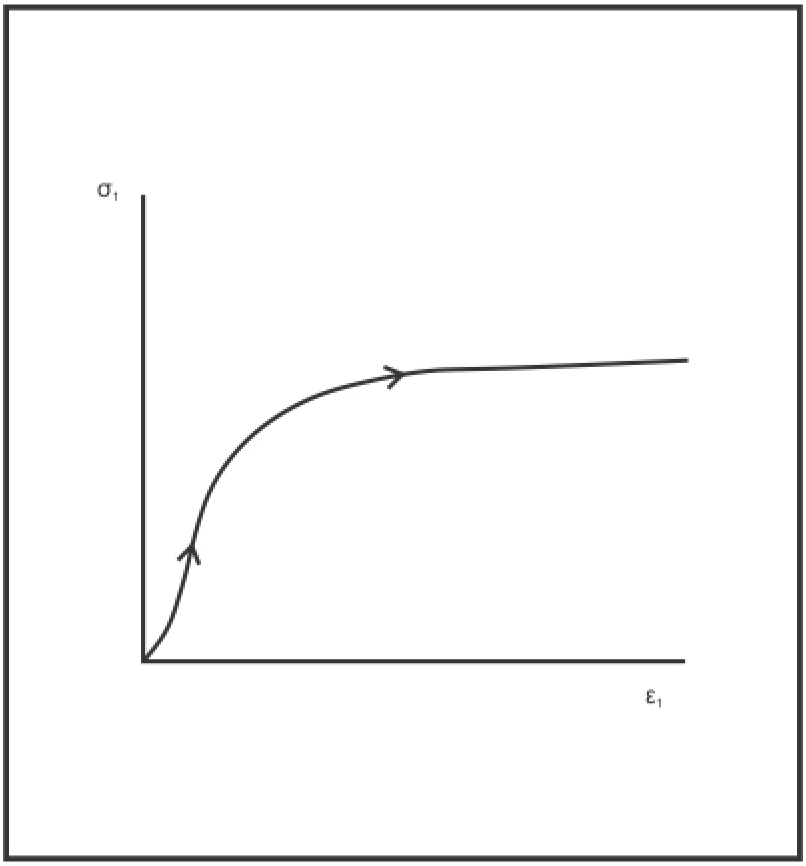

Figure 7: Non-linear elastic material

Plastic behaviour normally occurs in material that has a small elastic region, but where, at larger loads, the deformation is permanent. This is called plastic deformation when the material does not lose its ability to sustain load and perfectly plastic when the loads generated remain constant with increasing deformation.

Plastic behaviour suggests continuous deformation after some critical stress (σ_c) value is achieved and maintained. Permanent or irrecoverable deformation results.

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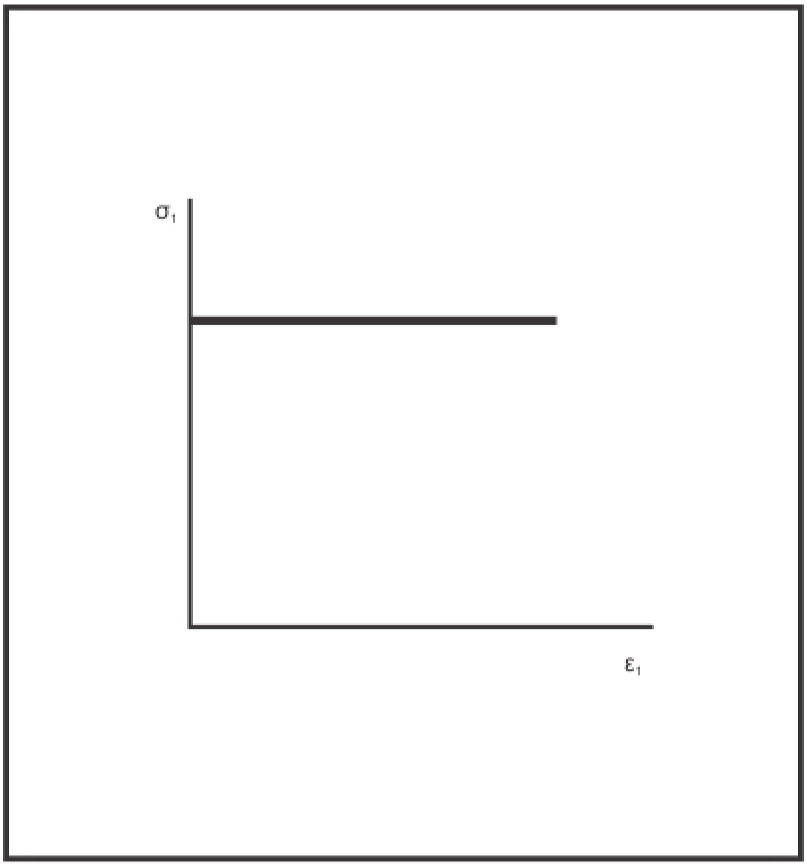

Figure 8: Perfectly plastic behaviour

Rupture deformation includes the creation of visible fractures and is irreversible, i.e. not recoverable strain.

Elasto-plastic behaviour

IMPORTANT

Many materials undergo elastic deformation at small loads and deform plastically at larger loads. At a certain stress level, the material starts to deform plastically rather than elastically.

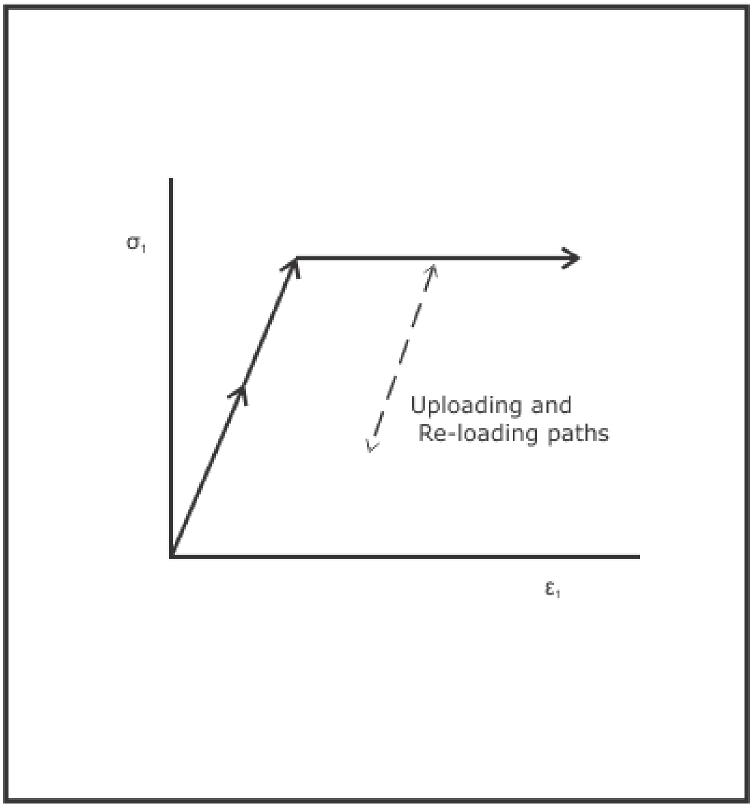

Figure 9: Elasto-plastic behaviour

Strain softening

Further deformation of the sample beyond its peak strength by continued loading results in an increase in strain but a decrease in the stress applied to the sample. The strain softening portion of the test graph is the line segment from A to B where the stress decreases (Figure 10) with increased strain.

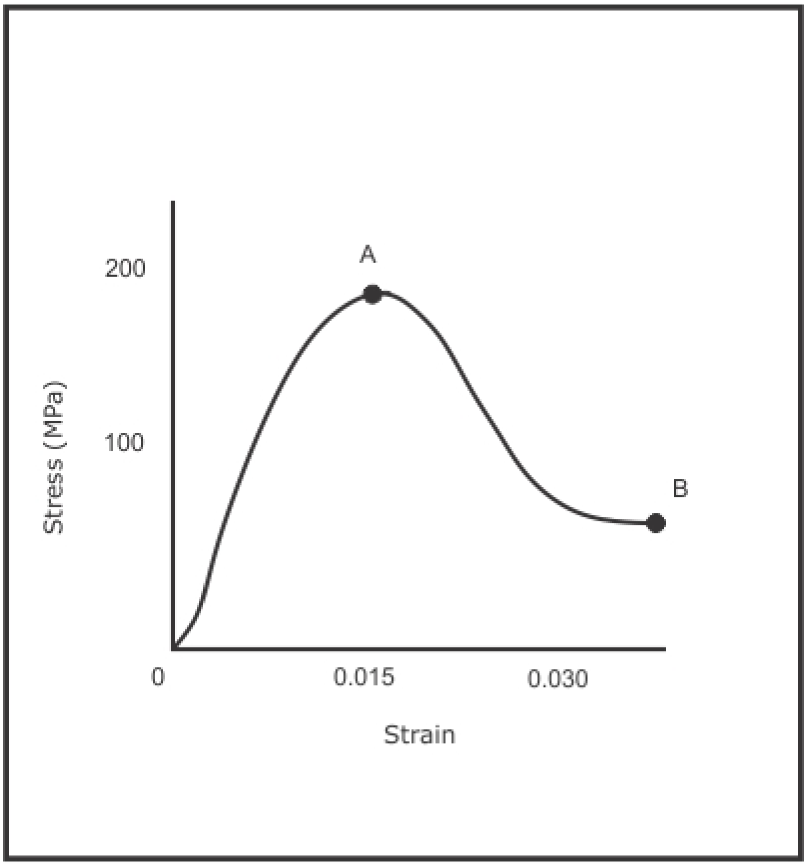

Figure 10: Strain softening behaviour

Strain hardening

IMPORTANT

The increase in applied stress results in an increase in resultant strain. The strain hardening portion of the test graph is the line segment from B to C where the stress again increases (Figure 7)

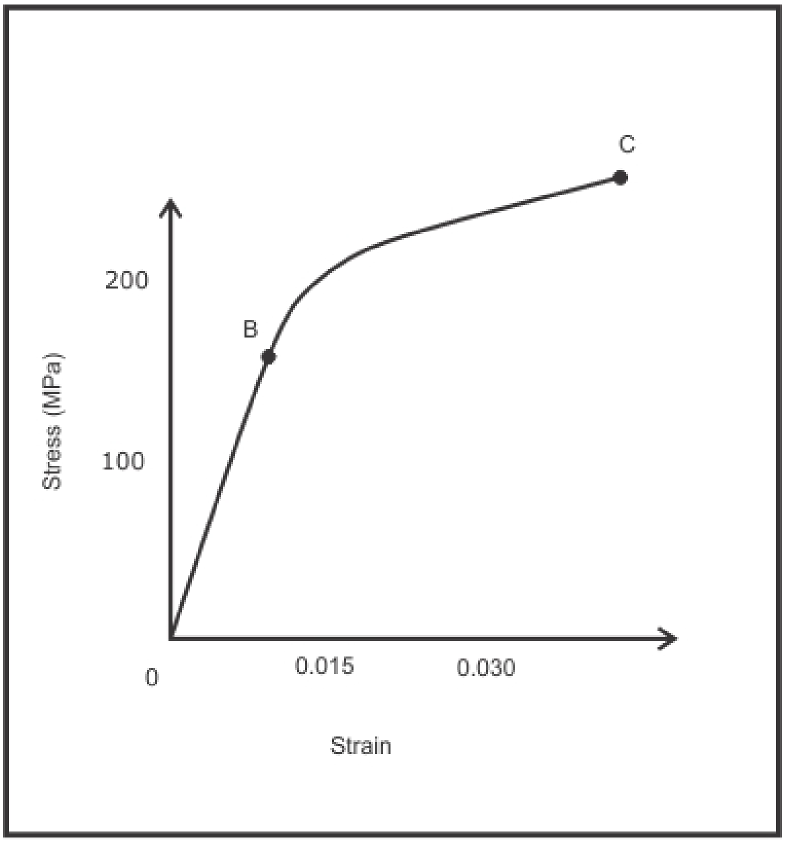

Figure 11: Strain hardening behaviour

REFERENCE

- Page 83,147 - 188,84,83, A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 252,106 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 3,159,160,180,179, 178 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 17,24,35 - Rock mechanics in mining practice, 1983, S Budavari et al.
- Page115,89 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown
- Page 5 - Rock mechanics in coal mining, 1976, MDG Salamon and KI Oravecz
- Page 19 - Handbook on hard rock strata control, 1995, AJS Spearing
- Page 26 Chapter 11 - Practical rock engineering, 2006, E Hoek

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LEARNING OUTCOME 3.2

Interpret these relationships citing material examples: Elastic, Plastic, Elasto-plastic, Strain softening, Strain hardening

- Elastic

  Most hard rock types such as dolerite, quartzite, norite, pyroxenite, manganese and marble indicate elastic behaviour in the first part of the stress/strain graph. For this elastic portion the loading and unloading paths are the same. Rock types typically found in South African hard rock mines around the mineral bearing orebody, inhibit some initial elastic behaviour. The stiffness (slope of stress-strain relationship) or the yield limits (the point at which linear elastic behaviour changes into non-linear behaviour) may, however, not be similar and will probably vary significantly.

- Plastic

  Plastic behaviour of rocks in the normal mining environment is normally associated with intersections with or mining into clay (as found in Kalahari formation), soft shale (as found in Khaki shale in the Free State Goldfields), smectite (intersections inFree State Goldfields) or some highly weathered rock types that exhibit plastic deformation with an increase in stress. These intersections experience closing up of excavations, punching of pillars, high levels of closure etc, all driven by the excessive straining of the material under constant (perfectly plastic) or increasing (ductile, strain hardening) stress levels.

- Elasto-plastic

  Jointed rock mass environments are often simulated as elasto-plastic materials due to their tendency to behave elastically at low stress levels (behaviour is controlled by the fact that stresses are too low to induce failure on joint surfaces) and plastically (behaviour is controlled by displacement occurring on the large number of joints) at larger stress levels. Rheological (time based) behaviour of some rock materials can fall into this category where initial behaviour is elastic but strength degradation with time, allows the onset of plastic behaviour.

- Strain softening

  Most rock types indicate a strain softening behaviour past the peak load as the rupture point is passed and cohesion in the material is lost, allowing failure to occur. The stress/strain curve has a negative trend (reduction in stress with increasing deformation).

- Strain hardening

  A good example of a material that indicates strain hardening behaviour is backfill material used for support in underground workings. The stress/strain curve for backfill material has a positive (upward) trend indicating strain hardening.

REFERENCE

- Page 176 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

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LEARNING OUTCOME 3.3

CALCULATE STRAIN COMPONENTS FROM STRESS COMPONENTS FOR ELASTIC MATERIALS IN TWO DIMENSIONS USING HOOKE'S LAW

IMPORTANT

In the case of linearly elastic substances, the stress/strain relationship is given by a straight line. In such a case the proportionality between stress / strain can be given by the equation shown below.

This equation is known as the Hooke's law in one dimension.

$\sigma = E\varepsilon$ where E = Young's modulus also known as the modulus of elasticity and where the stress applied and resulting strain occur in the same direction.

EXAMPLE

Assume a force of 100kN acts on the end-surface of a piece of core with original diameter 34mm and length 68mm. If the core sample decreases in length to 63mm and increases in diameter to 35mm, determine the modulus of elasticity of the material.

To calculate E, the stress acting on a sample and the axial strain are required:

The radius r of the core
= ½ x diameter = 17mm (or 0.017m).
The area on which the force works
= πr^2
= $3.14 \times (0.017)^2$
= $3.14\ (0.000289)\text{m}^2$
= 0.000907m^2
Stress = Force / Area
= $100\text{kN}/0.000907\text{m}^2$
= <u>110.253 MPa</u>

Axial strain (ε_a) = (change in dimension/original dimension)
= (Δ Length / original length)
= (68mm-63mm)/68mm
= 0.0735
E = Stress / Axial strain
= 110.253MPa / 0.0735
= <u>1.5 GPa.</u>

When a linearly elastic body is loaded uniaxially, it will also strain in the direction perpendicular to the applied stress.
This relationship is given by the equation shown below, giving the Poisson's ratio (ratio between the strains in the two mutually perpendicular directions) even under uniaxial stress.

$V = \left|\frac{\varepsilon_t}{\varepsilon_a}\right|$ where v= Poissons ratio, ε_t = Transverse strain (perpendicular to applied stress), and ε_a = Axial strain (in direction of applied stress).

EXAMPLE

Assume ε_t= 0.00025 and ε_a = 0.0005. Calculate the Poisson's ratio.

$V = \frac{\varepsilon_t}{\varepsilon_a}$ = (0.00025)/(0.0005)
= <u>0.5</u>

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EXAMPLE

Using the core example given in the section on Young's Modulus above, calculate the Poisson's ratio for the rock sample.

The axial strain is known to be ε_a = 0.0735 (see example).

The transverse strain ε_t=change in dimension / original dimension

= (Δ Diameter / original diameter)

= (34mm-35mm)/34mm

= -0.0294

BRIDGING GAP 5

"Absolute Values" on page 230

NOTE

In rock engineering compressive strain, i.e. sample getting smaller is positive and a sample that increases in dimension has a negative strain.

$$V = \frac{\varepsilon_t}{\varepsilon_a} = (-0.0294)/(0.0735) = -0.4$$

but Poisson's ratio is reported to be 0.4 (which is the absolute value of the value calculated).

NOTE

The Poisson's ratio is always positive, independent on the signs of the axial and transverse strains. It is critical that the 'absolute value' of the Poisson's ratio is reported as the final value.

When linearly elastic materials are loaded bi-axially (in the two dimensional XY coordinate system) σ_x and σ_y are applied simultaneously and strain occurs in the axial and transverse directions. The equations shown below were derived from the basic equations for biaxial loading conditions to calculate normal and shear strain components from stress components in two dimensions using the Hooke's law.

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x)$$

REFERENCE

- Page 19 - Rock mechanics in mining practice, 1983, S Budavari et al.
- Page 161, 164 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 21 - Handbook on hard rock strata control, 1995, AJS Spearing

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EXAMPLE

Assume
$v = 0.2$;
σ_x = 45 MPa:
σ_y = 80MPa;
E = 70,000 MPa and
τ_{xy} = 5 MPa.

Using the 2D representation of Hooke's Law formulae shown above, calculate ε_x and ε_y.

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y)$$

= 1/70,000MPa(45MPa-0.2(80MPa))

= 0.00041429

$$\varepsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x)$$

= 1/70,000MPa(45MPa-0.2(45MPa))

= 0.00101429

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EXAMPLE

Calculate the principal strains from principal stress components. Given:
$v = 0.2$;
$\sigma_x = 100$ MPa;
$\sigma_y = 46$ MPa;
$E = 70{,}000$ MPa

Using $\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y)$ and $\varepsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x)$ calculate ε_x and ε_y.

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y)$$

= 1/70,000MPa(100MPa-0.2(46MPa))
= <u>0.001297</u>

$$\varepsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x)$$

= 1/70,000MPa(46MPa-0.2(100MPa))
= <u>0.00037143</u>

REFERENCE

- Page 19 - Rock mechanics in mining practice, 1983, S Budavari et al.

The calculations above allow the determination of normal strains from normal stresses. However, if we use the modulus of rigidity we can calculate the shear strain γ_{xy} from the shear stress τ_{xy}.

The constant G that relates shear stresses to shear strain is called the shear modulus or modulus of rigidity, and is found using the equation shown below.

EXAMPLE

$$G = \frac{70000}{2(1+0.2)}$$
MPa
=29,167MPa

$G = \frac{E}{2(1+v)}$ Where G= modulus of rigidity;
E= Young's modulus and v= Poissons ratio.

EXAMPLE

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$
=5MPa/29,167MPa
=0.00017143MPa

Calculating the shear strain γ_{xy} from the shear modulus G is then possible
by using the following equation $G = \frac{\tau_{xy}}{\gamma_{xy}}$ where shear strain is given by γ_{xy} and shear stress by τ_{xy}.

Refer to data given at beginning of example.

Reorganising the equation provides: $\gamma_{xy} = \frac{\tau_{xy}}{G}$

REFERENCE

- Page 21 - Rock mechanics in mining practice, 1983, S Budavari et al.
- Page 11 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

NOTE

If we have the magnitude of the principal stresses or principal strains, the formulae shown below can be used to calculate the shear strain components using θ as the angle between the principal and normal stress axes.

γ_{xy} = $-(\varepsilon_1-\varepsilon_2)\sin 2\theta$
γ_{xy} = $1/E(\sigma_1-v\sigma_2)-(\sigma_2-v\sigma_1)\sin 2\theta$

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LEARNING OUTCOME 3.4

Calculate stress components from strain components for elastic materials in two dimensions using Hooke's law

The equations derived to calculate the stress components from strain components for elastic materials in two dimensions are shown below and are explained making use of examples.

- Page 19,21 - Rock mechanics in mining practice, 1983, S Budavari et al.

The equations listed below are used when principal stress components are calculated from principal strain components.

$$\sigma_1 = \frac{E}{1 - v^2}(\varepsilon_1 + v\varepsilon_2)$$

$$\sigma_2 = \frac{E}{1 - v^2}(\varepsilon_2 + v\varepsilon_1)$$

- Page 19 - Rock mechanics in mining practice, 1983, S Budavari et al.

EXAMPLE

Calculate principal stress components from the principal strains, Poissons ratio and Young's modulus when it is given that
$\varepsilon_1 = 2.5 \times 10^{-3}$;
$\varepsilon_2 = 0.5 \times 10^{-3}$;
$\gamma_{xy} = 1.2 \times 10^{-3}$;
$v = 0.25$ and
E = 60GPa
E = 60,000 MPa

$$\sigma_1 = \frac{E}{1 - v^2}(\varepsilon_1 + v\varepsilon_2)$$

= 60,000/1-(0.25)^2(2.5x10^{-3}+(0.25)(0.5x10^{-3}))MPa
= 168 MPa.

$$\sigma_2 = \frac{E}{1 - v^2}(\varepsilon_2 + v\varepsilon_1)$$

= 60,000/1-(0.25)^2(0.5x10^{-3}+(0.25)(2.5x10^{-3}))MPa
= 72MPa

The equations listed below are used when normal and shear stress components are calculated from normal and shear strain components.

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \quad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) \quad \tau_{xy} = G\gamma_{xy}$$

- Page 21 - Rock mechanics in mining practice, 1983, S Budavari et al.

EXAMPLE

Calculate the normal and shear stresses from the normal and shear strains given below.
$\varepsilon_x = 2 \times 10^{-3}$;
$\varepsilon_y = 1.5 \times 10^{-3}$;
$\gamma_{xy} = 1.2 \times 10^{-3}$;
$v = 0.25$ and
E = 60GPa
E = 60,000 MPa

The equations shown below are used to perform these calculations.

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \quad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) \quad G = \frac{E}{2(1+\nu)} \quad \tau_{xy} = G\gamma_{xy}$$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

= 60,000/1-(0.25)^2(2x10^{-3}+(0.25)(1.5x10^{-3}))MPa
= 152MPa.

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

= 60,000/1-(0.25)^2(1.52x10^{-3}+(0.25)(2x10^{-3}))MPa
= 128 MPa.

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The constant G (shear modulus) that relate the shear stresses and strains is called the shear modulus or also called the modulus of rigidity.

$$G = \frac{E}{2(1+v)}$$

where G= modulus of rigidity, E= Young's modulus and v= Poisson's ratio.

$$G = \frac{E}{2(1+v)}$$

$= 60,00/2(1+0.25)$MPa.

$=$ <u>24,000 MPa</u>

Calculate the shear stress

$\tau_{xy} = G\gamma_{xy}$

$= 60,000(1.2 \times 10^{-3})$MPa

$=$ <u>72MPa</u>

- Page 21 - Rock mechanics in mining practice, 1983, S Budavari et al.
- Page 11 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

LEARNING OUTCOME 3.5

Calculate strain components from stress components for elastic materials in three dimensions using Hooke's law

In this outcome strain components are calculated in three dimensions from stress components. In order to do this the formulae and references below were used.

- Page 22 - Rock mechanics in mining practice, 1983, S Budavari et al where single subscripts are used.

$$\varepsilon_x = \frac{1}{E}((\sigma_x - v(\sigma_y + \sigma_z)) \quad \varepsilon_y = \frac{1}{E}((\sigma_y - v(\sigma_x + \sigma_z))$$

$$\varepsilon_z = \frac{1}{E}((\sigma_z - v(\sigma_x + \sigma_y)) \quad \gamma_{xy} = \frac{\tau_{xy}}{G} \quad \gamma_{yz} = \frac{\tau_{yz}}{G} \quad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

- Page 148 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager, where double subscripts and a different definition of shear strain are used.

$$\varepsilon_{xx} = \frac{1}{E}((\sigma_{xx} - v(\sigma_{yy} + \sigma_{zz})) \quad \varepsilon_{yy} = \frac{1}{E}((\sigma_{yy} - v(\sigma_{xx} + \sigma_{zz})) \quad \varepsilon_{zz} = \frac{1}{E}((\sigma_{zz} - v(\sigma_{xx} + \sigma_{yy}))$$

$$\Gamma_{xy} = \frac{\gamma_{xy}}{2}$$

where

$$\Gamma_{xy} = \frac{\gamma_{xy}}{2}$$

EXAMPLE

Calculate the normal and shear strains from the normal stresses in three dimensions given the data listed below.
σ_x = 60 MPa;
σ_y = 42 MPa;
σ_z = 32 MPa;
E = 55 GPa;
v = 0.2

ε_x = 1/E((σ_x-v(σ_y+σ_z))
= 1/55,000(60-0.2(42+32))
= $\underline{8.218 \times 10^{-4}}$

ε_y = 1/E((σ_y-v(σ_x+σ_z))
= 1/55,000(42-0.2(60+32))
= $\underline{4.291 \times 10^{-4}}$

ε_z = 1/E((σ_z-v(σ_x+σ_y))
= 1/55,000(32-0.2(60+42))
= $\underline{2.109 \times 10^{-4}}$

To calculate the principal strains from principal stresses the equations shown below can be used.

- Page 149 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

$$\varepsilon 1 = \frac{\sigma 1 - \nu(\sigma 2 + \sigma 3)}{E}$$

$$\varepsilon 1 = \frac{\sigma 1 - \nu(\sigma 2 + \sigma 3)}{E}$$

$$\varepsilon 1 = \frac{\sigma 1 - \nu(\sigma 2 + \sigma 3)}{E}$$

Written in the same way as illustrated in Rock mechanics in mining practice, 1983, S Budavari et al – (Page 19) the equations can be simplified as:

$$\varepsilon 1 = \frac{1}{E}((\sigma 1 - \nu(\sigma 2 + \sigma 3))$$

$$\varepsilon 2 = \frac{1}{E}((\sigma 2 - \nu(\sigma 1 + \sigma 3))$$

$$\varepsilon 3 = \frac{1}{E}((\sigma 3 - \nu(\sigma 1 + \sigma 2))$$

EXAMPLE

Calculate the principal strains from the principal stresses in three dimensions given the data listed below.
σ_1 = 72 MPa;
σ_2 = 64 MPa;
σ_3 = 46 MPa;
E = 50GPa;
v = 0.2

ε_1 = 1/E((σ_1-v(σ_2+σ_3))
= 1/50,000(72-0.2(64+46))
= $\underline{1.0 \times 10^{-3}}$

ε_2 = 1/E((σ_2-v(σ_1+σ_3))
= 1/50,000(64-0.2(72+46))
= $\underline{8.08 \times 10^{-4}}$

ε_3 = 1/E((σ_3-v(σ_1+σ_2))
= 1/50,000(46-0.2(72+64))
= $\underline{3.76 \times 10^{-4}}$

- Page 22 - Rock mechanics in mining practice, 1983, S Budavari et al
- Page 164 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden

LEARNING OUTCOME 3.6

Calculate stress components from strain components for elastic materials in three dimensions using Hooke's law

The formulae shown below are used to calculated normal- and shear stress components from normal-and shear strains.

Page 22 - Rock mechanics in mining practice, 1983, S Budavari et al , where stress and strain components are indicated with single subscripts.

$$\sigma_x = \frac{2G}{1-2\nu}((1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z))$$

$$\sigma_y = \frac{2G}{1-2\nu}((1-\nu)\varepsilon_y + \nu(\varepsilon_x + \varepsilon_z))$$

$$\sigma_z = \frac{2G}{1-2\nu}((1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y))$$

The formulae shown below are used to calculated normal- and shear stress components from normal-and shear strains.

Normal stresses are indicated with double subscripts namely σ_{xx}, σ_{yy} and $\sigma_z z$.

Normal and shear stresses from normal and shear strains

$$\sigma_{xx} = [E/(1+v)(1-2v)]\,[(1-v)\,\varepsilon_{xx} + v(\varepsilon_{yy} + \varepsilon_{zz})]$$

$$\sigma_{yy} = [E/(1+v)(1-2v)]\,[(1-v)\,\varepsilon_{yy} + v(\varepsilon_{xx} + \varepsilon_{zz})]$$

$\sigma_{zz} = [E/(1+v)(1-2v)]\,[(1-v)\,\varepsilon_{zz} + v(\varepsilon_{xx} + \varepsilon_{yy})]$ as well as

$$\tau_{xy} = 2G\lambda_{xy}\tau_{yz} = 2G\lambda_{yz}\tau_{zx} = 2G\lambda_{zx}$$

Principal stresses from principal strains

$$\sigma_1 = [E/(1+v)(1-2v)]\,[(1-v)\,\varepsilon_1 + v(\varepsilon_2 + \varepsilon_3)]$$

$$\sigma_2 = [E/(1+v)(1-2v)]\,[(1-v)\,\varepsilon_2 + v(\varepsilon_1 + \varepsilon_3)]$$

$$\sigma_3 = [E/(1+v)(1-2v)]\,[(1-v)\,\varepsilon_3 + v(\varepsilon_1 + \varepsilon_2)]$$

- Page 149 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

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EXAMPLE

Calculate the normal and shear stress components from the normal and shear strain components in three dimensions given the data listed below.

$\varepsilon_x = 2.4 \times 10^{-3}$;
$\varepsilon_y = 1.8 \times 10^{-3}$;
$\varepsilon_z = 1.2 \times 10^{-3}$;
$\gamma_{xy} = 1.2 \times 10^{-3}$;
$\gamma_{yz} = 0.3 \times 10^{-3}$;
$\gamma_{zx} = 1.9 \times 10^{-3}$;
v =0.25 and
E=60GPa

$G = E/2(1+v)$
$= 60{,}000{,}000/2(1+0.25)$
$= \underline{24GPa}$

$\sigma_x = [2G/(1-2v)][(1-v)\varepsilon_x+v(\varepsilon_y+\varepsilon_z)]$
$= [2(24{,}000MPa)/(1-2(0.25))]$
$[(1-0.25)(2.4 \times 10^{-3})+0.25((1.8 \times 10^{-3})+(1.2 \times 10^{-3}))]$
$= \underline{244.8MPa}$

$\sigma_y = [2G/(1-2v)][(1-v)\varepsilon_y+v(\varepsilon_x+\varepsilon_z)]$
$= [2(24{,}000MPa)/(1-2(0.25))]$
$[(1-0.25)(1.8 \times 10^{-3})+0.25((2.4 \times 10^{-3})+(1.2 \times 10^{-3}))]$
$= \underline{216MPa}$

$\sigma_z = [2G/(1-2v)][(1-v)\varepsilon_z+v(\varepsilon_x+\varepsilon_y)]$
$= [2(24{,}000MPa)/(1-2(0.25))]$
$[(1-0.25)(1.2 \times 10^{-3})+0.25((2.4 \times 10^{-3})+(1.8 \times 10^{-3}))]$
$= \underline{187.2MPa}$

$\tau_{xy} = 2G\gamma_{xy}$
$= 2(24{,}000MPa)(1.2 \times 10^{-3})$
$= \underline{57.6MPa}$

$\tau_{yz} = 2G\gamma_{yz}$
$= 2(24{,}000MPa)(0.3 \times 10^{-3})$
$= \underline{14.4MPa}$

$\tau_{zx} = 2G\gamma_{zx}$
$= 2(24{,}000MPa)(1.9 \times 10^{-3})$
$= \underline{91.2MPa}$

LEARNING OUTCOME 3.7

Describe and explain the concept of plane strain

DEFINITION

Plane strain: *Plane strain is a state of strain in a solid body in which all strain components normal to a certain plane are zero. (From: International society for Rock Mechanics - Commission on terminology, symbols and graphic representation – 1975)*

If one dimension of an excavation is very large compared to the other two, the strain in the direction of the longest dimension is constrained and can be assumed to be zero, creatinga plane strain condition.
A tunnel under external loading in an underground situation is an example of a plane strain condition. In this example, stress levels in all directions are non-zero, i.e. $\sigma_z, \sigma_x, \sigma_y \neq 0$, but the strain normal to the XY plane, in the z-axis orientation is zero, i.e. $\varepsilon_z = 0$.

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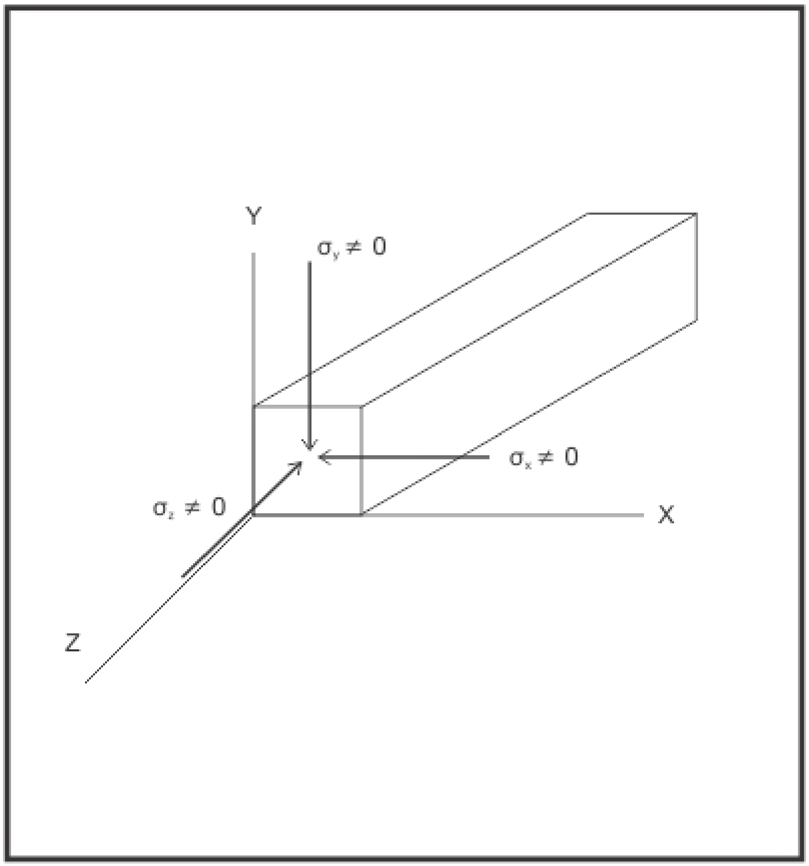

Figure 12: Plane strain

For the plane strain conditions shown in Figure 12, it can therefore be assumed that $\varepsilon_z = 0$, but the stress in the z-direction cannot be zero, as it is required to ensure that the strain is zero.The following equations will then be used to calculate the stress in the z-direction required to maintain zero strain in that direction and the normal strains in the other directions, from normal stress levels.

$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}((1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y))$$

$$\varepsilon_y = \frac{1}{E}((1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x))$$

NOTE

Keep in mind that $\varepsilon_z = 0$.

REFERENCE

- Page 22 - Rock mechanics in mining practice, 1983, S Budavari et al
- Page 168 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown
- Page 113 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 135, 151 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 167 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 91 - Underground excavations in rock, 1980, E Hoek and ET Brown

LEARNING OUTCOME 3.8

CALCULATE THE ELASTIC STRAIN ENERGY DENSITY IN ELASTIC MATERIALS

Energy: *Energy is a fundamental concept in physics. Energy takes on many different forms and is part of almost all physical process. On a basic level, it can be defined as 'stored work', suggesting that energy can be converted into work: The kinetic (motion related) energy in flowing water can drive a saw mill; on impact, a moving hammer can drive a nail into wood.*

BRIDGING GAP 6

"Energy" on page 231

Strain energy: *Strain energy is the work done in deforming a body.*

In order to strain an elastic material, work must be done on the material. In basic physics (as discussed above), the work done is the product of the forces that act on the body and the displacement undergone by the body due to the external forces.

In rock mechanics, we relate the work to strain and elastic material to the deformation (strain) undergone by a body due to the external stresses working on the body. This energy is stored in the form of potentially recoverable elastic strain energy.

The strain energy density W at a given point in the body is given by $W = \int \sigma \, d\varepsilon [J/m^3]$.

The unit of J/m^3 indicates 'energy per volume' which suggests an 'energy density' similar to rock density which is given as tonnes/m^3.

For a linear elastic material, it follows that the strain energy density is a function of all the stress and strain components working on a three dimensional body and results in

$$W = ½\ 1/2(\sigma_1\varepsilon_1+\sigma_2\varepsilon_2+\sigma_3\varepsilon_3)$$

or

$$W = ½\ 1/2(\sigma_{xx}\varepsilon_{xx}+\sigma_{yy}\varepsilon_{yy}+\sigma_{zz}\varepsilon_{zz}+2\tau_{xy}\lambda_{xy} + 2\tau_{yz}\lambda_{yz} + 2\tau_{zx}\lambda_{zx})$$

The total strain energy (Q) stored in the body can now be determined from:

Q = Volume of body x strain energy density = Vol x W (Joule)

The product of density and volume gives the total weight of a certain volume of rock. In a similar way, the product of the body volume and energy density gives the total energy stored in the body.

- Page 135 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

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EXAMPLE

Calculate the amount of stored strain energy per volume (under uni-axial loading) in a rock specimen at the point of failure when the length of rock specimen = 140mm; diameter of specimen = 70mm;
E =78GPa and
UCS = 152MPa.

Assume that the specimen remain elastic to the failure point. Now calculate the maximum strain energy in the specimen

Using the equation for strain energy density for three dimensions, namely

$W = ½(\sigma_1\varepsilon_1+\sigma_2\varepsilon_2+\sigma_3\varepsilon_3)$

the equation can be simplified to

$W=½(\sigma_1\varepsilon_1)$

for uni-axial loading conditions, when σ_2 and $\sigma_3 = 0$.

To be able to calculate W, both σ_1 and ε_1 should be known. Since σ_1 will equal the uni-axial stress at failure, 152 MPa, ε_1 can be calculated using Hooke's law in one dimension.

Hooke's Law states that:

$$\begin{aligned} E &= \sigma/\varepsilon \text{ therefore} \\ &= 152\text{MPa}/78{,}000\text{MPa} \\ &= \underline{0.00194872\text{MPa}} \end{aligned}$$

Now :

$$\begin{aligned} W &= ½(\sigma_1\varepsilon_1) \\ &= ½(152\text{MPa})(0.00194872\text{MPa}) \\ &= \underline{0.1481\text{MJ/m}^3} \end{aligned}$$

NOTE

Since the stress units are MPa, the energy units will be MN/m³, while the density remains 'per m³' in spite of the uni-axial loading conditions.

The total strain energy (Q) stored in the sample can now be calculated from:

$$\begin{aligned} Q &= \text{Volume x W} \\ &= (\pi D^2)/4 \text{ x } 0.1491 \text{ MJ/m}^3 \\ &= (\pi/4.(0.07)2)(0.1481)\text{MJ} \\ &= 0.0056966 \text{ MJ} \\ &= \underline{5.6966 \text{ kJ}} \end{aligned}$$

REFERENCE

- Page 23,72 - Rock mechanics in mining practice, 1983, S Budavari et al
- Page 128 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman

LEARNING OUTCOME 3.9

DESCRIBE, EXPLAIN AND DEFINE THE FOLLOWING TERMS:

DEFINITION

Dilation and dilatation: *Dilatation is defined as the quotient of the change in volume to the original volume of a body submitted to stress. Dilatation is therefore also referred to as volumetric strain and is usually applied to linear elastic material behaviour.*

Dilation is referred to when the radial strain deviates from linearity close to and after the peak strength point of a sample under stress. Dilation is measured in terms of the dilation angle.

Both concepts are indications of the change in sample volume when stressed.

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DEFINITION

Dilation angle: *Angle of dilation (ψ) can be defined as the ratio of incremental plastic volumetric strain plastic shear strain and is usually determined at the peak strength point from the slope of the radial : axial strain relationship.*

REFERENCE

- Page 82, A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

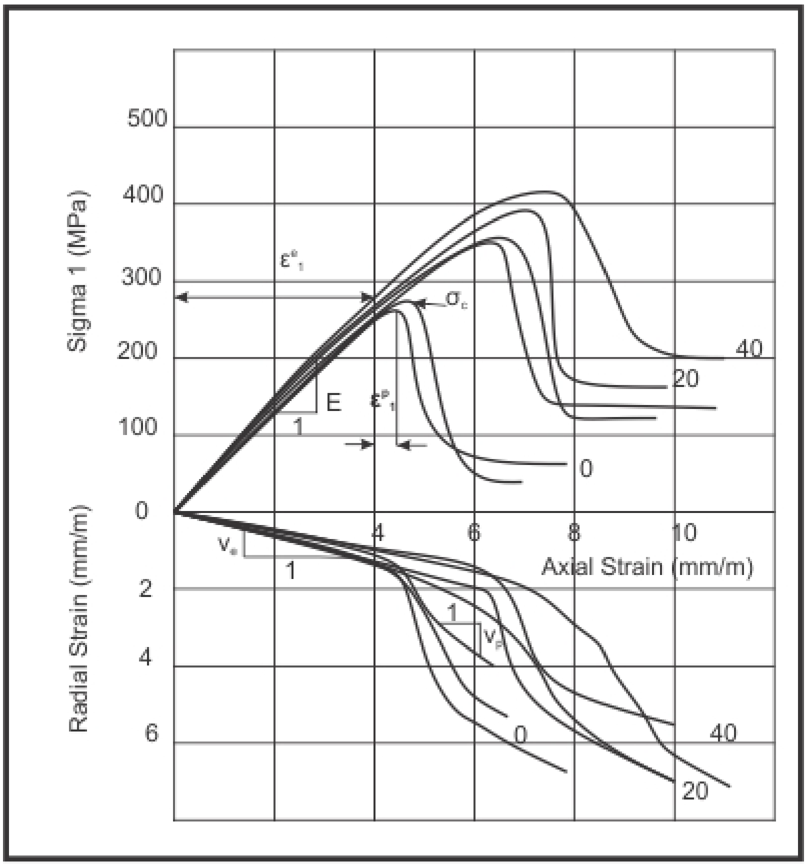

Figure 13: Complete tri-axial stress-strain curves for a typical quartzite (CSIR 1999)

Since dilation angle refers to the characteristic of the material to strain plastically, the area of interest on the stress:strain relationship shown in Figure 9 is that close to and after the peak strength point and therefore not within the linear elastic behaviour range.

Plastic strain normally commences when the stress:strain relationship deviates from linearity, indicating the onset of permanent deformation due to micro-fracturing. The ratio between ε_{1p} and ε_{1e} is called the strain ratio and is an indication of the strain behaviour of a material. A high strain ratio indicates that the plastic strain at peak strength is much larger than the elastic strain and suggests ductile behaviour as fracturing commences. A low strain ratio indicates that the plastic strain undergone is only slightly larger than the elastic strain and that the material fails in a brittle manner.

Dilation angle is an indication of the plastic dilatancy of the material. Its value is therefore determined by the plastic material behaviour and indicates a change in volume close to or at the peak strength point. A large dilation angle would indicate a very dilatant material that will experience a large increase in volume for a small disruption in shape close to peak strength point.

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The dilation angle can thus be determined by applying the ratio of the radial to axial strains close to the peak strength point (v_p) in the following equation (Figure 13):

$$\mathrm{Sin}\varphi=(2v_p-1)/(2v_p+1)$$

NOTE

Typical hard rock dilation angles range between 15° and 20° and will always be lower than the material friction angle.

EXAMPLE

Calculate the dilation angle for a material with a plastic radial : axial strain ratio of 0.9 at the peak strength point.

Using

$\mathrm{Sin}\varphi = (2v_p-1)/(2v_p+1)$ it follows that

$\varphi = \mathrm{ASin}((2v_p-1)/(2v_p+1))$

$= \mathrm{ASin}((2(0.9)-1)/(2(0.9)+1))=\mathrm{ASin}(0.286)=16.6°$

Angle of dilation (ψ) can also be defined in terms of dilation on a plane along which shear occurs. In the case of shear along a pre-existing plane, the dilation occurs due to the roughness of the plane surface and the dilation angle can be determined from Figure 14.

$$\mathrm{Tan}\varphi=(\text{Normal displacement})/(\text{Shear displacement})=\delta_n/\delta$$

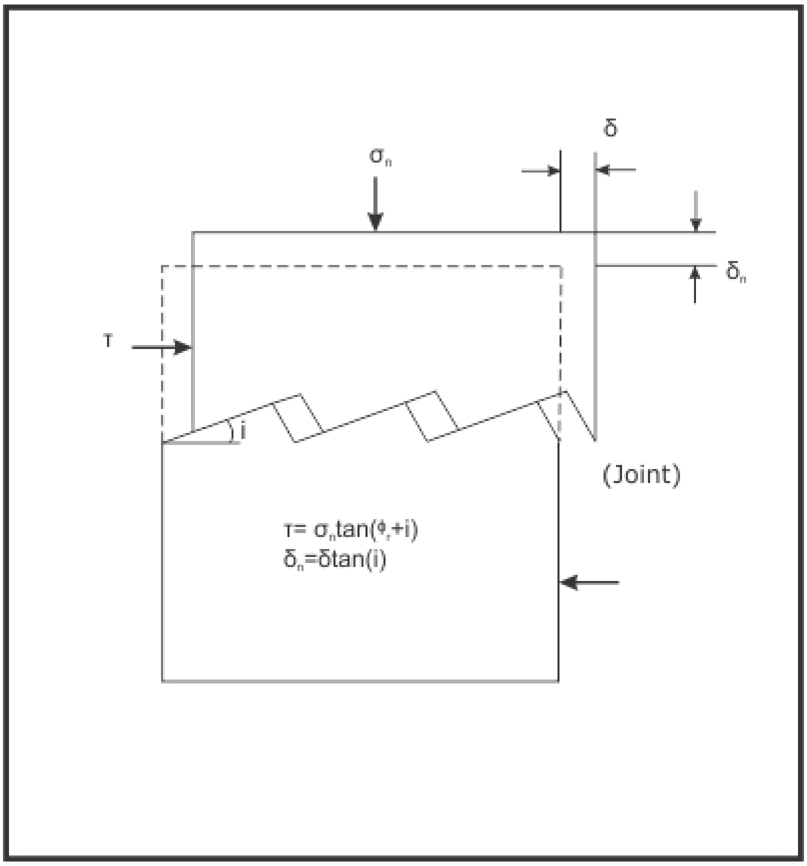

Figure 14: Dilation angle of joint surfaces (Ryder and Jager)

REFERENCE

- Page 103 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

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Associated and non-associated flow

Both associated and non-associated flows are forms of plastic behaviour exhibited by material. When a rock sample is stressed it behaves in an elastic manner (rock is only truly ductile at high temperatures and stress) where strain is recoverable up to a certain point where after it behaves inelastically. This inelastic behaviour has been found to be well approximated by using concepts from plasticity theory.

Tests have shown that a failure surface (also called yield surface) exists that separates the elastic and inelastic behaviour regions. Stress points below this yield surface experienceelastic behaviour while stress paths on or outside the yield surface experience inelastic or plastic behaviour.

- Page 423 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

The yield surface, as presented by the function F in Figure 15, indicates the stress state when the material will enter into plastic behaviour. Plastic flow potential, as represented by the function Q in Figure 15, indicates the stress state at which a flow surface allows plastic strain orthogonal to the surface.

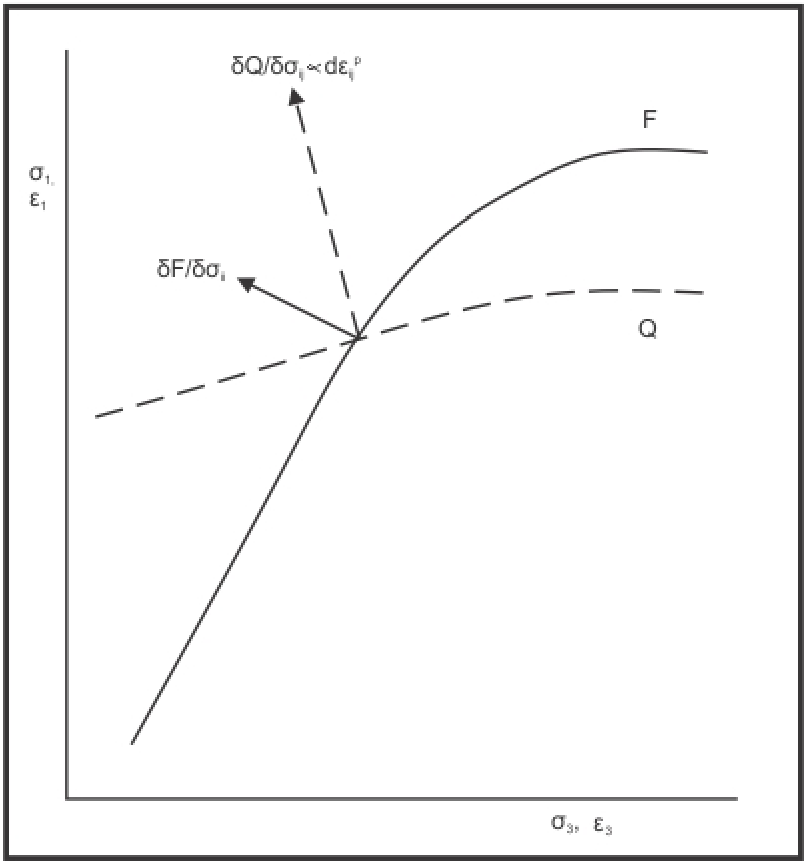

Figure 15: Yield and flow surfaces (Ryder and Jager)

Associated flow: *When the F and Q functions discussed above and shown in Figure 15 are the same, the flow is called 'associated'. All metals and homogeneous material exhibits associated flow. This suggests that flow is unique and that solution of the material behaviour can be guaranteed.*

The dilation angle for material that exhibits associated flow is equal to the material friction angle.

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Non-associated flow: *As soon as material is inhomogeneous, significant confinement strengthening occurs at failure and dilation occurs such that the dilation angle is much lower than the material friction angle.*

This behaviour is called non-associated flow and indicates behaviour where the F and Q functions (Figure 15) are not the same. This behaviour suggests that unique flow is not possible and that a unique solution cannot be guaranteed. Since constitutive equations for this type of flow are not symmetric, conventional solution techniques cannot be employed and more complex algorithms must be sought.

It should be clear that all rocks fall into this flow category and that plastic behaviour of rocks is more complex than those of metals and homogeneous materials.

- Page 267 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 82,422 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

LEARNING OUTCOME 3.10

DESCRIBE, EXPLAIN AND INDICATE ON A STRESS-STRAIN GRAPH FOR ROCK MATERIAL THE ONSET OF INELASTIC BEHAVIOUR, DILATION AND STRAIN SOFTENING

During rock tests, a stress-strain graph is drawn similar to that shown in Figure 12. On this graph, the axial and radial (transverse)strains exhibited by the sample during loading are plotted against the applied stress levels.

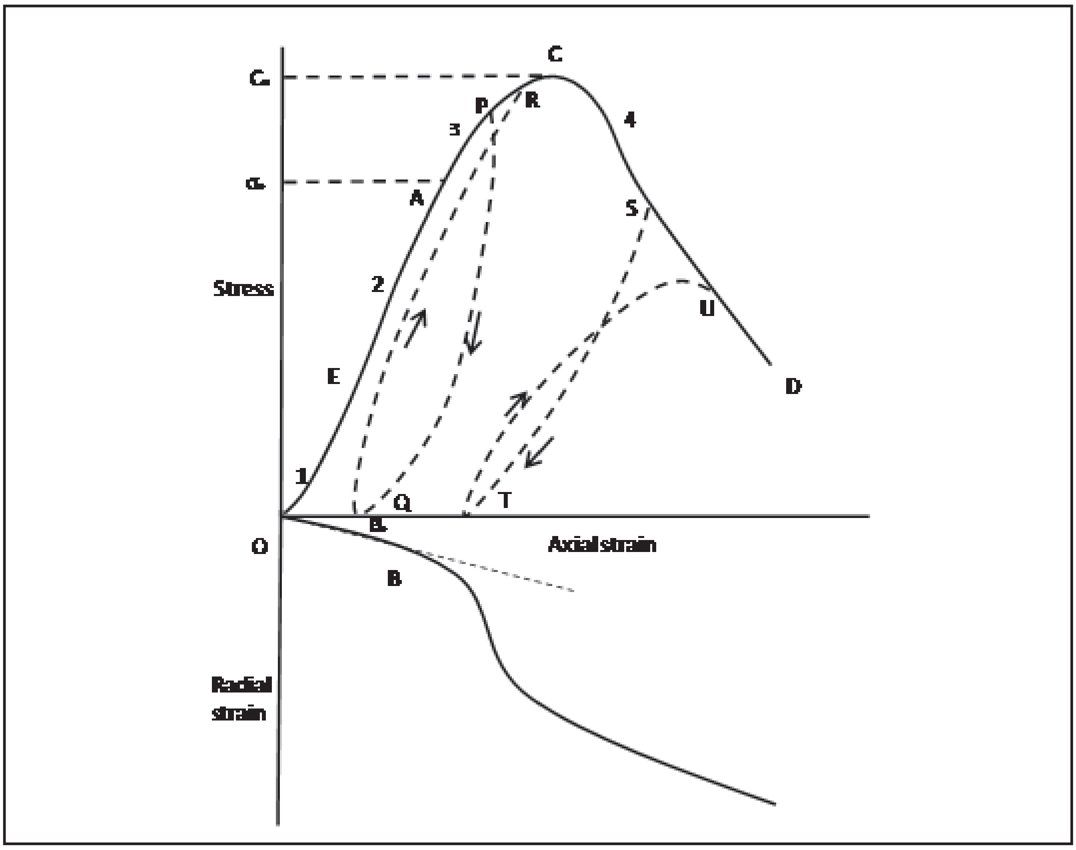

Figure 16: Stress – strain relationship during rock sample testing

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Inelastic behaviour
When a rock sample is subjected to stress, initial behaviour is linearly or non-linearly elastic, indicating that all strain is recoverable at first. When the sample experiences permanent strain (i.e. non-recoverable), such as when micro-fracturing occurs, inelastic behaviour commences (point A, Figure 16).

Dilation
Dilation occurs when the rock sample deforms axially and radially. Elastic dilatation occurs as soon as the volume of the sample changes (results in volumetric strain), while inelastic dilation commences when the axial - radial strain relationship deviates from linearity (point B, Figure 16)

Strain softening
Strain softening occurs when the deformation of the sample beyond its peak strength by continued loading, results in an increase in strain but a decrease in the stress sustained by the sample (line segment from C to D, Figure 16).

LEARNING OUTCOME 3.11

Identify, describe and explain time-dependent behaviour

Time dependent behaviour is known as creep. Creep is the tendency of a material to slowly move or deform permanently under the influence of constant stress and is also known as non-recoverable viscous flow. This phenomenon is more pronounced in softer rock types such as potash.

The rate of deformation is a function of the material properties, exposure time, exposure temperatures and the applied stress levels.

Figure 17 below illustrates a typical laboratory creep curve for rock suddenly subjected to a fixed elevated axial stress; with the primary, secondary and tertiary creep phases visible. An instantaneous elastic strain first takes place, governed by the familiar elastic modulus E. Thereafter, primary creep involves an initially rapid increase in strain with time. This soon dies out to reveal, at elevated stress levels, a long-term sustained secondary creep phase involving a constant strain rate. After the passage of sufficient time, an acceleration of strain rate (tertiary creep) may occur, leading to rapid failure of the specimen. If the specimen is unloaded at some stage during secondary creep, there is a rapid recovery of strain apart from a relatively small level of irrecoverable residual strain.

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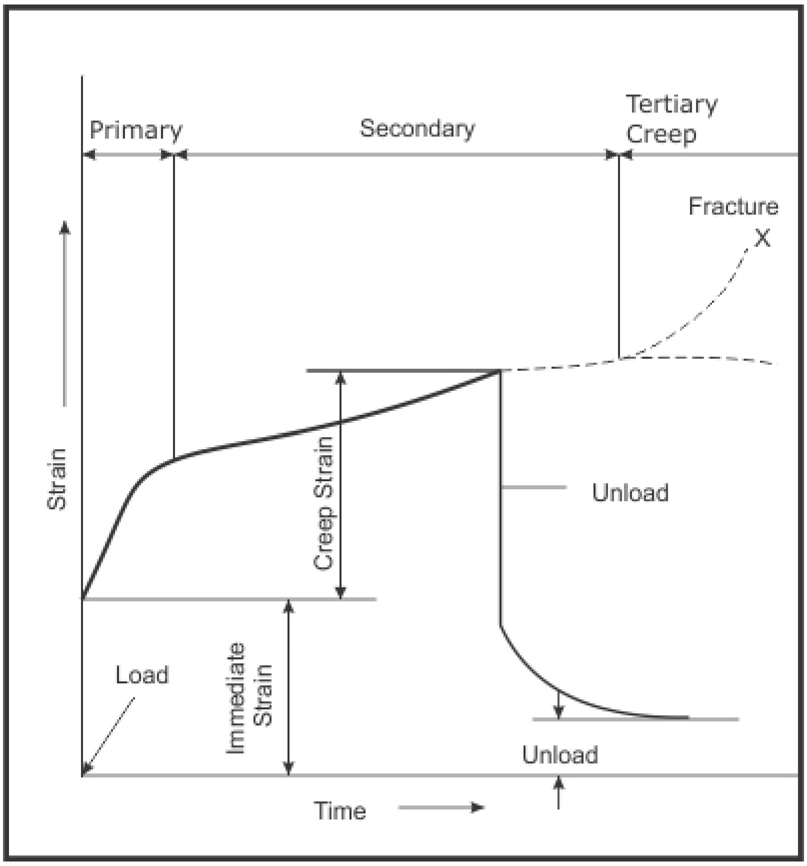

Figure 17: Idealised strain-time (creep) and strain rate-time curves (S Budavari)

The stages of creep are:

Primary creep
The primary creep stage is the early stage of deformation where straining becomes permanent (inelastic). Initially, the strain rate is high but it decreases with time.

Secondary creep
As soon as the strain rate reaches a constant value, the creep is stated to have entered the secondary stage. Straining of the material continues in this stage, but occurs at a constant rate.

Tertiary creep
In the tertiary creep stage the strain rate increases rapidly until complete failure occurs.

REFERENCE

- Page 24,30,31,37 - Rock mechanics in mining practice, 1983, S Budavari et al
- Page 269 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 106,183 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

LEARNING OUTCOME 3.12

INTERPRET THIS (CREEP) BEHAVIOUR CITING MATERIAL EXAMPLES

NOTE

Practical definition and interpretation of creep
Creep is the tendency of a solid material to slowly move or deform permanently under the influence of stresses. It occurs as a result of long term exposure to high levels of stress that are below the yield strength of the material.

Creep is more severe in materials that are subjected to heat for long periods, and near melting point. Creep always increases with temperature.

The rate of this deformation is a function of the material properties, exposure time, exposure temperature and the applied load. Depending on the magnitude of the applied stress and its duration, the deformation may become so large that the material cannot maintain its incipient shape and form.

Creep is a deformation mechanism that may or may not constitute a failure mode. Moderate creep in concrete is sometimes welcomed because it relieves tensile stresses that might otherwise lead to cracking.

Unlike brittle fracture, creep deformation does not occur suddenly upon the application of stress. Instead, strain accumulates as a result of long-term stress. Creep is a "time-dependent" deformation with stages shown in the figure below that have been explained in preceding sections of this material.The amount of creep is primarily influenced by stress level, but also by temperature, humidity and chemical environment.

Secondary creep, where present, is of the utmost importance in practical rock engineering since it can lead to delayed failure for stresses as much as 30 % below the tested laboratory strengths, i.e. the long-term strength of rock can be as little as 70 % of the UCS. This is illustrated in Figure 18 below, which shows the creep of quartzite and lava specimens under laboratory conditions. For both rock types, the amount of primary creep and the rate of creep in the secondary phase are dependent on the stress level; and significant secondary creep is visible only for stresses exceeding about 80 % of the UCS. Creep effects will be most pronounced close to the exposed rockwall of excavations where the deviatoric stresses are large.

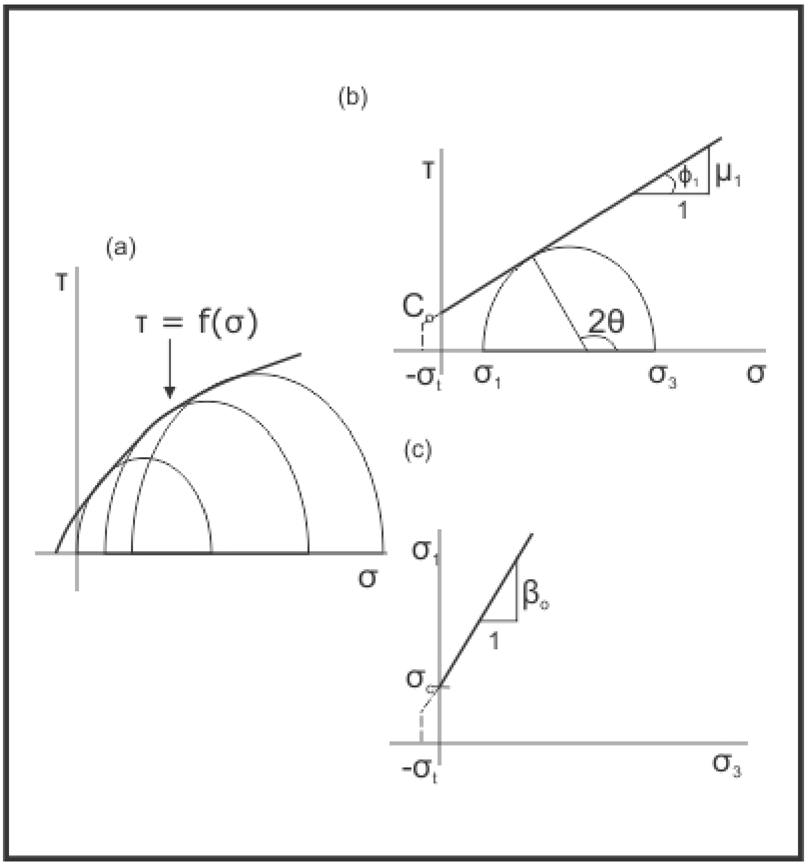

Figure 18: Creep of quartzite and lava under constant uniaxial loading

Creep within mining practice

In terms of strata control there are several important consequences of time-dependent processes:

- In stopes, slow or intermittent rates of face advance lead to a deterioration in hangingwall conditions and enhanced amounts of stope closure. Maintenance of steady advance rates reduces these difficulties.
- Gullies cut into argillaceous ground can suffer excessive rates of sidewall closure, which can be controlled by the installation of gully sidewall support, or sometimes only by cutting sacrificial parallel gullies to absorb the excessive lateral movements involved.
- If tunnels and other long-life service excavations are developed in rock that is prone to excessive time-dependent behaviour, adequate clearances must be provided and support of appropriate quality and yieldability needs to be installed, preferably from the outset. These excavations should ideally be developed in overstoped ground to reduce the effects of principal stress loading and thereby reduce the magnitude of the imposed deviatoric stresses.
- Pillars can slowly crush, or sometimes worse, can slowly punch into an argillaceous footwall leading to excessive footwall heave. Pillar sidewall support, enlargement of pillar dimensions and use of backfill are possible means of redress.
- Shaft barrels can be progressively destabilised when sections are developed in material prone to time dependent behaviour as illustrated in the figure below which depicts the progressive movement of a section of a shaft barrel developed in Westonaria formation lava

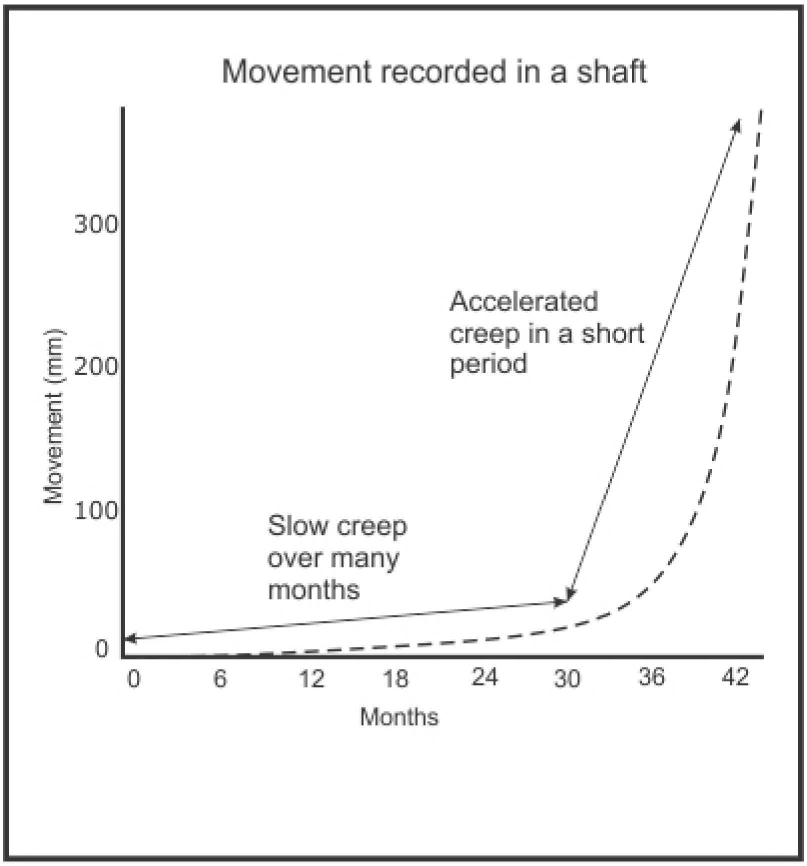

Figure 19: Creep of a section of shaft developed in weak lava

Rock materials prone to creep
All rock materials, when subjected to loading close to their yield point and sustained incipient temperature and pressure levels are prone to creep. Creep could occur over a few days or over millions of years depending on the chemical constituents of the rock, the magnitude of applied load, and the rate of loading and temperature and pressure levels. For the most part and bulk of rock types this movement is so insignificant that for all design and practical purposes, it can be ignored. There are rock types that are particularl prone to creep within a relatively short space of time. Within the context of these notes, 'short space of time' is defined as within the operating life cycle of a mine i.e. 0-25 years. These rock types are described below.

Halite (Figure 20)
Halite, commonly known as rock salt, is the mineral form of sodium chloride (NaCl). Halite forms isometric crystals. The mineral is typically colourless or white, but may also be light blue, dark blue, purple, pink, red, orange, yellow or grey, depending on the amount and type of impurities. It commonly occurs with other evaporite deposit minerals such as several of the sulphates, halides, and borates.

Due to the weak bonds between individual crystals and the hexagonal structure of each crystal, interfaces are easily broken and even under very low ambient or applied loads, the entire mass or individual crystals of halite can creep.

Figure 20 Halite or rock salt crystals

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Vesiculated and talcose basalts (Figure 21)

Basalt is a common extrusive volcanic rock. It is usually grey to black and fine-grained due to rapid cooling of lava at the surface of a planet. It may be porphyritic containing larger crystals in a fine matrix, or vesicular, or frothy scoria. Unweathered basalt is black or grey. According to the official definition, basalt is defined as an aphanitic igneous rock that contains, by volume, less than 20% quartz and less than 10% feldspathoid and where at least 65% of the feldspar is in the form of plagioclase. Basalts formed as a result of pyroclastic flow or settlement or rapidly cooled under water are often highly vesiculated (Figure 18) and contain alteration minerals such as talcose, tremolite and carbonates. These impurities serve to lower the strength of these basalts and alter the crystalline structure thereby making them prone to creep.

The sequence and type of rock horizons that encompass weaker horizons also influence the onset, rate and magnitude of creep. This is prevalent in geological settings that contain varying types of VCR host rock types and/or strengths.

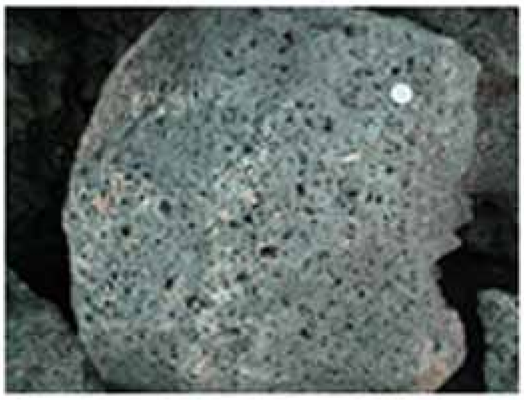

Figure 21: Vesiculated basalt (air pockets)

Foliated rock masses (Figure 22)

Foliation is any penetrative planar fabric present in rocks. Foliation is common to rocks affected by regional metamorphic compression typical of orogenic belts.

Rocks exhibiting foliation include the typical sequence formed by the metamorphism of mudrocks; slate, phyllite, schist and gneiss. The slatey cleavage typical of slate is due to the preferred orientation of microscopic phyllosilicate crystals. In gneiss, the foliation is more typically represented by compositional banding due to the segregation of mineral phases. Passive or active loading over time degrades the shear strength of foliation interfaces, the movement of which, causes creep of the entire rock mass.

Figure 22: Banded foliation in a gneiss

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Shale (Figure 23)

Shale is a fine-grained, clastic sedimentary rock composed of mud that is a mix of flakes of clay minerals and tiny fragments (silt-sized particles) of other minerals, especially quartz and calcite. The ratio of clay to other minerals is variable.Shale is characterised by breaks along thin laminae or parallel layering or bedding less than one centimetre in thickness, called fissility.Mudstones, on the other hand, are similar in composition but do not show the fissility.

Figure 23: Shale exhibiting fissility

In South African mines, various examples of significant creep are associated with the Khaki Shale overlying the Basal Reef in the Free State goldfields. Shale's are normally soft and deform easily. Time dependant creep deformations result in the reduction in sizes of excavations and clearance problems.

Mylonite (Figure 24)

Mylonite is fine grained foliated rock found in narrow planar zones of localised ductile deformation i.e. fault infilling. Mylonite is therefore a very soft material that has a tendency to creep when loaded for long periods of time. From the photo below of Mylonite it can be seen that the rock sample has weak layers.

Figure 24: Mylonite within the contact surfaces of a fault

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High frequency bedded rocks

Rocks with high frequency bedding are prone to creep when subjected to sustained loading which is a manifestation of the beds sliding or shearing across each other. The application of bed classification schemes can provide an indication of the ability of a bedded rock mass to creep.

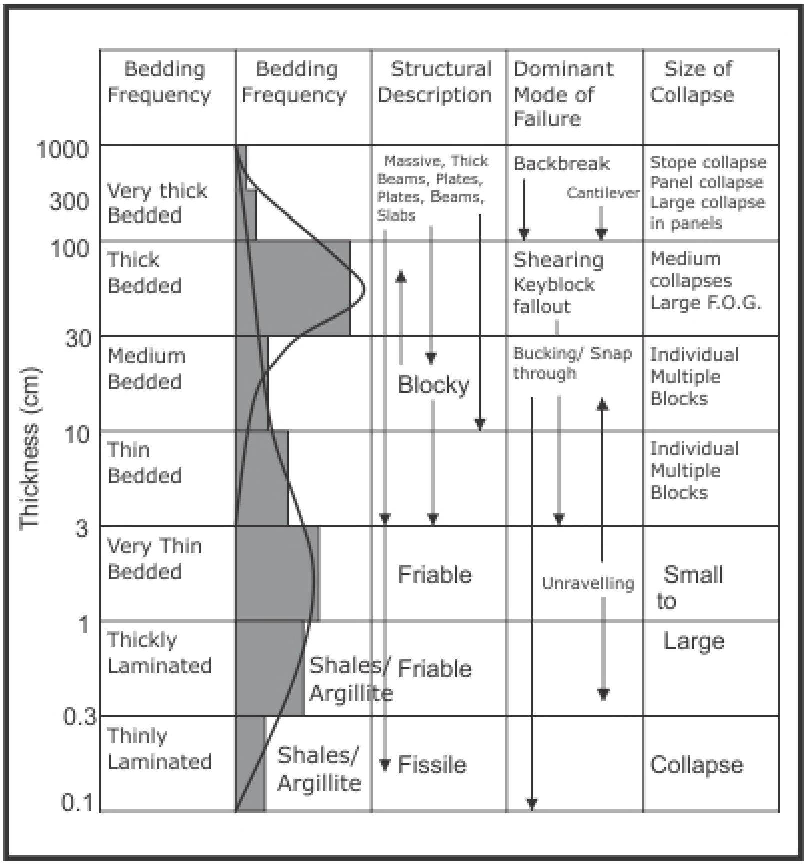

Figure 25: Typical bed classification schemes

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CHAPTER

4

ROCK STRENGTHS

INTRODUCTION

During the discussion of constitutive relationships (Chapter 3), the failure of rocks was discussed briefly as part of the behaviour of the rock materials under stress. However, rock failure is critical to the rock engineer as it provides an understanding of the limits to which materials can be stressed and which should be taken into account during design phases. It was also noted that jointing affects failure in rock mass, requiring a more detailed look at the strengths of joints.

This chapter leads the rock engineering practitioner to understand the failure of rock material.

LEARNING OUTCOMES

The candidate must be able to demonstrate knowledge and understanding of the above subject area by being able to:

4.1 ROCK PROPERTIES

4.1.1 Identify, describe and explain the possible modes of failure of rock in uni-axial compression
4.1.2 Describe and explain the effect of orientation on strength results for anisotrophic rocks such as shales
4.1.3 Sketch, describe and explain the complete axial stress-strain graph for rock in uni-axial compression
4.1.4 Sketch, describe and explain the complete axial stress-strain graph for rock in tri-axial compression
4.1.5 Indicate the following for the above two cases :

- the associated radial strain
- the associated volumetric strain
- the associated brittle-ductile transition
- the associated residual strength
- hysteresis in the post peak portion of the curve

4.1.6 Describe and explain brittle behaviour of rocks based upon Griffith's theory
4.1.7 Explain the basis for the Griffith criterion
4.1.8 Explain the basis for the Coulomb criterion
4.1.9 Identify, describe and explain the assumptions of the Coulomb criterion
4.1.10 Describe and explain the orientation of the failure surface predicted by the Coulomb criterion
4.1.11 Describe and explain how the cohesion and friction angle required by the Coulomb criterion may be determined from rock tests
4.1.12 Explain the basis for the Hoek and Brown criterion
4.1.13 Describe and explain how to apply the Hoek and Brown criterion to predict rock failure

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4.1.14 Describe and explain how the 'm'-parameter for intact rock required for the Hoek and Brown criterion may be obtained
4.1.15 Sketch, describe and explain :

- The stress-strain behaviour of brittle rock in uni-axial compression
- The effect of confinement on rock strength
- The effect of confinement on the stress-strain behaviour of rock samples
- What happens to rock after it has reached its peak strength
- 4.1.16. Sketch, describe and explain :
- Rock fracture, brittle failure
- Ductile deformation, yield
- Peak strength, residual strength
- Effective stress, pore pressure
- The volume effect on the strength of intact rock.

4.2 JOINT STRENGTH

The candidate must be able to demonstrate knowledge and understanding of the above subject area by being able to :

4.2.1 Represent and describe the occurrence of rock jointing, the number of joint sets and their orientations
4.2.2 Describe and explain the effect of friction angle on joint strength
4.2.3 Describe and explain the effect of infilling on joint strength
4.2.4 Describe and explain the effect of water on joint strength
4.2.5 Sketch, describe and explain the typical shear resistance-displacement behaviour of smooth joints
4.2.6 Sketch, describe and explain the typical shear resistance-displacement behaviour of rough joints
4.2.7 Sketch, describe and explain the effect of surface roughness on joint shear strength
4.2.8 Compare and contrast typical shear resistance versus shear displacement behaviour of smooth and rough joints
4.2.9 Describe and explain the effect of increasing the normal stress on the behaviour of joints under shear loading
4.2.10 Describe and explain the concept of roughness angle and its effect on initial shear resistance according to Patton's theory
4.2.11 Describe and explain the effect of surface roughness on dilation during shear on a joint
4.2.12 Describe and explain the effect of the above on rock stability in unconfined situations
4.2.13 Describe and explain the effect of the above on rock stability in confined situations
4.2.14 Apply Barton's equation to estimate the peak strength of joints
4.2.15 Describe and explain the difference between joints, fractures and cracks.

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REFERENCE

- Page 13 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 9, 10, 157 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 7 - Rock mechanics in coal mining, 1976, MDG Salamon and KI Oravecz

ROCK STRENGTH

LEARNING OUTCOME 4.1.1

IDENTIFY, DESCRIBE AND EXPLAIN THE POSSIBLE MODES OF FAILURE OF ROCK IN UNI-AXIAL COMPRESSION

CONNECTION 8

See UCS tests, Paper 2 Guide, Outcomes 6.1.1.1, 6.1.1.2 and 6.1.1.3,

Failure of the rock sample placed in a uni-axial test can occur in either of the following two modes:

Indirect tension: If the platens are lubricated, confinement on the ends of the samples is nullified and the sample is allowed to radially strain along its whole length. Under these conditions, a specimen splits axially and near vertical fractures area formed. The 'extension' fractures relates to the fracturing seen on tunnel sidewalls, pillars and abutments in high stress environments (Point A, Figure 1).

Shear: When confinement of the sample ends (or the complete sample such as in a tri-axial test), one or more inclined 'shear' fracture planes are created and the sample fails by shear along these planes (similar to Point B, Figure 1).

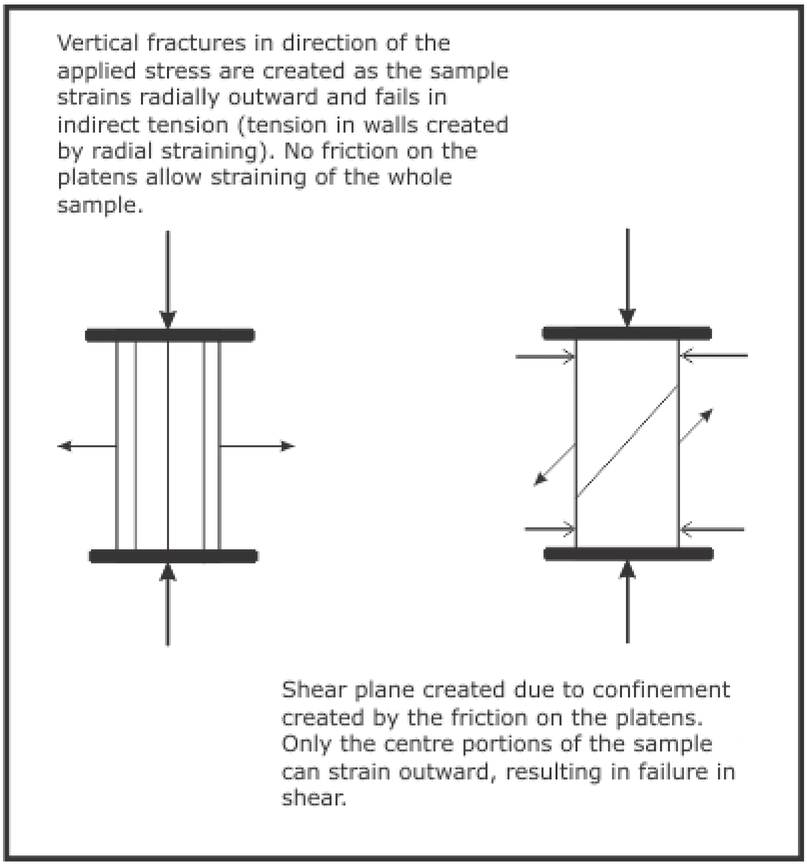

Figure 1: Failure modes in uni-axial compression

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REFERENCE

- Page 88 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 59 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

LEARNING OUTCOME 4.1.2

DESCRIBE AND EXPLAIN THE EFFECT OF ORIENTATION ON STRENGTH RESULTS FOR ANISOTROPHIC ROCKS SUCH AS SHALES

Anisotropic and foliated rocks that are dominated by closely spaced planes of weaknesses such as slates, schists and phyllites, give large discrepancies in uni-axial compressive strength results when tested at different directions to the planes of weaknesses.

INTERESTING INFO

Salcedo (1983) published the results of a set of directional uni-axial compressive tests on a graphitic phyllite from Venezuela. These results are shown in Figure 2 below. It is clear that the uni-axial compressive strength of this material varies by a factor of approximately 5, depending upon the direction of loading compared to the foliation orientation.

Transversely isotropic rock masses do not have the same properties when tested in different directions across the material. Test results at various orientations across the sample will therefore have different results. Below are some examples of rock types that are anisotropic and have vastly different uni-axial strength results when tested in different directions to the planes of weaknesses.

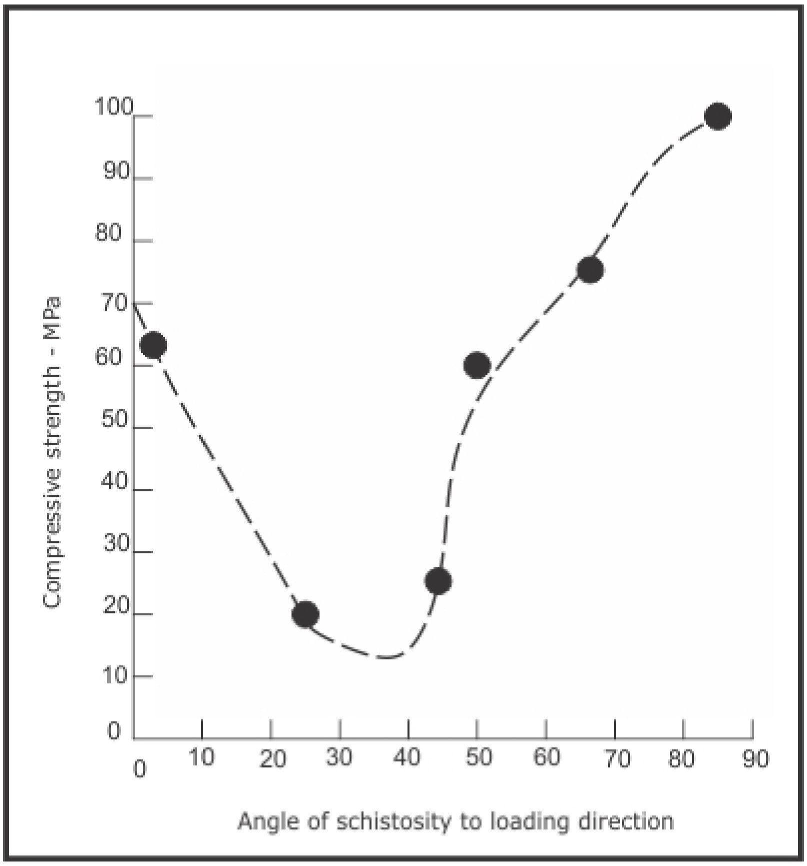

Figure 2: The effect of loading direction on the strength of graphitic phyllite(Salcedo 1983)

Strongly laminated rock
Strongly laminated rocks may have good rock strengths when tested at 90° to the laminations but are extremely weak when tested in the direction of the laminations

Well bedded rocks
Well bedded rocks may also have larger rock strengths when tested at 90° to the bedding.

Rocks with preferential mineral alignments
Schistosityis defined as "the foliation in schist's or other coarse grained rock due to the parallel, planar arrangement of mineral grains of the platy, prismatic, or ellipsoidal types, usually mica. This foliation results in the weakening of the rock mass when tested at different orientations to the preferential mineral alignments.

Gneissocity or gneissic structure can be defined in the following manner: "In a metamorphic rock, commonly gneissocity or gneissic, textural lineation or banding of the constituent minerals into alternating silica and mafic layers having different material properties and rock strengths". For this reason the test results in different orientations to the lineation or banding will reveal vastly different results

CONNECTION 9

Using one of the failure modes discussed in "LEARNING OUTCOME 4.1.1", it is easy to show that testing of material samples with loads perpendicular to already existing weakness planes (weakness planes as discussed above) within the sample will result in higher strength results than when the applied load is orientated parallel to the weakness planes.

In an effort to describe this, load orientations are shown in Figure 3 below to be either parallel to the weakness planes or perpendicular to them. If the sample is loaded and indirect tension is created in the straining walls of the sample, the sample with weaknesses parallel to the applied load will allow outward straining when the tensile stress overcomes the tensile strength of the weakness plane. Failure loads will now be determined by the tensile strength of the weakness planes and not that of the rock material. When the loads are applied perpendicular to the weakness planes, new fractures (vertical) have to be created due to the indirect tension created before the sample will fail.

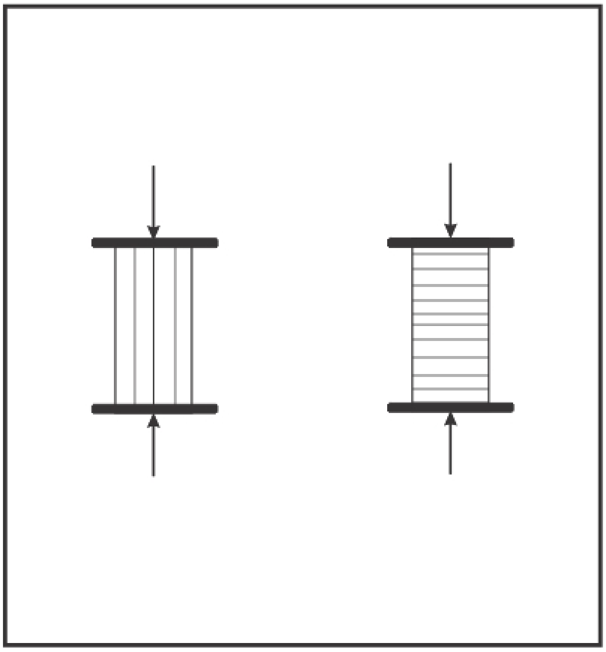

Figure 3: Testing parallel and perpendicular to inherent material weakness planes

REFERENCE

- Page 103,137 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 15 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 8 Chapter 11 - Practical rock engineering, 2006, E Hoek
- Page 157 - Underground excavations in rock, 1980, E Hoek and ET Brown
- Page36,117 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 4.1.3

SKETCH, DESCRIBE AND EXPLAIN THE COMPLETE AXIAL STRESS-STRAIN GRAPH FOR ROCK IN UNI-AXIAL COMPRESSION

Figure 4 shows a typical axial stress-strain graph for rock under uni-axial compression.

NOTE

Both the axial and the radial strains can be recorded during a typical uni-axial loading test.These tests are called UCM tests and are uni-axial compressive tests (UC) 'with the measuring of strain to allow the derivation of moduli' (M).

From such an uni-axial compressive test result, the following are determined:

Elastic range and yield stress
The elastic range is the initial portion of the stress-strain curve where the strain is fully recoverable (line segment E - A on Figure 4). Yield stress is the stress at which the stress/strain graph deviates from the elastic behaviour during strain hardening and strain becomes non-recoverable (point A in Figure 4).

Peak strength
The peak strength or uni-axial compressive strength is the highest point on the stress/strain graph. The peak strength is at point C on the graph in Figure 4 below

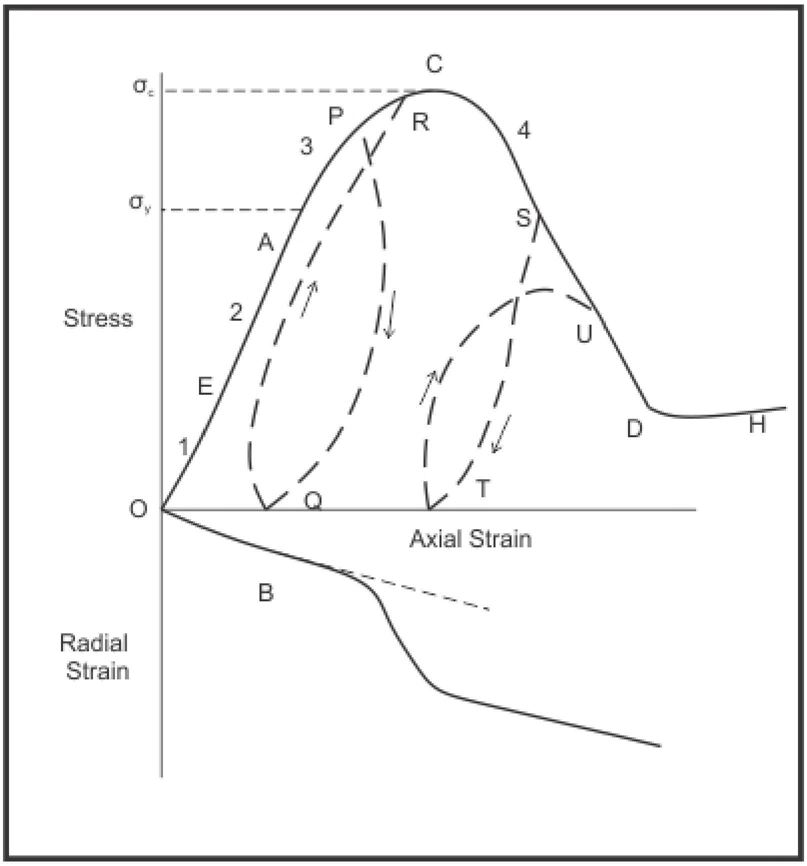

Figure 4: Stress-Strain graph under uni-axial loading conditions

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Strain softening
The line segment between C and D in Figure 4 shows strain softening of the test sample.

Residual strength
The residual strength on the test graph in Figure 3 above is the line segment D to H where the stress remains constant with increasing strain.

Irrecoverable strain
When a test sample is loaded past its yield point and then unloaded, some irrecoverable strain occurs (ε_o at point Q, Figure 4).

Dilation
Dilation (point B, Figure 4) occurs when the radial : axial strain relationship deviates from linearity. Hysteresis loading and unloading cycles (STU and PQR)

Poisson's ratio
Poisson's ratio is defined as the ratio of the transverse (radial) strain to axial strain
We can calculate the Poisson's ratio of the example shown below in Figure 5 by using the equation below.

$$v = -\varepsilon_{trans}/\varepsilon_{axial} = -\varepsilon_x/\varepsilon_y$$

where the (-) sign ensures that since one of the strains measured will be 'expansion' and therefore negative (-), the Poisson's ratio will be positive.

NOTE

The Poisson's ratio can also be calculated using an 'absolute value' sign, forcing all negative values to become positive, ensuring a positive value for Poisson's ratio.

BRIDGING GAP 6
"Absolute Values" on page 230

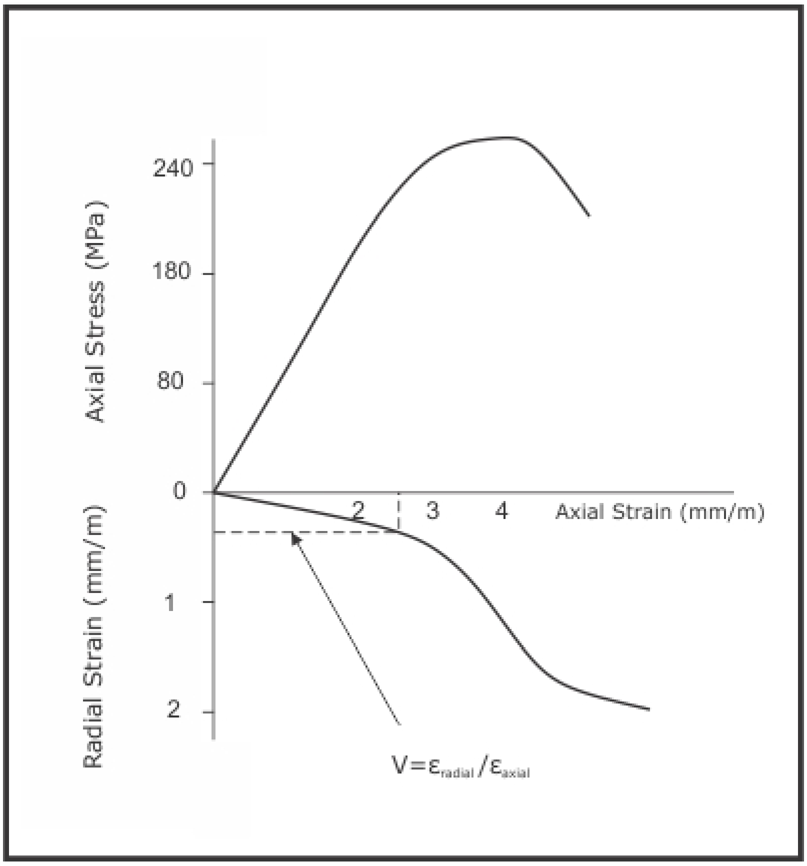

Figure 5: Poisson's ratio

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Secant modulus of elasticity (E_{sec}) is the slope (σ/ε) of a line connecting the origin and a given point on the stress-strain curve (Figure 6).

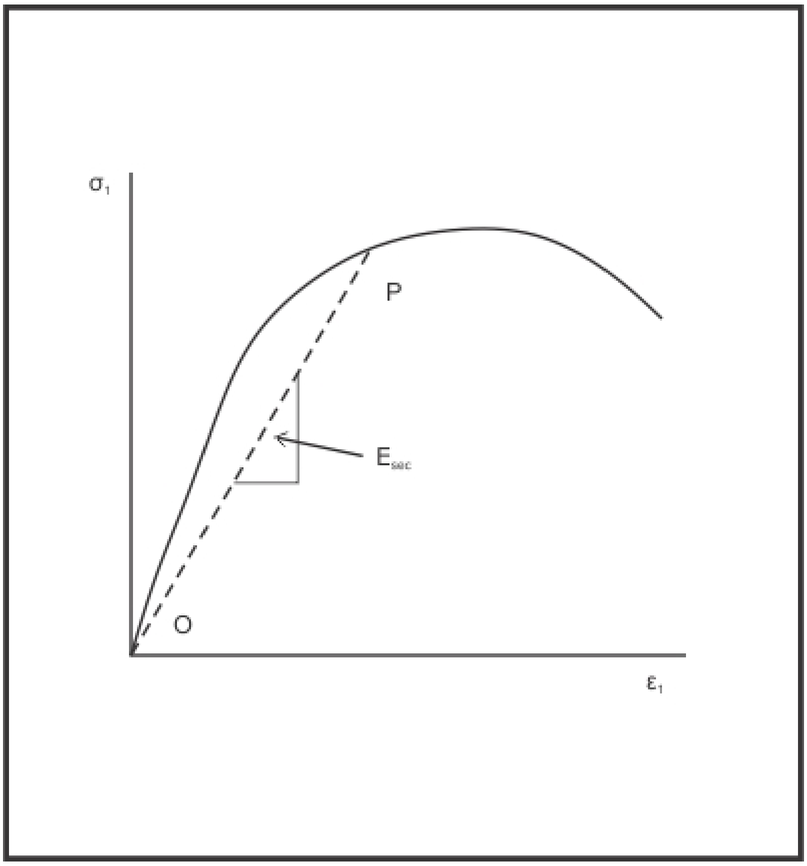

Figure 6: Secant modulus

Tangent modulusof elasticity (E_{tan}) is the slope ($d\sigma/d\varepsilon$) of the line that is drawn tangent to the curve of stress versus strain at a given stress value (Figure 7).

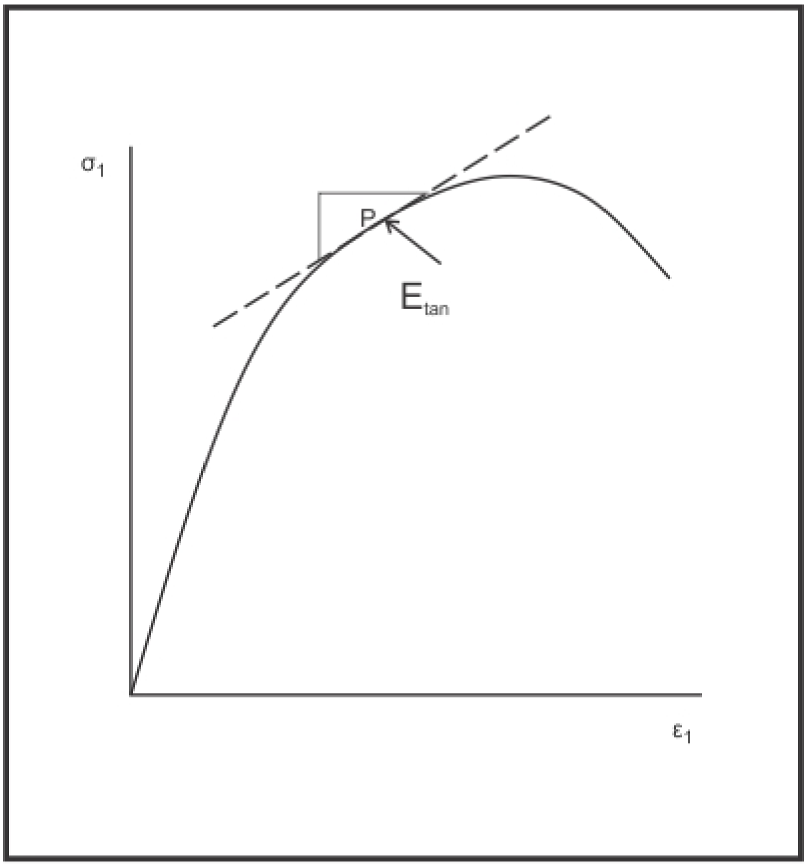

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Figure 7: Tangent modulus.

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REFERENCE

- Page 60,81 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 82 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 97 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 4.1.4

SKETCH, DESCRIBE AND EXPLAIN THE COMPLETE AXIAL STRESS-STRAIN GRAPH FOR ROCK IN TRI-AXIAL COMPRESSION

CONNECTION 10

See shear box tests, Paper 2 Guide, Outcome 6.1.1.5 and 6.11.6

'Tri' means three. A tri-axial test means all three sides (directions) of a sample are being loaded.In this type of test, a rock sample is prepared and placed in the testing machine and is then pressed from all three direction. The stresses applied to the specimen are recorded when the sample fails. A tri-axial cell that can be used for this test is illustrated in Figure 8 below.

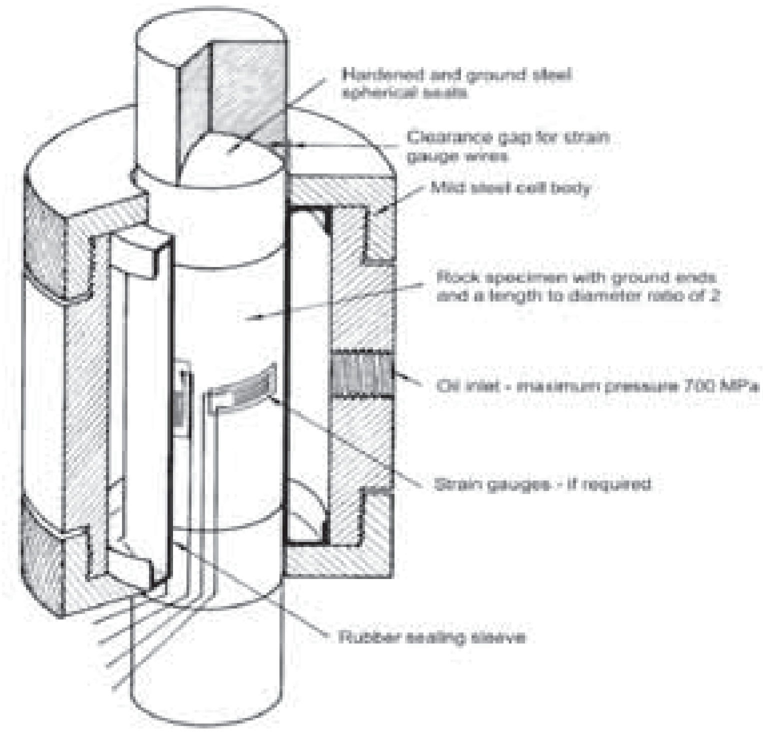

Figure 8: Cut-away view of a tri-axial cell for testing rock specimens (Hoek's notes)

When a test sample is confined its strength increases with an increase in confining stress (Figure 8).

In Figure 9 the confining stresses(σ2 and σ3) increased from3 MPa, 7 MPa and 14 MPa resulting in an increase in the peak and residual strengths increases. The initial part (elastic) of the graphs are not affected by the confinement stresses, but the yield point (where inelastic behaviour commences), peak and residual strengths are affected. It is important to note that the post-failure curves or behaviour (past point A, Figure 9) are also affected by the confinement stress levels, indicating that increased confinement could change brittle rock behaviour into ductile behaviour (Figure 10).

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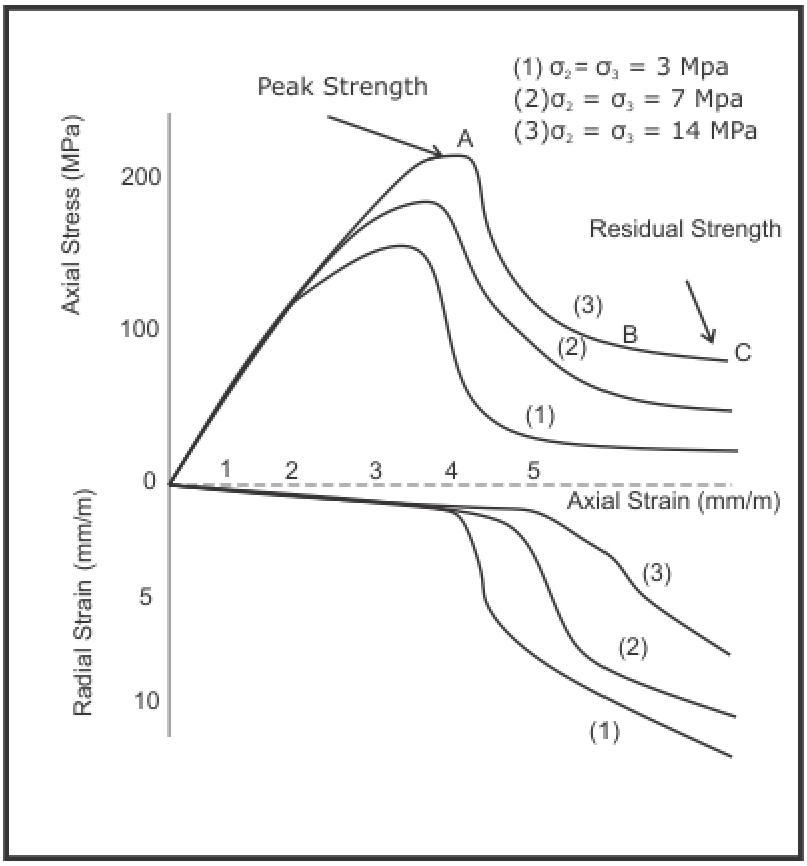

Figure 9: Complete (tri-axial) axial stress vs axial strain radial curves

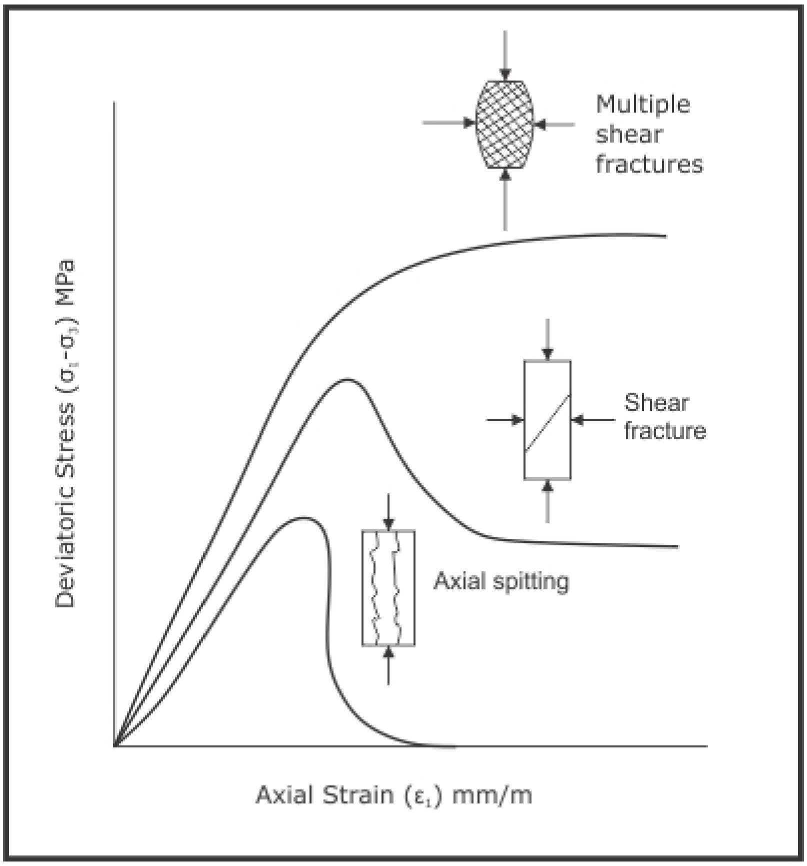

Figure 10: Change in post failure behaviour with an increase in confining pressure (Ryder and Jager)

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CONNECTION 11

An increase in confinement during the test also changes the failure mode on the sample, as shown in Figure 10. Lower confinement allows indirect tensile failure, while high confinement causes the creation of multiple shear fractures within the sample. “LEARNING OUTCOME 4.1.5” on page 104.

Peak strength
The peak strength is the highest stress sustained by the sample during a test and can readily be read from the stress/strain graph.

Residual strength.
The residual strength is the line segment B to C in Figure 9, where the stress remains constant with increasing strain.

REFERENCE

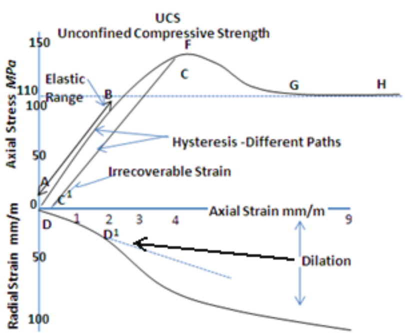
- Page 85, 100 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman

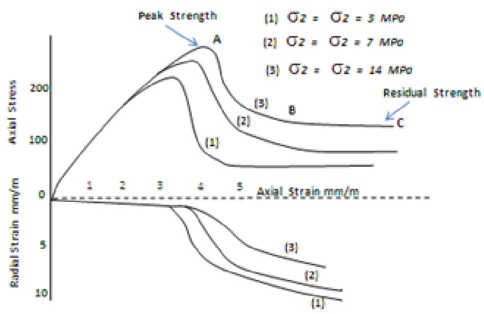
LEARNING OUTCOME 4.1.5

INDICATE THE FOLLOWING FOR THE ABOVE TWO CASES:

- the associated radial strain
- the associated volumetric strain
- the associated brittle-ductile transition
- the associated residual strength
- hysteresis in the post peak portion of the curve

Radial strain (UCS)	Radial strain (Tri-axial)
Under uni-axial compression conditions, the radial strain is not limited by confinement stresses. The rock test specimen therefore deforms easily and fails when the stress exceeds the strength. Once the rock specimen has been stressed past its elastic zone the radial strain increases rapidly ,indicating dilation.	Under tri-axial conditions, an increase in confinement results in a stronger rock sample. Radial deformations are therefore reduced as they are inhibited by the confinement stresses. Once the rock specimen has been stressed past its elastic zone the radial strain increases rapidly but follows different paths depending on the confinement stress





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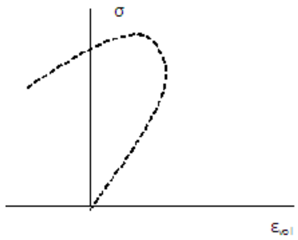
Volumetric strain (UCS)

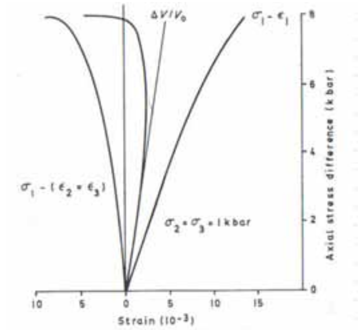
Volumetric strain is the change of volume per unit volume, given by $\varepsilon_v=\Delta V/V_O$ and is therefore the change in volume divided by the original volume of the rock specimen. When a sample is compressed, its volume decreases (positive ε_{vol}) and then starts to increase when inelastic behaviour commences, leading to a sample with increased volume (negative εvol) which can not sustain the high loads anymore (drop in σ).



Volumetric strain (Tri-axial)

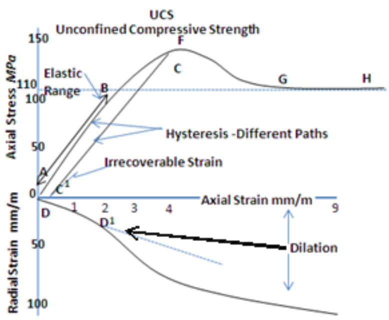
The graph above shows the axial stress-volumetric strain behaviour of Westerly granite in a tri-axial test at a confining pressurre of 1kbar. Under tri-axial compression the volumetric strain is the change in the volume under loading divided by the original volume for each loading condition. The volumetric strain under tri-axial loading is the sum of the principal strains for each loading condition.
Eg. $\varepsilon_{v1}=\varepsilon_1+\varepsilon_2+\varepsilon_3=\Delta V/V_O$

Page 84 Fundamentals of Rock Mechanics –JC Jager and NGW Cook.



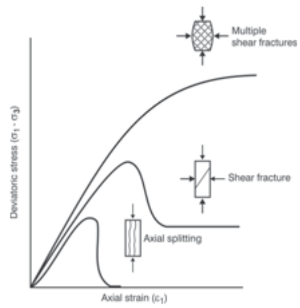
Brittle-ductile transition (UCS)

Under uni-axial compression conditions behaviour for most rock material is brittle (line segment FG) and a sudden loss of ability to sustain load is reported after the failure points (point F) have been reached.



Brittle-ductile transition (Tri-axial)

Under tri-axial compression conditions the brittle-ductile transition is affected by the confinement stresses. The line segment AB reports a brittle behaviour at no or little confinement and this behaviour changes as the confinement stresses increase towards a more ductile behaviour.

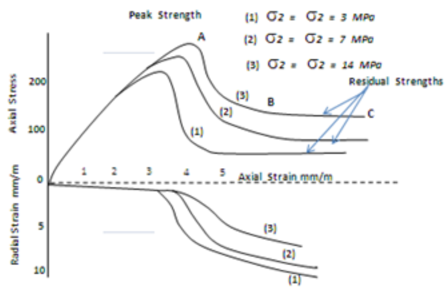


Residual strength (UCS)

Under uni-axial compression conditions the residual strength is shown as the line segment G to H above. This does not occur when testing strong brittle rock and is therefore mostly observed in soft rocks.

Residual strength (Tri-axial)

Under tri-axial conditions with an increase in confinement the rock test specimen becomes stronger. The residual strengths are shown as the line segments B to C above where test 3 having the highest confinement stress of 14 MPa has the highest residual strength.



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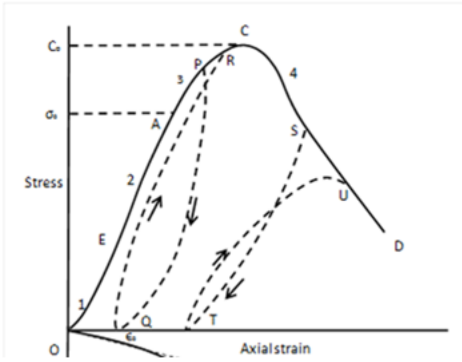
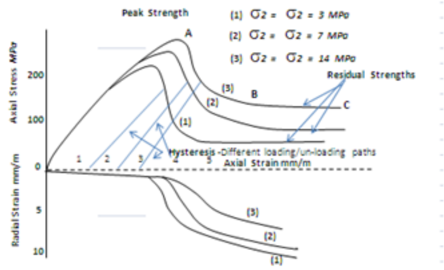
<table>
<tr>
<th>Hysteresis (UCS)</th>
<th>Hysteresis (Tri-axial)</th>
</tr>
<tr>
<td>Hysteresis in the post peak portion occurs when the stress is relieved at any position between line segmentC to D. In such a case the loading and unloading paths followed are different but re-loading would still approximate the original stress-strain graph. Non-recoverable strain is reported in post yield point and post failure hysteresis

</td>
<td>Hysteresis in the post peak portion of various tests will follow different paths if the unloading cycle was initiated at the same stress level. As the confinement level increases, hysteresis will indicate the occurrence of larger portions of non-recoverable strain.

</td>
</tr>
</table>

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- Page 59 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 133 - Underground excavations in rock, 1980, E Hoek and ET Brown
- Page 103 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 4.1.6

DESCRIBE AND EXPLAIN BRITTLE BEHAVIOUR OF ROCKS BASED UPON GRIFFITH'S THEORY

INTERESTING INFO

The Griffith failure criterion was introduced by Griffith in 1921 after a series of experiments on glass rod fracturing. His hypothesis was that materials are pervaded with microcracks that can grow when the stresses at crack tips are favourable for growth. The theory explained why materials often failed at lower stresses than allowed for if the solid was unflawed.

The criterion suggests that failure occurs at the tips of the critically orientated cracks, failure of the entire sample occurs at once, suggesting extreme brittle behaviour (sudden and complete loss of ability to sustain loads) at the onset of damage within the sample. Since it is known that in compressive stress environments, micro-fracturing commences when the stress levels exceed the yield point of the material and result in yielding (plastic behaviour) of the sampling prior to failure when the peak strength is exceeded, Griffith's theory appears to be more appropriate in explaining the initiation of damage (yielding) within the sample rather than complete failure of, or the failure mode of the material.

REFERENCE

- Page 307,314 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 75 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 40 - Rock mechanics in mining practice, 1983, S Budavari et al.
- Page 24 - Handbook on hard rock strata control, 1995, AJS Spearing
- Page 107 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

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LEARNING OUTCOME 4.1.7

EXPLAIN THE BASIS FOR THE GRIFFITH CRITERION

Griffith's original study was aimed at explaining specifically tensile failures (Figure 11a). An isolated 2D open crack of flattened elliptical cross-section was analysed (Jaeger & Cook 1979). Critical values of crack orientation and position of incipient failure near the crack tip where the skin stresses are mostly tensile were established mathematically.

The resulting failure criterion has two sections (Figure 11c): a straight line 'tension cutoff' portion passing through the uni-axial tension point at $-\sigma_t$, and a parabolic portion which is tangent to this line and which cuts the σ_1 axis at $8\sigma_t$.

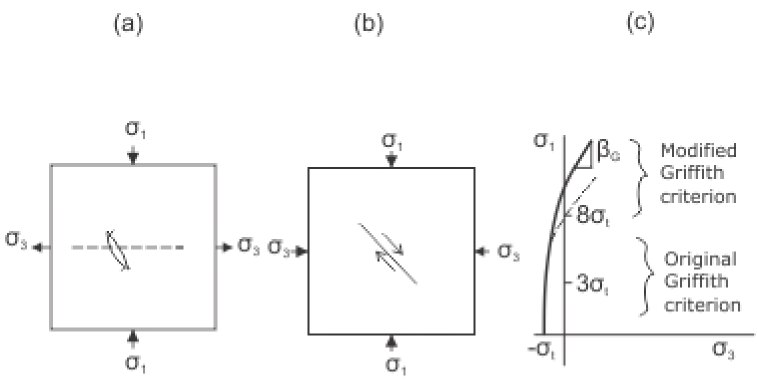

Figure 11: Griffith Criterion (Ryder and Jager)

Griffith suggests that failure takes place:

- In direct tension when the major principal stress is not significantly larger than the confinement stress ($\sigma_1+3\sigma_3<0$), and is then based on the existence of a tensile confining stress ($\sigma_3=-T_o$) that exceeds the tensile strength of the material and

- In compression (or indirect tension) if the major principal stress is significantly larger than the confinement stress ($\sigma_1+3\sigma_3>0$).

The Griffith theory predicts a parabolic concave-down failure surface that can be presented in shear and normal stress space, given by the equation: (Refer to figure 12)

$$\tau^2=4T_o\,(\sigma_n+T_o)$$

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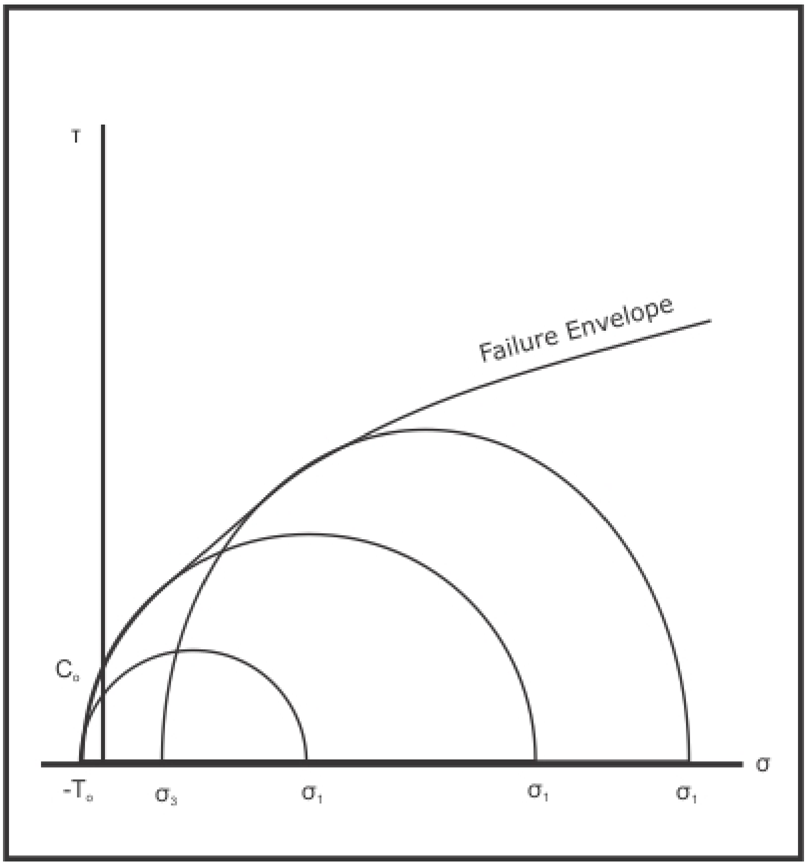

Figure 12: Parabolic failure envelope predicted by Griffith criterion

The above failure envelope can not be accurately fitted to rock failure behaviour for the following reasons:

- Best-fit envelopes to rock strength data are typically concave downwards, but this concavity lies somewhere between a straight line and a parabola.
- The Griffith criterion is based on the physical process of crack opening and extension. However, closing of cracks, crack shear under compression and crack-crack interactions are not accounted for.

REFERENCE

- Page 307,314 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 75 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 40 - Rock mechanics in mining practice, 1983, S Budavari et al.
- Page 24 - Handbook on hard rock strata control, 1995, AJS Spearing
- Page 107 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 4.1.8

EXPLAIN THE BASIS FOR THE COULOMB CRITERION

In 1773 Coulomb postulated that the shear force required to cause slip on a plane (i.e. shear strength of the plane) in a solid material is governed by the cohesive force of the plane and the normal force acting on it, i.e. $|F_s|=C_o+\mu F_n$

where F_s is the absolute value of the shear force necessary to cause slip; C_o is the cohesive force on the plane; μ is the coefficient of friction on the

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plane; and F_n is the normal force acting on the plane (compressive force positive).

NOTE

The absolute value sign around F_s indicates that the shear strength is non-directional and that shear stress acting in any direction on the plane would need to overcome the shear strength value to allow slip. If F_n is negative (i.e. tensile force openingthe plane), the shear strength would be lower and shear would be easier along the plane (until F_n becomes large enough tensile to overcome the cohesion on the plane, at which point the shear strength would be zero).

Figure 13 shows details of the failure model. The sense of the shear force (whether it makes the plane slip to the right or the left) is not important; only its magnitude is taken into account.

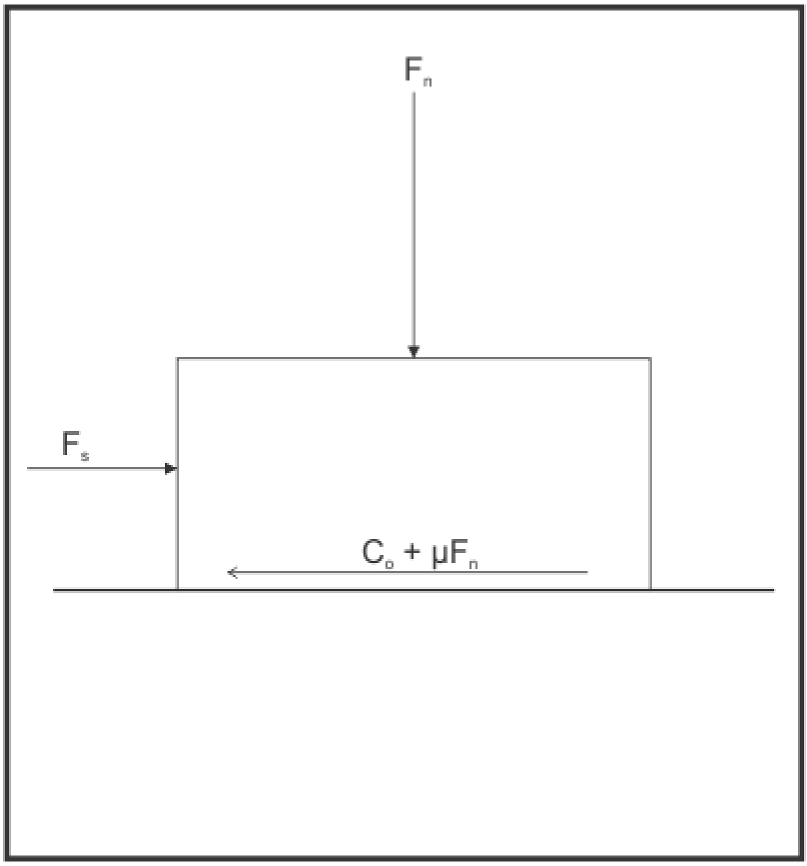

Figure 13: Details of the failure model

CONNECTION 12

Mohr circle discussed in "LEARNING OUTCOME 2.9" on page 47

In 1900 Mohr proposed that when shear failure takes place on a plane in a solid, the stress state in the solid at the time of failure is given by $|\tau|=f(\sigma)$. This was based on his geometrical representation of stress known as the Mohr circle, forming the basis for the failure criterion.

The Mohr failure envelope is shown graphically in Figure 14.

NOTE

'Failure envelope' refers to the line on a graph that separates states of stress that cause 'failure' of the rocktype from 'no failure'. In Figure 14, any state of stress (plot a point that represents a ($\tau;\sigma$)state or draw a Mohr circle that represents a ($\sigma_1;\sigma_2$)state) that is above the envelope or intersects the envelope, will suggest failure of the rock sample, while any state of stress below the envelope or that does not intersect the envelope will suggest non-failure of the sample.Smaller and larger combinations of stresses can give rise to circles that touch the failure envelope at other points, i.e. there is an infinite number of special stress combinations that can lead to failure.

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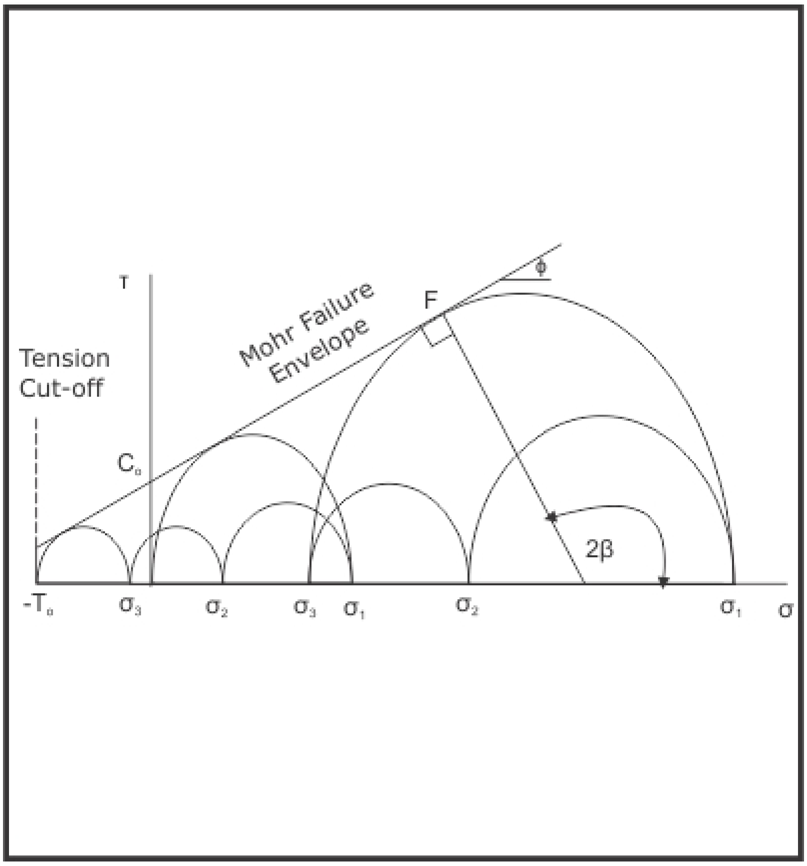

Figure 14: Mohr circles with failure envelope

The failure envelope is shown to be a straight line, represented by $\tau = C_o + \sigma_n \, Tan\Phi$ where $\mu = Tan\Phi$ is the coefficient of friction. The relationship $\tau = C_o + \mu\sigma_n$ therefore represents a linear Mohr-type relation in shear stress: normal stress space, and is therefore also called the 'Mohr-Coulomb' criterion after Coulomb's contribution. This criterionalso translates into a linear relation $\sigma_1 = \sigma_c + \beta\sigma_3$ in the principal stress space (Figure 15).

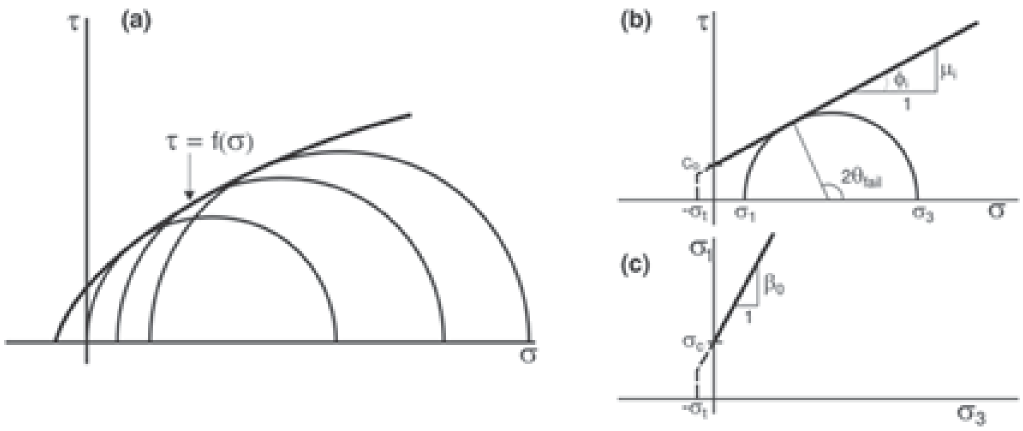

Figure 15: a): Mohr's hypothesis, b): Mohr-Coulomb in shear-normal stress space, c): Mohr-Coulomb in principal stress space (Ryder and Jager)

From the linear relationship, the following useful inter-relationships are derived:

$$\sigma_c = 2C_o\sqrt{\beta_o}, \quad \beta_o = \frac{(1 + \mathrm{Sin}\phi_i)}{(1 - \mathrm{Sin}\phi_i)}, \quad \mathrm{Sin}\phi_i = \frac{(\beta_o - 1)}{(\beta_o + 1)}, \quad C_o = \frac{\sigma_c}{\sqrt{\beta_o}}$$

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When a near-linear strengthening relationship applies in a $\sigma_1:\sigma_3$ plot, it is not then necessary to construct a set of Mohr circles. It is acceptable to establish a best-fit straight line in the $\sigma_1:\sigma_3$ plot and read off the values of the σ_1 axis intercept σ_c and the slope β_o to calculate the angle of internal friction Φ_i and cohesion C_0 from the relationships above.

In using the Mohr-Coulomb criterion, a 'tension cut-off' at -σt (dashed line in Figure 15c) is introduced to improve the description of the low tensile strength of rocks, as the linear failure envelope overestimates the tensile strength (intersection with normal stress axis).

REFERENCE

- Page 40 - Rock mechanics in mining practice, 1983, S Budavari et al
- Page 90,94 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 67 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 170 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 18 Chapter 11 - Practical rock engineering, 2006, E Hoek
- Page 22 - Handbook on hard rock strata control, 1995, AJS Spearing
- Page 105 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 4.1.9

IDENTIFY, DESCRIBE AND EXPLAIN THE ASSUMPTIONS OF THE COULOMB CRITERION

The Coulomb failure criterion has the following implications and assumptions:

- The value of the intermediate principal stress σ_2 does not affect failure but is a function of the major (σ_1) and minor (σ_3) principal stresses only;
- C_o and μ are interpreted as the only material properties, namely the cohesive strength of the material and angle of internal friction respectively, that affect failure and
- The rock material has a tensile strength given by $-T_o$, which is interpreted as a tension cut-off.

CONNECTION 13

See "LEARNING OUTCOME 4.1.8" on page 108

CONNECTION 14

See "LEARNING OUTCOME 4.1.10" on page 112

No processes leading to failure take place in the rock or soil until the Mohr circle that represents the state of stress touches the failure envelope. At this stage the failure takes place instantaneously along a plane and in shear only;

Failure can only take place in shear on a plane that either develops in the solid material or precedes the loading (geological discontinuity) and is favourably orientated to permit shear failure.

REFERENCE

- Page 40 - Rock mechanics in mining practice, 1983, S Budavari et al
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- Page 67 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

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- Page 22 - Handbook on hard rock strata control, 1995, AJS Spearing
- Page 105 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 4.1.10

DESCRIBE AND EXPLAIN THE ORIENTATION OF THE FAILURE SURFACE PREDICTED BY THE COULOMB CRITERION

A literal prediction of Coulomb's analysis is that the failure rupture plane should be oriented in the plane of the intermediate principal stress σ_2 while the shear fracture forms an angle of θ_{fail}=45°-Φ/2 with the major principal stress σ_1 direction.

NOTE

The angle that the shear plane makes with the major principal stress can be given in two equal forms, θ_{fail}=45°-Φ/2 as indicated above or θ_{fail}=1/2(90°-Φ) (see Figure 16).

INTERESTING INFO

The orientation of this plane can be determined from the Mohr circle using the principles established in Outcome 2.1.9. If failure occurs when the circle representing a state of stress intersects the failure envelope, the line connecting the centre of the Mohr circle with the 'circle-failure envelope intersection point' indicates the orientation of the 'normal' to the failure plane from σ_1.

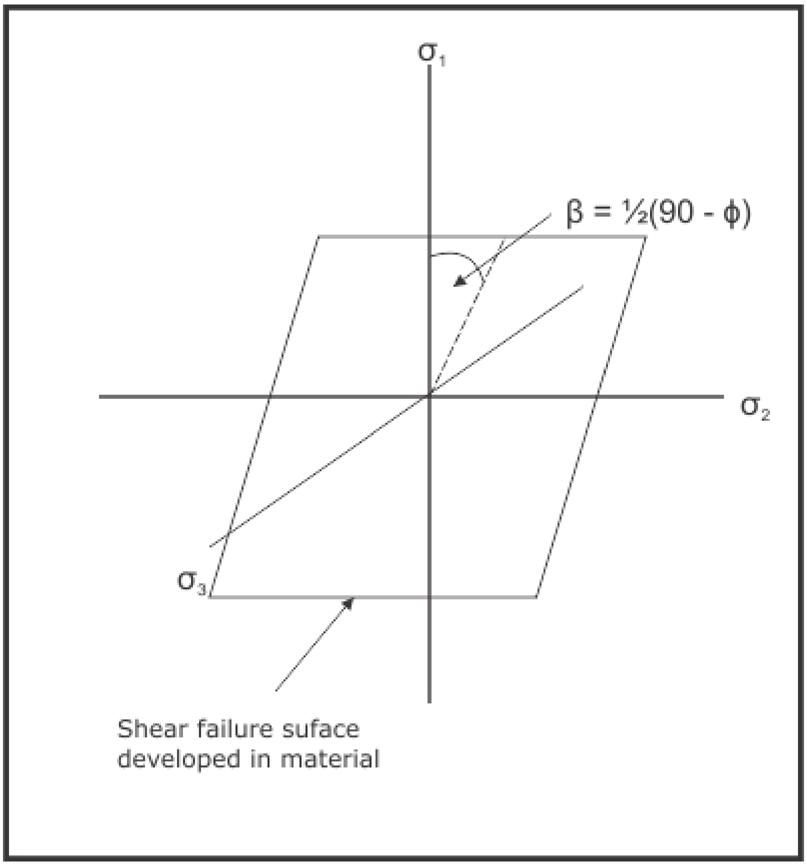

Figure 16: Geometrical relationship between principal stresses and shear rupture surface predicted by the Mohr-Coulomb failure criterion

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CONNECTION 15

Mohr circle discussed in "LEARNING OUTCOME 2.9" on page 47

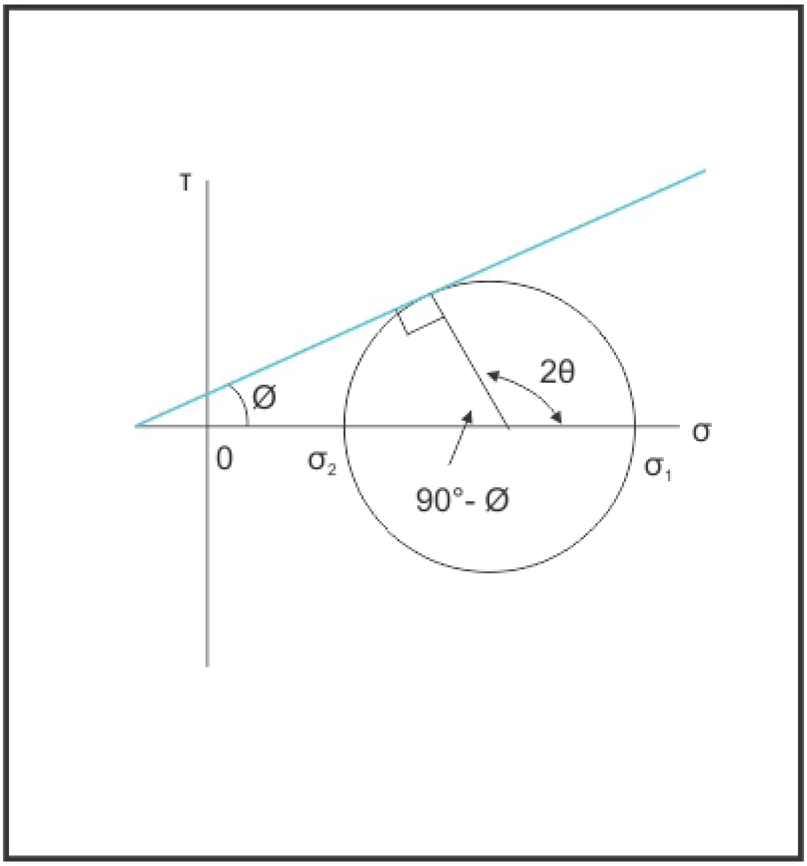

Figure 17: Stress state at failure (Mohr-Coulomb failure criterion)

According to Mohr circle theory, the angle between σ_1 and the normal to the plane on which failure takes place is 2θ (Figure 16). Since the failure envelope is orientated at an angle Φ to the normal stress axis, and the line representing the normal to failure plane is 90° to the failure envelope, it follows that

$$2\theta = 180° - (90° - \Phi)$$
$$= 90° + \Phi.$$
Solving for θ indicates that
$$\theta = ½\,(90°+\Phi) = 45° + \Phi/2$$

This is the angle between σ_1 and the normal to the plane.
The orientation of the plane itself from σ_1, is therefore 90° - (45°+Φ/2) = 45° - Φ/2.

- Page 91 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman

LEARNING OUTCOME 4.1.11

DESCRIBE AND EXPLAIN HOW THE COHESION AND FRICTION ANGLE REQUIRED BY THE COULOMB CRITERION MAY BE DETERMINED FROM ROCK TESTS

Tri-axial tests must be conducted on a number of rock samples during which the σ_1 and σ_3 relationships at failure are recorded. This data can be presented in two different stress spaces, namely the shear-normal stress space and the principal stress space.

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In the shear-normal stress space, the test results are presented in the form of a series of Mohr circles, plotting the stress levels as the circle intercepts on the normal stress axis (Figure 18a). An envelope (straight line touching all the circles) is drawn and extended to intersect the shear stress axis (Figure 18b). From this information, the angle of internal friction can be measured as the angle that the envelope makes with the normal stress axis. The cohesion can be read as the envelope intercept with the shear stress axis (Figure 18b).

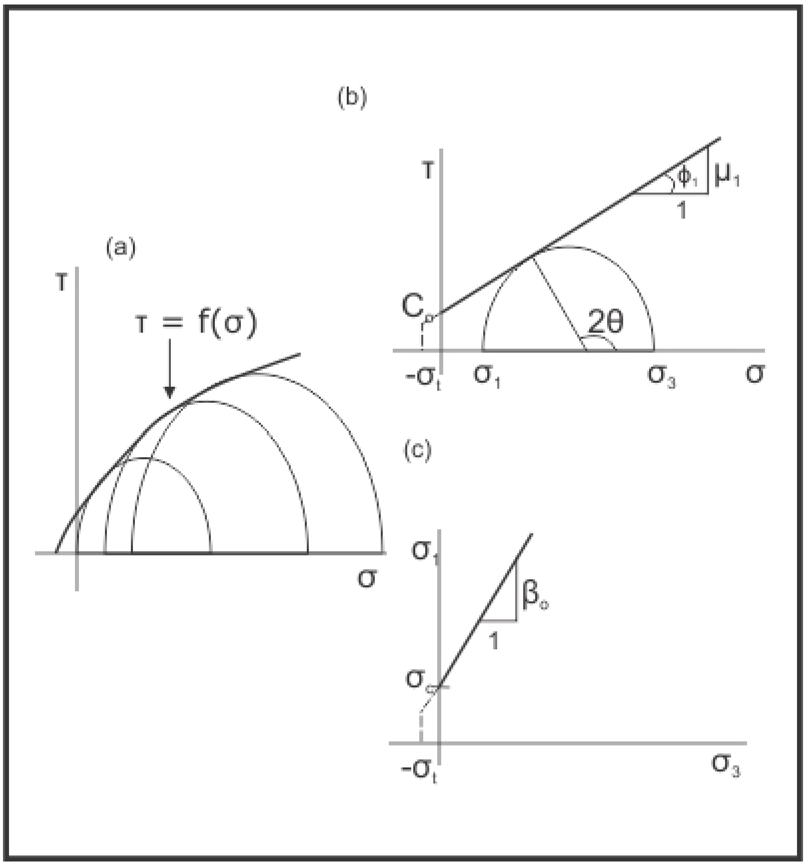

Figure 18: Deriving rock properties

When the test results are plotted in the principal stress space, it is not necessary to construct a set of Mohr circles but the $\sigma_1 : \sigma_3$ data sets are plotted as points within the principal stress space and a straight line is constructed through all the points (Figure 17c). The UCS of the material is determined by the σ_1 axis intercept while the slope β_o of the line is utilised to calculate the angle of internal friction Φ_i and cohesion C_0 from the relationships below.

CONNECTION

See "LEARNING OUTCOME 4.1.8" on page 108

$$C_0 = \frac{\sigma_c}{\sqrt{\beta_0}} \text{ and } \sin\phi_i = \frac{(\beta_0 - 1)}{(\beta_0 + 1)}$$

LEARNING OUTCOME 4.1.12

EXPLAIN THE BASIS FOR THE HOEK-BROWN CRITERION

Hoek and Brown recognised that there was no satisfactory failure criterion for rock and rock masses that satisfied the following requirements

- It should adequately describe the response of an intact rock sample to the full range of stress conditions likely to be encountered underground. These conditions range from uni-axial tensile stress to tri-axial compressive stress.

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- It should be capable of predicting the influence of one or more sets of discontinuities on the behaviour of a rock sample. This behaviour may be highly anisotropic, i.e. it will depend on the inclination of the discontinuities to the applied stress direction.
- It should provide some sort of projection, even if approximate, for the behaviour of a full-scale rock mass containing several sets of discontinuities.

The authors went on to introduce an empirical failure criterion that was sufficiently accurate for most underground excavation design processes. Using their experience in experimental and theoretical aspects of rock behaviour, they determined, by trial and error, an empirical relationship between the principal stresses associated with rock failure and presented it as follows:

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}$$

where σ_1 and σ_3 are the major and minor principal stresses respectively; σ_c is the laboratory-measured uni-axial compressive strength of the rock; and m and s are constants that depend on the properties of the rock and on the extent to which it has been broken before being subjected to the stresses σ_1 and σ_3.

Like the Mohr-Coulomb criterion, σ_2 is not relevant to failure, falling in magnitude between the maximum and minimum principal stresses. However, the Hoek-Brown criterion does not make any assumptions about the failure mechanisms that are responsible for the yield of rock under stress. It is a fairly general criterion that can be used for materials ranging from soils to hard rock in all stress conditions, thereby meeting the requirements set out by the authors.

Hoek and Brown (1980) presented a series of yield curves for different rock types, showing that there is reasonable agreement between laboratory results and fitted yield curves. In this way they were able to obtain ranges of values for m and s, and to compile these into a table for use by engineers to estimate rock mass strength for all stress states, given only σ_c and rock mass quality.

The increasing presence of fracture and weathering in the rock (i.e. a reduction in rock mass quality) brings about a reduction in s, while the rock type has an effect on the value of m. Figure 19 shows that an increase in the m value maintains a constant UCS but increases the strengthening parameter (slope of the failure envelope) and the compressive strength while decreasing the tensile strength of the rock. An increase in the s value maintains a constant strengthening parameter, but increases the expected UCS, tensile and compressive rock strength.

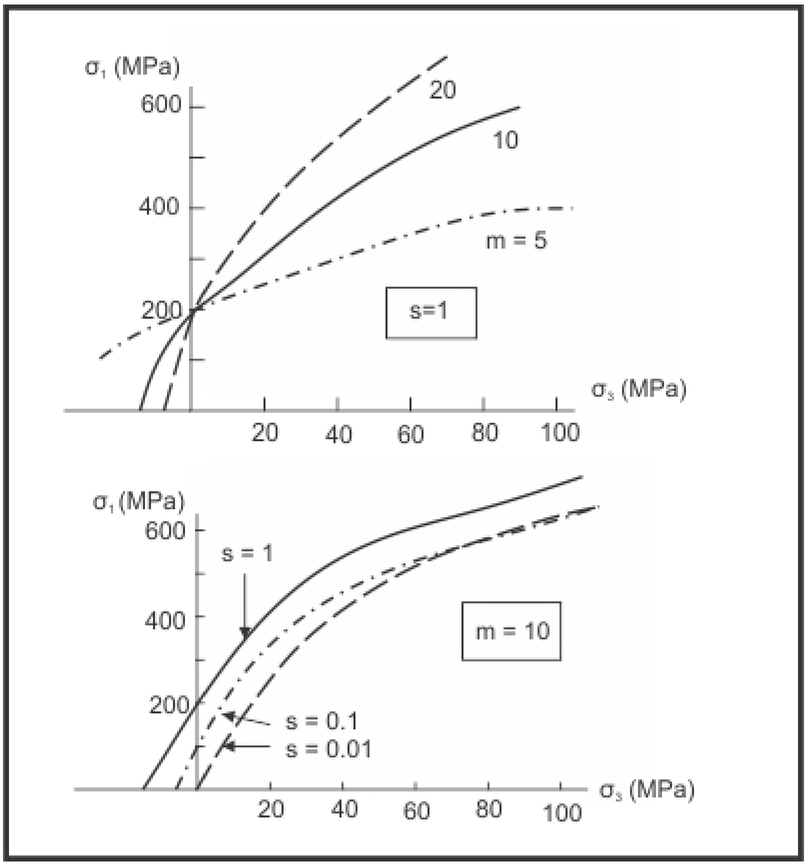

Figure 19: Properties of the Hoek-Brown strength criterion

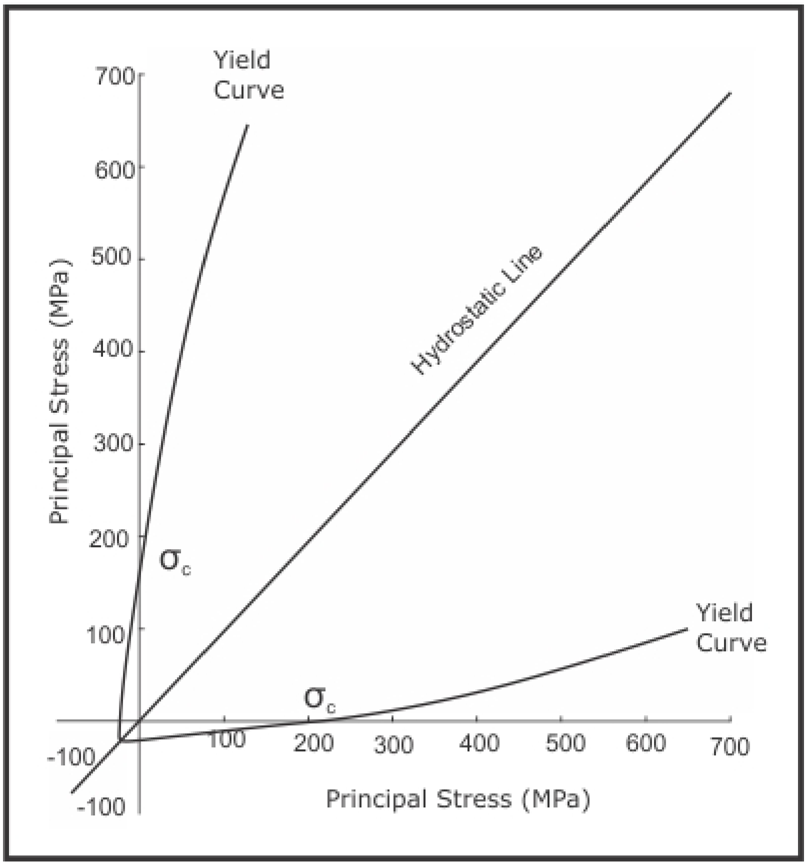

Figure 20: Hoek-Brown yield curve for m = 20, s = 1 and σ_c = 200 MPa

There are limitations to the use of the Hoek-Brown criterion, mainly because it assumes implicitly that rock masses have isotropic strength, i.e. the strength of the rock mass does not vary with direction. As a result, the failure criterion should not be used when the rock mass is anisotropic (fractures/joints not randomly orientated, e.g. in shales), or when the rock mass responds to loading anisotropically. However, it can be used in heavily jointed rock masses where joints/fractures are more or less randomly orientated, i.e. when the rock mass as a whole is isotropic. Under uni-axial conditions $\sigma_2 = \sigma_3 = 0$, the Hoek-Brown criterion simplifies to $\sigma_1=(\sqrt{s})\sigma_c$. The s parameter expresses the downgrading of uni-axial strength due to large-scale geological weaknesses. By definition, s = 1 implies laboratory-scale (intact) rock, allowing the Hoek-Brown criterion to estimate the strength of the rock sample in a laboratory to be equal to the UCS. An s value of 0.01, for example, indicates in situ rock whose unconfined strength can be expected to be only 10% of the intact laboratory UCS.

The appropriate values for m and s can be determined from empirical rock mass quality data. For example, given that a 'Q' or 'RMR' assessment of rock mass quality has been established, the following equations have been suggested (Hoek& Brown 1988) for estimating in situ m and s values appropriate for underground mining:

$$RMR=9Log_eQ+44$$

The RMR and Q correlation shown above should preferably be established for specific datasets as the correlation is empirical and may vary from dataset to dataset.

$$s=exp(RMR-100/9) \text{ and } m=m_i\, e(RMR-100/28)$$

Updated work yielded similar equations to determine the m and s values, but using GSI ratings instead on RMR, as the RMR values failed to provide good correlation under certain conditions, while a rating to indicate blast damage to the rock mass was also included.

- Page 68 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 173 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 1 Chapter 11, Practical rock engineering, 2006, E Hoek
- Page 137 - Underground excavations in rock, 1980, E Hoek and ET Brown
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LEARNING OUTCOME 4.1.13

DESCRIBE AND EXPLAIN HOW TO APPLY THE HOEK-BROWN CRITERION TO PREDICT ROCK FAILURE

NOTE

An example is used to explain this outcome.

EXAMPLE

Five cylindrical samples of rock are tested in compression in a rock testing machine under tri-axial loading conditions. The first sample is tested uniaxially and the UCS was found to be 200 MPa. The results from the other four tests with increasing confinement are listed in the table on the right

Confining stress (MPa)	Peak strength (MPa)
5	228
10	264
15	300
20	315

All samples showed brittle behaviour in the post-failure regime.

(a) Determine the Mohr-Coulomb parameters of the rock.

(b) If one of the samples had Hoek-Brown m and s values of 15 and 1.0 respectively, what would the peak strength be if confinement were 20 MPa?

(c) How does this compare with the Mohr-Coulomb result?

(a) In order to determine the Mohr-Coulumb parameters the data can either be plotted in shear /normal stress space (Figure 21) and the Mohr-Coulomb values read from the graphs or the data can be plotted in principal stress space (Figure 22) and the Mohr-Coulomb parameters calculated from the UCS and strengthening parameter. Both these options are included below.

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Shear-normal stress space:

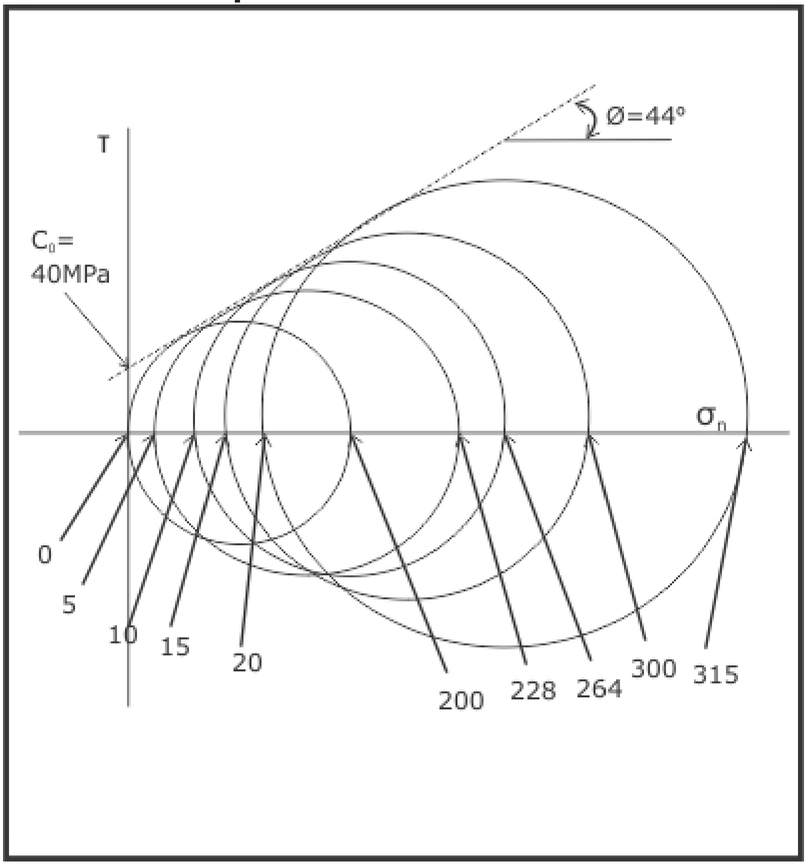

Figure 21: Shear – Normal stress space

Principal stress space:

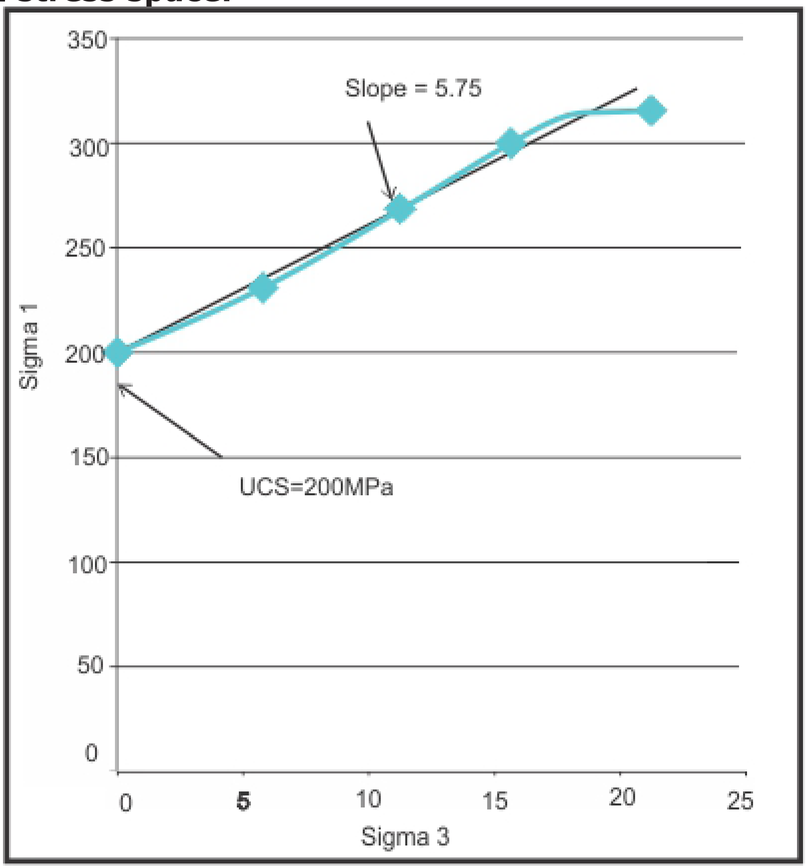

Figure 22: Principal stress space

Using the relationships below, it follows that:

$$\text{Sin}\phi_i = \frac{(\beta_0 - 1)}{(\beta_0 + 1)} \text{ and } C_0 = \frac{\sigma_c}{\sqrt{\beta_0}}$$

$$\begin{aligned} \text{Sin}\Phi &= (\beta_0-1)/(\beta_0+1) \\ &= (5.75-1)/(5.75+1) \\ &= \underline{0.7037} \end{aligned}$$

leading to

$$\begin{aligned} \Phi &= \text{ASin}(0.7037) \\ &= 44.7° \text{ and} \end{aligned}$$

$$\begin{aligned} C_o &= \sigma_c/(2\sqrt{(\beta_o)}) \\ &= 200\text{Mpa}/(2\sqrt{5.75}) \\ &= \underline{41.7}\text{ MPa} \end{aligned}$$

compared to the C_o of 40 MPa and Φ of 44° determined graphically (usually less accurate due to dependence on drawing accuracy) from the shear-normal stress space.

(b) Peak strength according to Hoek-Brown with m=15, s=1 and σ_3=20Mpa:

$$\begin{aligned} \sigma_1 &= \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2} \\ &= 20+\sqrt{(1(200)^2+(15)(200)(20))}\text{MPa} \\ &= \underline{336}\text{MPa} \end{aligned}$$

(c) Comparison of results

	(MPa)	(MPa)
Measured results	20	315
Hoek-Brown estimation	20	336

The Hoek-Brown estimation is 6% higher than the actual measurement indicating a slight overestimation of the intact rock strength for this rock type. The m-value is the critical parameter in this comparison as a slight reduction in this value will see a reduced strength at 20MPa confinement. However, if that is accepted to be accurate, good correlation between the actual and Hoek-Brown estimation is achieved.

LEARNING OUTCOME 4.1.14

DESCRIBE AND EXPLAIN HOW THE 'M'-PARAMETER FOR INTACT ROCK (M_I) REQUIRED FOR THE HOEK AND BROWN CRITERION MAY BE OBTAINED

Laboratory tests are carried out to determine the 'm'-parameter for intact rocks (m_i) keeping the moisture content similar to the actual in situ condition.Once 5 or more tri-axial tests are carried out they can be analysed to determine m_i value.

The method used by Hoek and Brown 1980 required the testing of 'n' samples, after which the σ_{ci}, m_i and coefficient of determination r^2 values can be calculated from the test data using the following:

$$\sigma_{ci}{}^2 = \frac{\Sigma_y}{n} - \left[\frac{\Sigma_{xy} - (\Sigma_x \Sigma_y / n)}{\Sigma_x{}^2 - ((\Sigma_x)^2 / n)}\right]\frac{\Sigma_x}{n}$$

$$m_i = \frac{1}{\sigma_{ci}{}^2}\left[\frac{\Sigma_{xy} - (\Sigma_x \Sigma_y / n)}{\Sigma_x{}^2 - ((\Sigma_x)^2 / n)}\right]$$

$$r^2 = \frac{[\Sigma_{xy} - (\Sigma_x \Sigma_y / n)]^2}{[\Sigma_x{}^2 - (\Sigma_x)^2 / n][\Sigma_y{}^2 - \Sigma_y{}^2 - (\Sigma_y)^2 / n]}$$

When laboratory test are not available, the m_i values can be estimated from historical data gathered by Hoek and showed in Figure 22.

REFERENCE

- Page 4,7,15 Chapter 11 - Practical rock engineering, 2006, E Hoek
- Page135 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

Rock type	Class	Group	Texture			
			Coarse	Medium	Fine	Very fine
SEDIMENTARY	Clastic		Conglomerate (22)	Sandstone 19	Siltstone 9	Claystone 4
				Greywacke (18)		
	Non-Clastic	Organic		Chalk 7		
				Coal (8-21)		
		Carbonate	Breccia (20)	Sparitic Limestone (10)	Micritic Limestone 8	
		Chemical		Gypstone 16	Anhydrite 13	
METAMORPHIC	Non Foliated		Marble 9	Hornfels (19)	Quartzite 24	
	Slightly foliated		Migmatite (30)	Amphibolite 25 - 31	Mylonites (6)	
	Foliated*		Gneiss 33	Schists 4 - 8	Phyllites (10)	Slate 9
IGNEOUS	Light		Granite 33		Rhyolite (16)	Obsidian (19)
			Granodiorite (30)		Dacite (17)	
	Dark		Diorite (28)		Andesite 19	
			Gabbro 27	Dolerite (19)	Basalt (17)	
			Norite 22			
	Extrusive pyroclastic type		Agglomerate (20)	Breccia (18)	Tuff (15)	

Figure 23: m_i Values for different rock types (Hoek, 2006)

LEARNING OUTCOME 4.1.15

SKETCH, DESCRIBE AND EXPLAIN:

- The stress-strain behaviour of brittle rock in uni-axial compression
- The effect of confinement on rock strength
- The effect of confinement on the stress-strain behaviour of rock samples
- What happens to rock after it has reached its peak strength

The stress-strain behaviour of brittle rock in uni-axial compression.

In a uni-axial compressive brittle rock reaches its peak strength and then fails completely. There is therefore a large drop in stress after the peak strength (UCS) is reached. In soft loading testing machines there is excess energy that may destroy the test sample completely. When tested in stiff loading machines the rest of the stress/strain curve after the peak

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CONNECTION 15

See uni-axial graphs in "LEARNING OUTCOME 4.1.5" on page 104

CONNECTION 16

Refer to "LEARNING OUTCOME 4.1.5" on page 104

CONNECTION 17

Refer to "LEARNING OUTCOME 4.1.4" on page 102

stress is reached will be plotted showing a large stress drop and then stabilising into a residual strength at some level.

The effect of confinement on rock strength

From Figure 24 below it can be seen that with an increase in confinement the peak and residual strengths of rock samples increase.

The effect of confinement on the stress-strain behaviour of rock samples In order to determine the effect of this confinement on the strength of the rock, tri-axial compressive tests are carried out on samples in a laboratory. The rock specimen is placed in a sealed pressure cell, and confining stress is achieved by pressurising oil in the cell. The axial strength is then tested at various confining pressures. As the amount of confinement increases, the strength of the material sample and its post failure behaviour change.

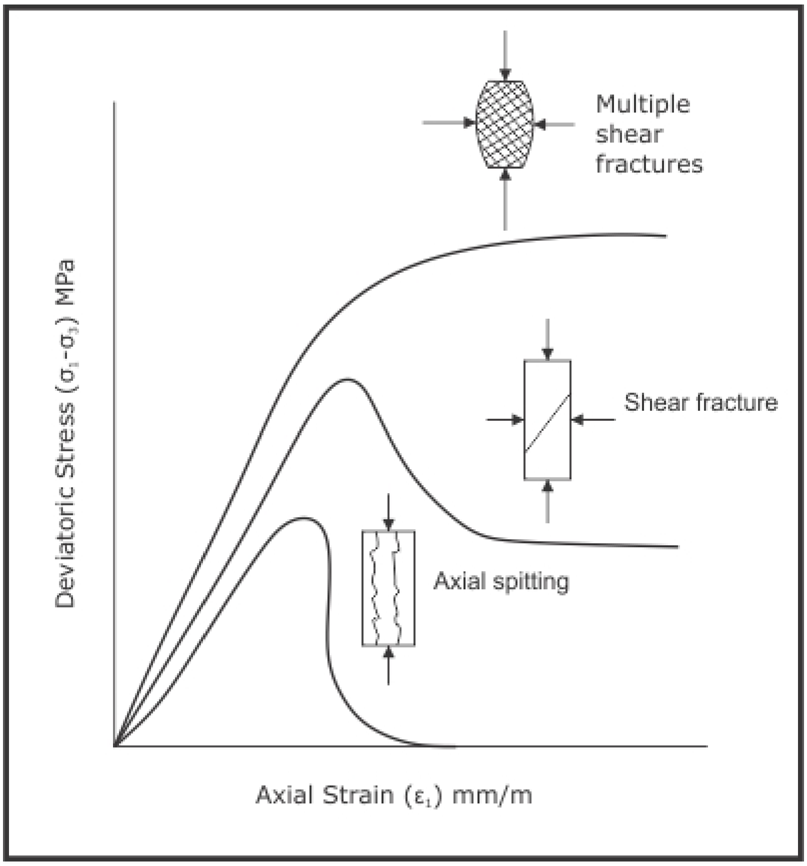

Figure 24: Stres sstrain curves of a specimen at increasing confinement

What happens to rock after it has reached its peak strength

By definition, reaching the peak strength should indicate that failure occurs when the sample is stressed further. The mode of failure, however, is different for different material types as well as confinement levels. The material will behave between these two extremes:

- The sample will strain and fail in a brittle manner, completely destroying the sample or
- The sample will continue to strain while not losing its ability to sustain load (ductile behaviour).

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REFERENCE

- Page 83,87,88 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 20 Chapter 11 - Practical rock engineering, 2006, E Hoek

LEARNING OUTCOME 4.1.16

SKETCH, DESCRIBE AND EXPLAIN:

- Rock fracture, Brittle failure
- Ductile deformation, Yield
- Peak strength, Residual strength
- Effective stress, Pore pressure
- The volume effect on the strength of intact rock.

CONNECTION 18

See "LEARNING OUTCOME 3.1" on page 61

ROCK FRACTURE

Rock fracturing occurs when the stress acting on a particular rock sample exceeds the strength of the material as determined by the material and the loading environment.

CONNECTION 19

See"LEARNING OUTCOME 3.1" on page 61

BRITTLE FAILURE

Under uni-axial loading, the sample fractures in an axial direction (if the platens are lubricated and therefore have no frictional forces preventing rock displacement in the horizontal direction) and experience a sudden load loss after the failure occurs. This sudden load loss is described as an important part of 'brittle' rock failure where a 'stress drop' occurs at failure. This 'stress drop' is representative of energy released at fracture creation and therefore, brittle failure is often experienced as a 'violent' reaction. Underground, this brittle, near vertical dipping fracturing is often seen in pillar sidewalls.

As the horizontal stress acting on the sample increases, i.e. confinement increases, inclined 'shear'; surfaces are created when the sample is overstressed. As with the vertical fractures, these shear fractures are also brittle and release energy with the drop in sample stress. This behaviour is very similar to shear fractures created in front of deep level mining faces, fracture formation that is often accompanied by the release of significant energy in the form of seismic events.

DUCTILE DEFORMATION

At very high confinement, the sample behaviour at failure (when fracturing occurs) changes to ductile behaviour, where the sample now crushes or yields in a controlled manner. The stress levels often do not reduce after failure, and the brittle (stress drop) behaviour of other loading conditions are absent.

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CONNECTION 20

See "LEARNING OUTCOME 3.1" on page 61

In Figure 25 below, the different fractures during rock testing are shown.

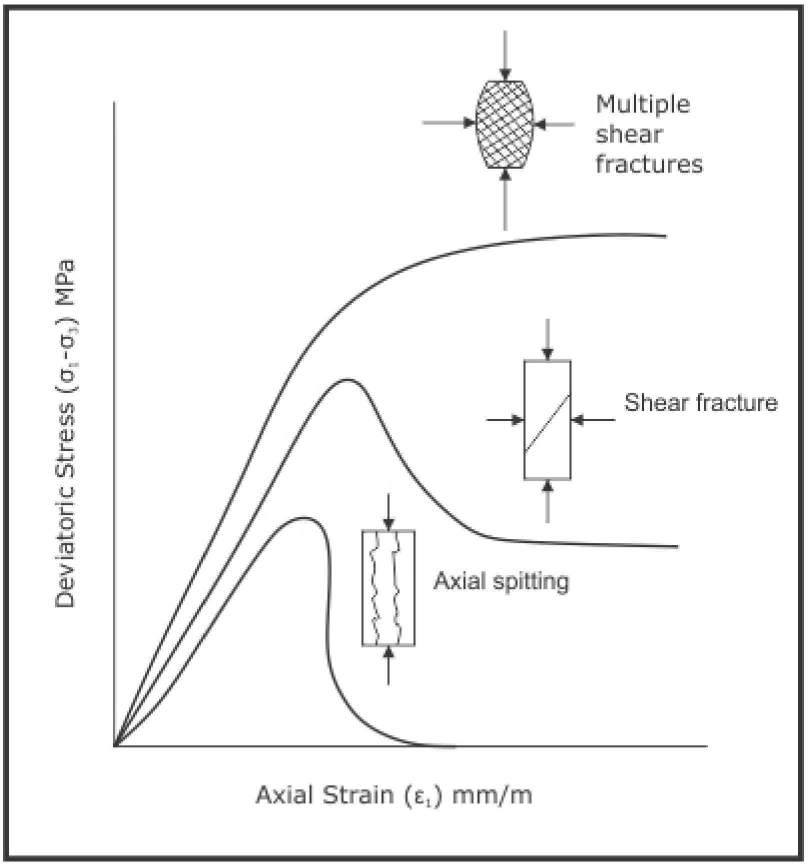

Figure 25: Change in nature of failure with increase in conining pressure; after Santarelli& Brown (1989) (ascribed to Jaeger & Cook 1979)

CONNECTION 21

"LEARNING OUTCOME 4.1.3" on page 99 and "Figure 24" on page 123

CONNECTION 22

See "LEARNING OUTCOME 4.2.4" on page 133 and "Learning Outcome 6.1.6" on page 198

CONNECTION 23

See "LEARNING OUTCOME 4.2.4" on page 133 and "Learning Outcome 6.1.6" on page 198

INTERESTING INFO

Karl von Terzaghi first proposed the relationship of effective stress in 1936. For him the term "effective stress" meant the calculated stress that was effective in moving soil, or causing displacements.

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YIELD

PEAK STRENGTH

RESIDUAL STRENGTH

PORE PRESSURE

Pore pressure refers to the pressure of ground water held within a soil or rock, in gaps between the particles (pores). Pore pressure present within a rock mass represents a level of stress (pressure) that exists on a specific structure or within a specific rock material prior to stress levels caused by external sources, such as mining induced or overburden stress.

EFFECTIVE STRESS

The pore pressure within the material or plane acts against external compressive stresses and since its direction is opposite to those stresses, an 'effective stress' value can be determined by subtracting the pore pressure from the 'external' compressive stress levels.

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NOTE

This reduction in normal stress reduces shear strength of the material / plane, an impact that is just as important for rocks as for soil.Saturated soils will have lower angles of repose as the shear strength of the material is reduced by the pore pressure impact. In the same way, saturated material forming highwalls in surface mineswill require shallower face and slope angles to remain stable.

EXAMPLE

If the external compressive stress acting normal to a fault plane, with a pore pressure of 5 MPa, is 35 MPa, the effective stress is given by $\sigma_{effective}=\sigma_{normal}$-Pore pressure which calculates to an effective stress of 30MPa.

THE VOLUME EFFECT ON THE STRENGTH OF INTACT ROCK

One type of scale effect is inherent in the rock structure due to heterogeneities (different properties at different points within the material) ranging from micro-to-macro scale while a second scale effect is involved with the scale of application.

Scale effect is evident when different sample sizes give a large variation in strength test results. For intact rock, heterogeneity is the most important scale effect and increases in a rock sample when the number of mineral components (each with its own potential behaviour) present in the sample, increases. Large variations in strength test results may occur when a concentration of a certain mineral (which is more evenly spread throughout the other samples) exists in the sample being tested.

Natural materials have shown the tendency to have lower cube strengths when large samples are tested, than when small samples of the same material is tested. This is due to the presence of a larger number of flaws in the larger sample, while the flaws could be absent from the more solid, smaller samples.

For example, a sample of rock of volume 0.05 m3 is far less likely to contain a large flaw than a sample of 1.0 m3 volume. The larger the flaw, the weaker the rock, and therefore the larger the volume of rock tested, the lower the strength. This is particularly evident in UCS tests and less so at high confining stresses in the tri-axial test where the stresses are more likely to suppress the effect of the flaw. UCS results are therefore invariably overestimating of the strength of rock material in the field, but so are tri-axial test results mainly because, the field strength (rock mass strength) is dominated by the presence of discontinuities, and therefore the strength of those discontinuities rather than by the strength of the rock material itself. Typically, UCS values obtained for 25mm diameter cores will be 25-35% larger than for 75-100 mm specimens.

Crystal size also has an effect on strength such as where a fine-grained granite and a coarse-grained granite, although both identical in composition, are tested, the coarse-grained granite is always weaker. This is because the yield process in low-porosity rocks is always associated with the generation and propagation of micro-fractures. Because these are almost always along the grain boundaries, the size of the crystals controls the size of the micro-fractures, which controls the magnitude of the stress concentration in the 'fracture', which controls the strength of the specimen.

Micro-fracturing generated by straining will mainly occur along the boundary between two different minerals because the moduli are different, therefore there is a local (micro) stress concentration. Once initiated, the flaw tends to propagate rapidly until it "butts" up against another crystal face. The length of the fracture (L_{crack}) approximates the diameter of the crystal and the larger L_{crack} is, the larger is the tensile stress at the tip of the fracture that drives fracture growth. In coarse-grained granite, the first few micro-fracturesthat are generated cause destruction, whereas in the fine-grained granite, many of the fractures have to be created before they coalesce into a rupture plane.

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- Page 87 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 4,5,86, 87,90 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown
- Page 153, 155 - Underground excavations in rock, 1980, E Hoek and ET Brown
- Page 78 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 9 Chapter 11 - Practical rock engineering, 2006, E Hoek
- Page 97 - Best practice rock engineering handbook for other mines, 2001, TR Stacey

JOINT STRENGTH

- Page 60 Chapter 4 - Practical rock engineering, 2006, E Hoek

LEARNING OUTCOME 4.2.1

REPRESENT AND DESCRIBE THE OCCURRENCE OF ROCK JOINTING, THE NUMBER OF JOINTSETS AND THEIR ORIENTATIONS

A joint is a natural discontinuity and shows no signs of shear displacement, can be open, filled or healed, with smooth or rough surfaces that can vary in persistency within the rock mass.

Jointed rock masses may have various joint sets striking and dipping at various dip angles where a 'set' represents all the joints that fall within a certain range of consistent orientations, but may vary in spacing and persistency.The existence of multiple joint sets at various angles to each otherresults in blocky ground conditions and can create potentially unstable wedges.The more closely spaced joints within the various sets are, the smaller the resulting block sizes that need to be effectively supported are.

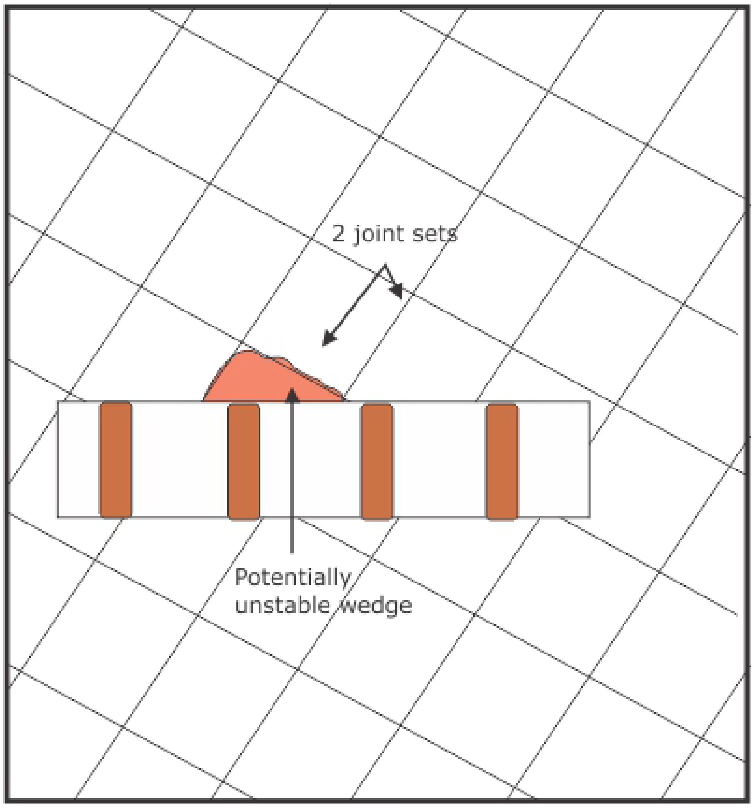

Figure 26: Example of joint sets creating unstable wedges.

Joint orientations are often parallel to other features such as faults or dykes and are therefore often called sympathetic jointing. The dip and strike of joints are critical features to know as they may be unfavourable relative to the mining direction or exposed excavation walls.

As examples, the following two cases are described:

- Flat dipping joint sets may pose a high risk to potential falls of ground when they are mined against their dip direction but are less of a problem when mined in the opposite direction.(Figure 26)

- In deep level mining, steeper dipping joints are usually more stable since they are being clamped (high normal stress) by the high horizontal stresses created in the hangingwall due to stress fracture dilation. In these cases the flat dipping joints can be destabilisedby this same horizontal stress.(Figure 27)

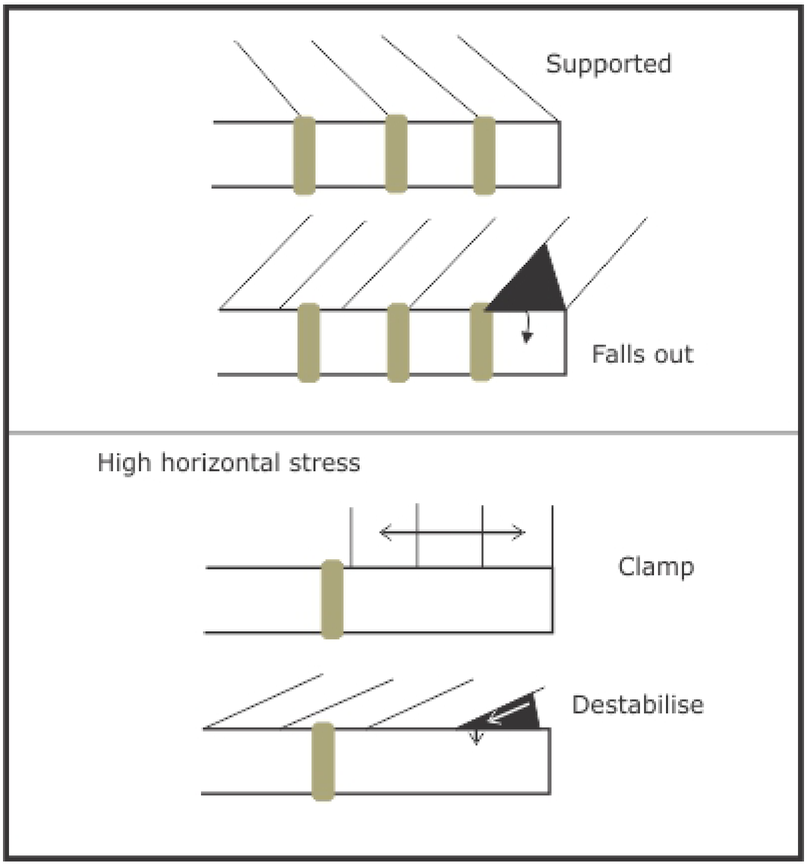

Figure 27: Impact of jointing on excavation stability

The mechanical behaviour of rock masses is dependent on the strength of the blocks created by random patterns of discontinuities and the strength of the discontinuities. For this reason joint sets and their orientations are taken up in rock mass quality systems utilised to design excavations and support, such as Barton's Q rating, the CSIR Rock Mass Rating, and Laubscher's adaptation of the Rock Mass Rating to evaluate caving.

Joints (spacing, persistence, density, roughness, alteration and orientation) can be presented in several formats to allow understanding of the jointed rock mass, such as:

- **Tables:** The use of tables to present data is the most basic form of presentation and as shown in Figure 28, can be used to indicate joint spacings within specific joint dip ranges (eg. joints within the 75°-90° joint dip range have an average spacing of 60.1cm).

Joint spacing (cm)					
75-90	60.75	45.6	30.45	<30	Average excluding bedding
60.1	153.7	293.3	98.9	25.9	151.5

Figure 28: Tabled joint spacing information

- **Graphs:**
  - Joint dip data (as for any of the other joint properties) can easily be graphed to indicate critical trends. Figure 29 shows that within a certain data set, the majority of joints have a similar dip angle of around 26°, with only a small number at steep dips of around 70°.

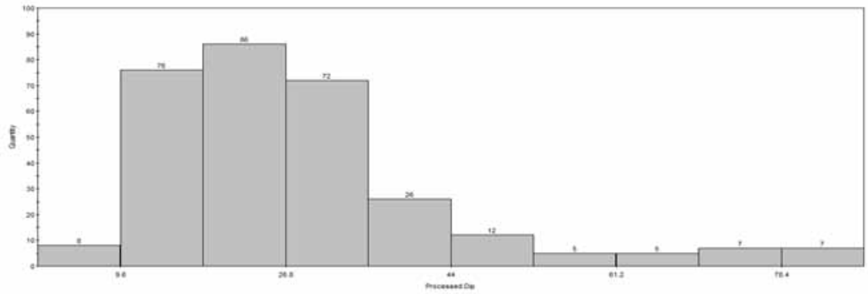

Figure 29: Graphical presentation of joint dip angle distribution

- Figure 30 shows the joint spacing distributions within the various joint dip angle ranges and indicates that the steep dipping joint sets (70°-90°) exist mostly at small spacings (0.5-1.0m), while the rest of the joint sets that dip at shallower angles exist within a larger spacing range of between 1.0 and 4.5m.

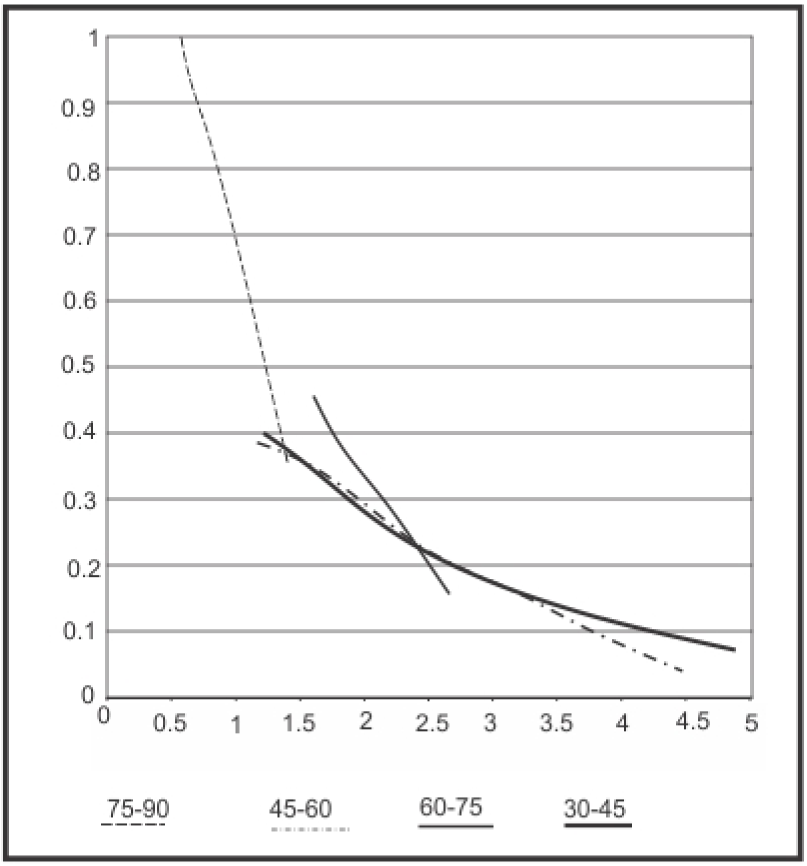

Figure 30: Graphical presentation of joint spacing

- **Stereonet:**Stereonet presentation is a very powerful method of presenting joint dip and dip direction data since it is possible to indicate the presence of specific groups of joints with similar dips and dip directions, called joint sets. The most appropriate dip and dip direction for such a joint set are also provided through this method.

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CONNECTION 24

See "Learning Outcome 6.2.1" on page 200 in Paper 1 guide and Outcome 3.1.1 in the Paper 2 guide.

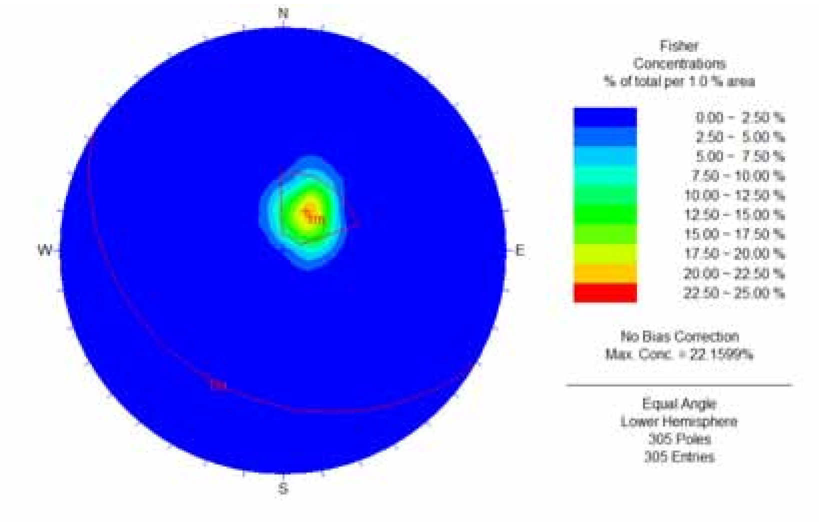

Figure 31: Stereonet presentation of joints

CONNECTION 25

See "Learning Outcome 6.2.1" on page 200 in Paper 1 guide and Outcome 3.1.1 and 3.2.1 in the Paper 2 guide.

- **Rock mass ratings:** Rock mass ratings include reference to jointing and are often used to represent the quality of the rock mass based on the discontinuities logged / mapped within the area of interest. From these ratings it is possible to establish commonalities between the observed joints such as type of infilling, joint surface roughness, joint surface alteration, joint spacings, level of water present on joints, etc. The joints can be presented by the individual ratings such as J_w or J_a, or as the quality of the material such as the Q Index or GSI. These presentations can be in table format or graphical, depending on the purpose of the data.

REFERENCE

- Page 2 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 18,91 - Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page
- Page51 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 4.2.2

DESCRIBE AND EXPLAIN THE EFFECT OF FRICTION ANGLE ON JOINT STRENGTH

'Joint strength' usually refers to the resistance a joint has to allow shear displacement on the joint. This resistance is affected by many parameters, but is most simply explained using the Mohr-Coulomb failure criterion. In this criterion, the strength (shear strength) of a joint is given by

$$\tau_{strength} = C_o + \mathrm{Tan}\Phi . \sigma_n$$

CONNECTION 26

See "LEARNING OUTCOME 4.1.10" on page 112.

where C_o is the cohesion on the joint plane, ø is the plane friction angle and σ_n is the normal stress clamping the joint plane.

Friction angles on joints, which are surfaces that exist in the rock mass prior to mining are determined from tilt tests or shear box tests, the friction angle is that property that indicates how difficult (or easy) shear displacement will occur on a joint plane and is expressed as an angle. High friction angles indicate that the joint surface has high friction and high resistance to shear displacement, while a low friction angle indicates that friction is easily overcome so that shear displacement can occur. The higher angle therefore indicates a higher resistance to shear displacement and therefore also higher joint strength.

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CONNECTION 27

See shear box tests, Paper 2 Guide, Outcome 6.1.1.11

The slope of the shear stress-normal stress representation of the state of stress at failure, gives the friction angle of the material (no pre-existing plane) or the plane (joint surface). Using simple geometry it is easy to show that the shear stress required to induce shear displacement on a joint, given the normal stress (clamping of the joint), can be determined by

$$\tau = \mathrm{Tan}\Phi . \sigma_n$$

where Tanø is the slope of the failure envelope

CONNECTION 28

See "LEARNING OUTCOME 4.1.11" on page 113

This means that the potential to have shear displacement on the joint is a function of the friction angle ø, where a higher angle would indicate a higher shear stress (τ) required before shear displacement will be allowed and therefore a higher joint strength.

The symbols most often used for friction angle include Φ_b = base friction angle in degrees and Φ_r= the residual friction angle of the discontinuity (in degrees), once all asperities have been sheared off.It is also important to note that $\mu = \mathrm{Tan}\Phi$ where μ = coefficient of friction, and are often used to communicate the friction angle.

REFERENCE

- Page 65,67 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 71 Chapter 4 - Practical rock engineering, 2006, E Hoek
- Page 97, Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page
- Page 100 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 6,170 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden

LEARNING OUTCOME 4.2.3

DESCRIBE AND EXPLAIN THE EFFECT OF INFILLING ON JOINT STRENGTH

'Joint strength' usually refers to the resistance a joint has to allow shear displacement on the joint. This resistance is affected by many parameters, but is most simply explained using the Mohr-Coulomb failure criterion. In this criterion, the strength (shear strength) of a joint is given by

$$\tau_{strength} = C_o + \mathrm{Tan}\Phi . \sigma_n$$

where C_o is the cohesion on the joint plane, ø is the plane friction angle and σn is the normal stress clamping the joint plane.

Cohesion can be viewed as the 'glue' that indicates how high the shear stress on the joint must become at zero normal stress (σ_n=0), before any shear displacement will occur on the joint. Cohesion ignores friction on the plane and indicates the strength of the 'glue' present on the plane.

CONNECTION 29

See "LEARNING OUTCOME 4.1.10" on page 112 – Cohesion value on τ (shear stress) axis of Mohr Coulomb failure criterion.

The cohesion on the plane is provided by the material that 'fills' the joint plane. Filling is meant to include any material different from the host rock, but which is thicker than a coating and which occurs between the joint planes.The type of infilling present on the joint determines the level of cohesion on the joint plane.
The infilling thickness has the added impact on joint strength that it affects the friction on the surface of the joint. Thick infilling removes

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most of the friction between joint surfaces at low normal (confinement) stresses while thin infilling allows the joint surface to determine the friction levels. Joint strength can therefore be further reduced when infilling material on the joint plane is thick.

REFERENCE

- Page 97 - Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page
- Page 100 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 57,129 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 4.2.4

DESCRIBE AND EXPLAIN THE EFFECT OF WATER ON JOINT STRENGTH

Water on a joint surface affects joint strength in two possible ways:

CONNECTION 30

See effective stress concept "LEARNING OUTCOME 4.1.16" on page 124

- The confinement or normal stress clamping the joint and which is responsible for increasing friction and therefore increased joint strength, can be reduced by the water pressure (pore pressure) on the joint. This reduction in normal stress to the joint reduces the friction component of the joint strength and therefore also reduces the joint strength and

- Water can assist in lubricating the joint surface, especially when infilling materials containing clay particles are present on the joint. This lubrication reduces the friction angle of the joint surface and therefore reduces the joint strength.

REFERENCE

- Page 97 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 97 - Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page
- Page 100 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 70 Chapter 4 - Practical rock engineering, 2006, E Hoek
- Page 25 - Handbook on hard rock strata control, 1995, AJS Spearing
- Page 153 - Underground excavations in rock, 1980, E Hoek and ET Brown
- Page 5,87 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 4.2.5

SKETCH, DESCRIBE AND EXPLAIN THE TYPICAL SHEAR RESISTANCE-DISPLACEMENTBEHAVIOUR OF SMOOTH JOINTS

In the case of smooth joint surfaces or polished surfaces, 'stickslip' shear displacement often occurs. When a shear force is applied at a constant normal force, minimal shear displacement occurs while the shear force steadily increases. When the shear force reaches a critical value

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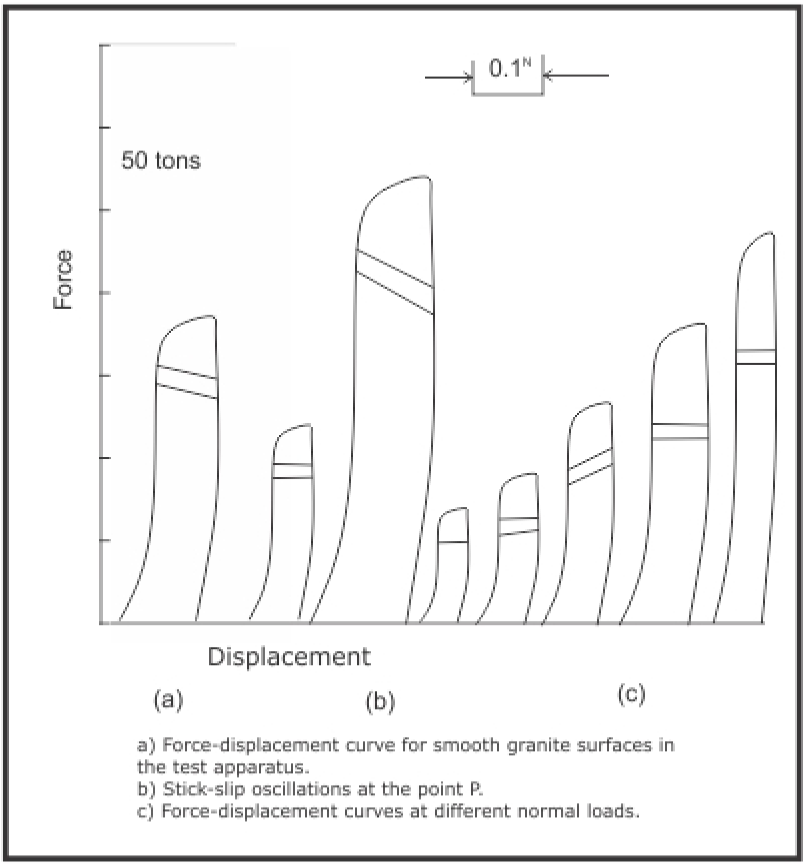
where the surface shear strength is overcome, sudden 'slip'or shear displacement occurs between the surfaces. The shear force drops and the interlocking of the surfaces inhibits shear displacement until the shear force is increased again (Figure 32).



a) Force-displacement curve for smooth granite surfaces in the test apparatus.
b) Stick-slip oscillations at the point P.
c) Force-displacement curves at different normal loads

Figure 32: 'Stickslip' phenomena on smooth joints

CONNECTION 31

See "LEARNING OUTCOME 4.2.6" on page 135

In the case of smooth joint surfaces slippage occurs at lower shear stress levels than would be the case for rough joint surfaces, mainly due to the difference in friction angle. Once cohesion on a smooth joint is broken, the surface will mobilise its residual friction angle (Φ_r) and slip will occur at $\tau_{max} = \sigma_{normal}.Tan\Phi_r$.(Figure 33)

The base friction (Φ_b) angle of a joint is easily determined by noting the tilt angle at which core pieces containing smooth surfaces starts to slide. For fresh and unweathered discontinuities, the base and residual fraction angles are equal, but in the case where infilling is present or where substantial weathering of the surface had occurred the residual friction angle could be up to 7° lower than the base friction angle.

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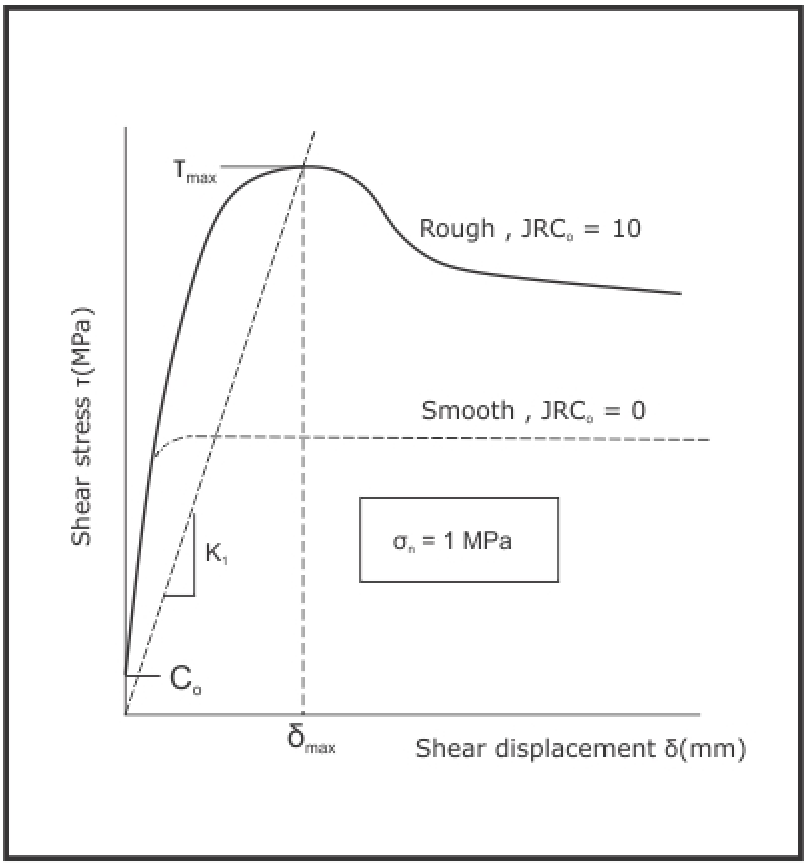


Figure 33: Shear stress vs shear displacement graph indicating smooth and rough joints (Ryder&Jager)

REFERENCE

- Page 61,70,375 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 103 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager

LEARNING OUTCOME 4.2.6

SKETCH, DESCRIBE AND EXPLAIN THE TYPICAL SHEAR RESISTANCE-DISPLACEMENTBEHAVIOUR OF ROUGH JOINTS

In the case of rough joint surfaces slippage occurs at much higher shear stress levels than would be the case for smooth joint surfaces.A much higher level of shear strength is attained on a rough joint and a large stress-drop may occur prior to the gradual attainment of the residual friction level of Φ_r , normally the friction angle attained after shear displacement had occurred along a rough joint.

Due to the rough joint surfaces, the level of dilation (Figure 34) is indicated by the vertical displacement δn caused during shear displacement δ due to the roughness profile of the joint) and occurs as shear displacement is induced on the joint.The dilation angle can be determined by the ratio of the normal displacement (δ_n) to the shear displacement (δ) using the following relationship $\tan\varphi=\delta_n/\delta$.

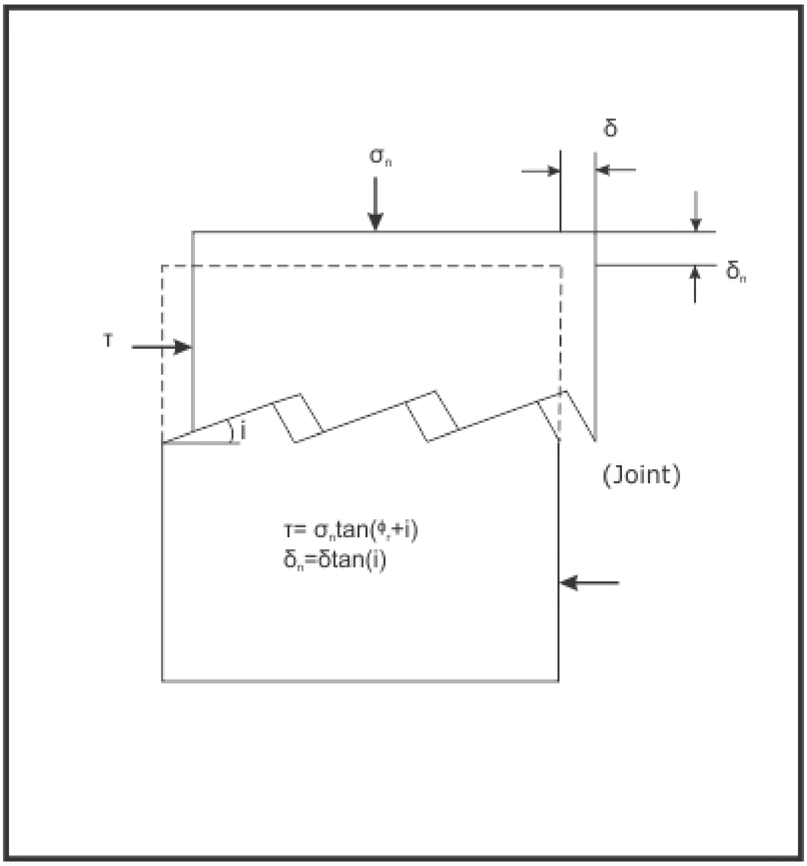
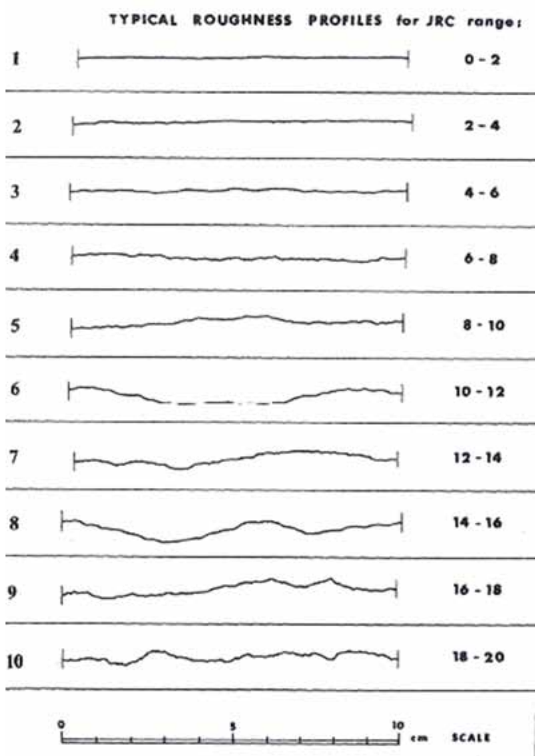


Figure 34: Conceptual model of joint surfaces riding over roughness asperities (Ryder&Jager)

When assessing rough joints it is therefore important that the terms JRC_n and JCS_n be understood, where:

- JRC_n is the scale dependent Joint Roughness Coefficient and is an indication of the roughness of a joint or discontinuity surface measured on a scale from 0-20.



- JCS_n is the scale dependent Joint Compressive Strength. This is also referred to as the joint wall compressive strength and is the UCS of the discontinuity surface or the infilling if it is weaker than the host rock.

When rough surfaces are forced to shear it must overcome the residual friction angle Φ_r while the surface must ride "uphill" over the asperity surface, causing the dilation. The resulting mobilised friction angle is therefore $\Phi_n=(\Phi_r + i)$ where (i) is suggested to be the 'roughness friction angle'. This 'roughness friction angle' increases by increasing geometrical roughness (represented by the JRC_n) and decreases by increasing normal stress on the joint surface (represented by the ratio of JCS_n and σ_n).

The factors described above indicate that scale dependency on joints is also critical to the roughness friction angle, and then through the following relationships:

$$JCS_n \approx JCS_o \, (L_n/L_o)^{-0.03JRCo}$$
$$JRC_n \approx JRC_o (L_n/L_o)^{-0.02JRCo}$$
$$i = JRC_n . Log_{10} (JCS_n/\sigma_n)$$

where L_n is the in-situ joint length and L_o is the length over which JRC_o and JCS_o were determined.

These parameters all have an effect on the shear stress to displacement behaviour of rough joints (Figure 35), where higher JRC values will require higher shear stresses to initiate failure on the joint (shear stress exceed shear strength and result in increased shear displacement) or lower JRC values will change the failure and post-failure behaviour of the joint.

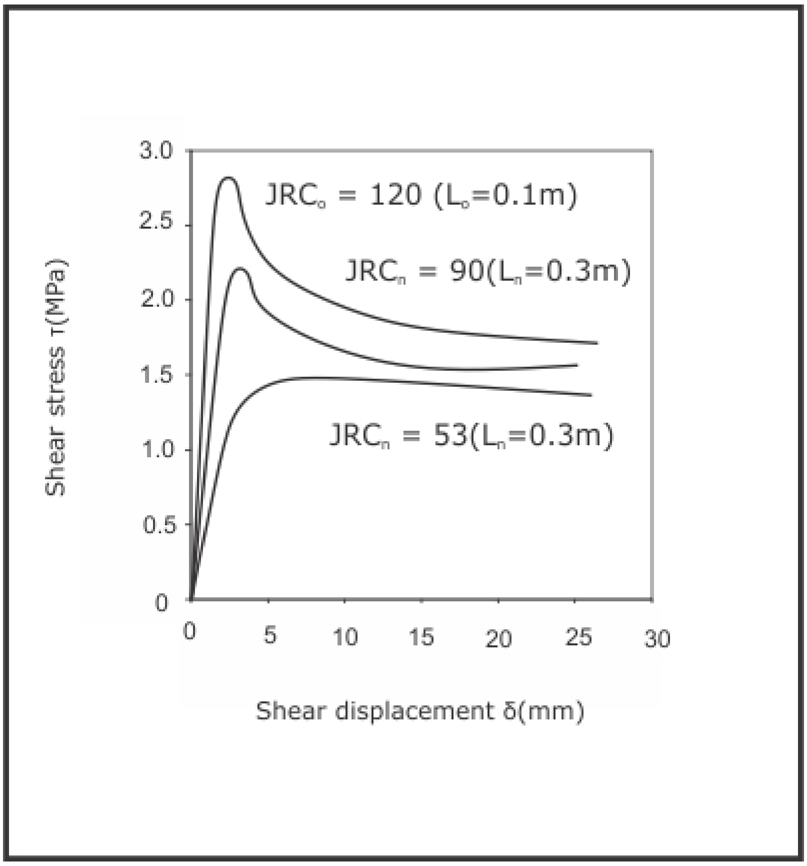


Figure 35: Simple Predicted scale effects on the shear behaviour of a joint

Figure 36 below shows the peak shear strength and normal stress relationships for smooth and rough joints. Note that for smooth joints the friction angle = (Φ_r) while for the rough joints the friction angle = (Φ_r + i).

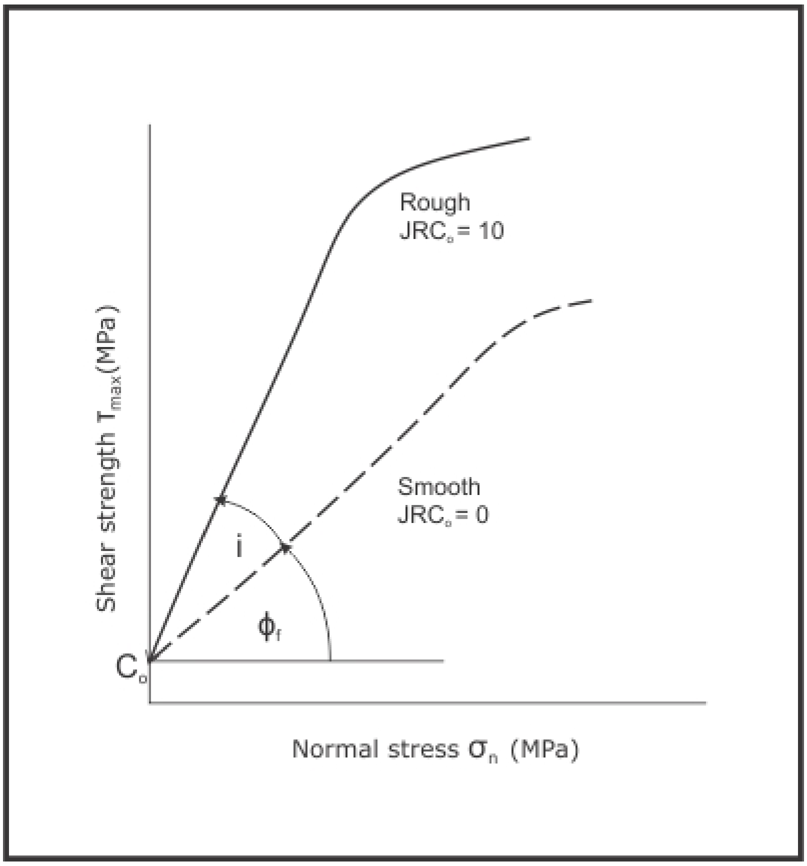


Figure 36: Peak shear strength as a function of normal stress graph showing smooth and rough joints (Ryder&Jager)

REFERENCE

-
- Page 70,395 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmermana
- Page 122 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown
- Page 64 Chapter 4 - Practical rock engineering, 2006, E Hoek

LEARNING OUTCOME 4.2.7

SKETCH, DESCRIBE AND EXPLAIN THE EFFECT OF SURFACE ROUGHNESS ON JOINT SHEARSTRENGTH

Most of the points relative to this outcome have already been discussed under the previous outcomes.The joint shear strength is determined from $\tau_{strength} = \sigma_{normal} \mathrm{Tan}(\Phi_r + i)$, from which it is clear that an increase in (Φ_r + i) angle will result inan increase in the shear strength. This means that with an increase in surface roughness (increase in the roughness friction angle i), the shear strength of the joint surface will also increase (Figure 35above).

REFERENCE

- Page 7,99 - Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page
- Page 101,103 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 54,122 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

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LEARNING OUTCOME 4.2.8

COMPARE AND CONTRAST TYPICAL SHEAR RESISTANCE VERSUS SHEAR DISPLACEMENTBEHAVIOUR OF SMOOTH AND ROUGH JOINTS

CONNECTION 32

‘

See “LEARNING OUTCOME 4.2.5” on page 133, “LEARNING OUTCOME 4.2.6” on page 135 and “LEARNING OUTCOME 4.2.7” on page 138

Most of the points relative to this outcome have already been discussed under the previous outcomes. However, from Figure 37 it can be shown that:

- The presence of some level of cohesion is indicated for both joints by the intersection of the graphs with the shear stress axis (C_o);
- The presence of a constant confinement stress (σn) on both joints is indicated;
- Roughness of the joints is indicated by the JRC$_o$ ratings of 0 (smooth) and 20 (rough);
- The initial joint stiffness (shear stress to shear displacement ratio) is similar for the joints assuming similar properties (cohesion, infilling), with the only difference being the roughness of the surfaces;
- The shear stress required to overcome the joint shear strengths is much larger for rough joints than for smooth joints;
- When the shear stress reaches the shear strength of the joint, the shear displacement increases substantially for both joints;
- Smooth joints allow shear displacement at the constant shear stress level when the joint shear strength is reached;
- Rough joints experience a shear stress drop when the shear strength is exceeded and settles on a lower shear stress level at which shear displacement occurs, indicating that asperities have been sheared off during the initial shear and that a smoother joint surface has been created at a lower shear strength than the initial joint surface;
- Continued shearing further reduces the residual friction angle on the rough joint, resulting in a post-failure behaviour that indicates a continued drop in shear stress to induce shear displacement, i.e. shear strength and
- The post-failure constant shear stress levelindicated for the smooth joints represents the creation of a residual friction angle that remains constant after shear displacement occurred.

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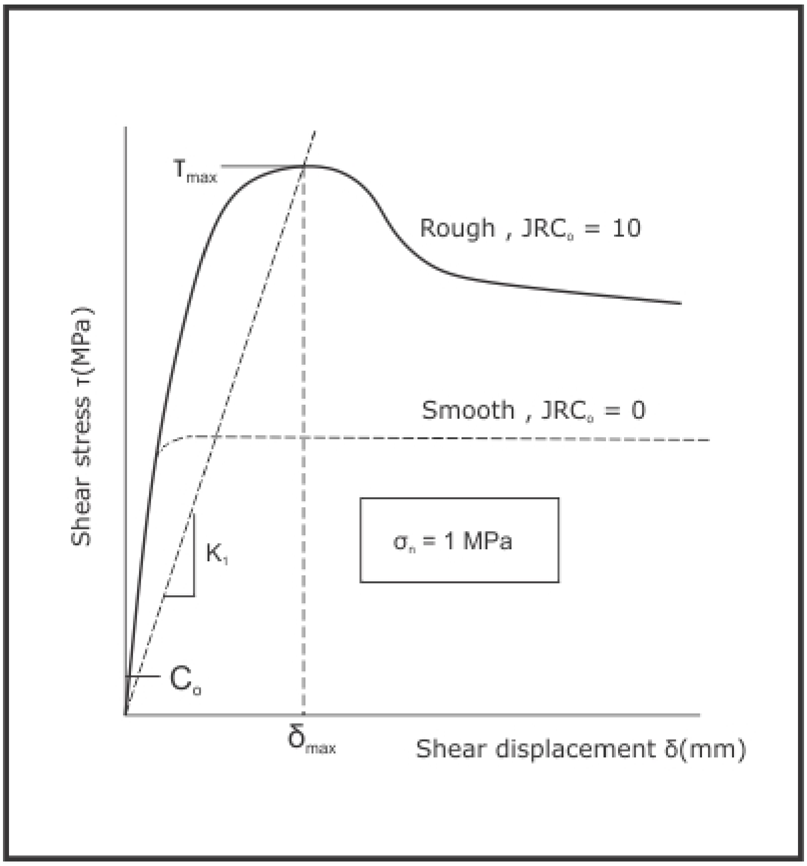

Figure 37 Shear stress vs Shear displacement graph comparing smooth and rough joints (Ryder&Jager)

LEARNING OUTCOME 4.2.9

DESCRIBE AND EXPLAIN THE EFFECT OF INCREASING THE NORMAL STRESS ON THEBEHAVIOUR OF JOINTS UNDER SHEAR LOADING

CONNECTION 33

See "LEARNING OUTCOME 4.2.4" on page 133, "LEARNING OUTCOME 4.1.16" on page 124 and 6.1.6

When the normal stress (clamping stress) on a joint surface is increased, it willrequire a higher shear stress to initiate shear displacement (slip) along the joint surface. This is clear from the joint strength equation (Mohr-Coulomb) $\sigma_{strength} = \sigma_{normal}.\mathrm{Tan}(\Phi_r + i)$.

In the underground situation, pillars and support elements are used to stabilise discontinuities by increasing the normal stresses acting on the discontinuity surfaces.

LEARNING OUTCOME 4.2.10

CONNECTION 34

See "LEARNING OUTCOME 4.2.7" on page 138

DESCRIBE AND EXPLAIN THE EFFECT OF SURFACE ROUGHNESS ON JOINT SHEAR STRENGTH

LEARNING OUTCOME 4.2.11

DESCRIBE AND EXPLAIN THE CONCEPT OF ROUGHNESS ANGLE AND ITS EFFECT ON INITIALSHEAR RESISTANCE ACCORDING TO PATTON'S THEORY

CONNECTION 35

The roughness angle has been described in "LEARNING OUTCOME4.2.6" on page 135 and will not be repeated here.

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According to Patton's criterion, the effect of joint surface roughness on the shear strength of rock fractures such as joints (as correlated by

Patton in 1966), assuming that the asperities on the fracture surface have identical shapeswith inclination angle i, was formulated as

$$\sigma_t^P=\sigma_n.\mathrm{Tan}(\Phi_b+i)$$

where $\Phi_b=\Phi_r$ and i is the effect of irregular asperities on the fracture surface and can also be equal to the dilation angle.

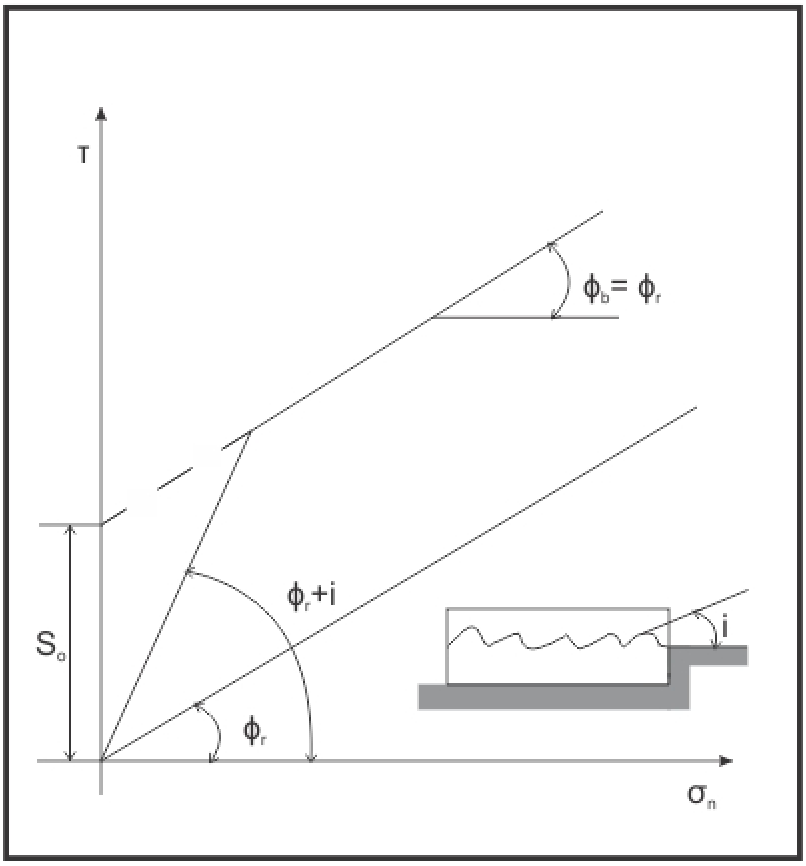

Figure 38: Patton's criterion

Patton's criterion can result in an overestimation of the cohesion (S_o) shown in Figure 38 and should be applied very carefully in practice. However, the criterion does explain the impact of surface roughness on joints shear strength as well as the cause and magnitude of dilation during shear displacement.

NOTE

Cohesion can be described as the 'initial shear resistance', i.e. the shear stress axis intercept in Figure 37 where the normal stress is zero.

REFERENCE

- Page 124, Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

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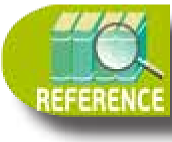
CONNECTION 36

As discussed in the ""LEARNING OUTCOME 4.2.6" on page 135

LEARNING OUTCOME 4.2.12

DESCRIBE AND EXPLAIN THE EFFECT OF SURFACE ROUGHNESS ON DILATION DURING SHEAR ON A JOINT



- Page 125 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 4.2.13

DESCRIBE AND EXPLAIN THE EFFECT OF THE ABOVE (SURFACE ROUGHNESS AND DILATION) ON ROCK STABILITY IN UNCONFINEDSITUATIONS

In the case of unconfined situations the normal or clamping stress on joint surfaces is zero. Based on the joint strength equation, this means that joint strength is determined by the cohesion on the joint only when

$$\tau_{strength}=C_o+\mathrm{Tan}\Phi.\sigma_n \text{ becomes } \tau_{strength}=C_o.$$

The absence of normal stress means that the friction angle (which is affected by joint roughness) and the dilation that occurs due to the roughness of the surface play no role in the joint strength. The 'friction' driven portion of the joint shear strength equation falls away and joint stability is therefore determined by the infilling type, strength and thickness. The absence of infilling, and therefore cohesion, would indicate very little resistance to shear displacement.

Jointed rock mass stability can therefore be seriously affected as slipalong joint surfaces, especially on steeply joint sets can occur easily, more so when the cohesion on the joint surface is very low or zero (no infilling).

CONNECTION 37

As discussed in the "LEARNING OUTCOME 4.2.6" on page 135

In the case of very rough surfaces, the shear displacement on the joint will cause dilation as ride across the asperities occur. This dilation could induce some form of 'clamping' of blocks and retain stability.

In the case of shallow dipping joints the mechanism of failure is not driven by shear failure on the joint. The normal stress acting on the joint becomes tensile (gravity pulling the surface open). The joint strengths are then determined by the cohesion on the surface and are independent of the surface roughness.

If the confining stress becomes tensile, i.e. the normal stress opens the joint surface, instability results in back breaks or massive falls of ground in jointed rock mass environments (Figure 39).

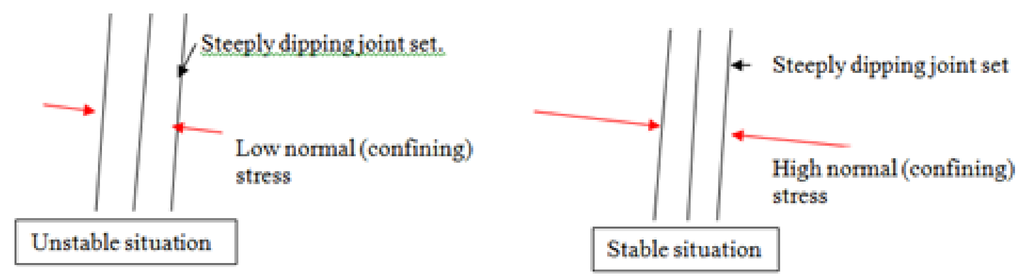


Figure 39: Effect of confinement on the stability of steeply dipping jointed rock masses

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LEARNING OUTCOME 4.2.14

DESCRIBE AND EXPLAIN THE EFFECT OF THE ABOVE (SURFACE ROUGHNESS AND DILATION) ON ROCK STABILITY IN CONFINEDSITUATIONS

In confined situations, the normal stress acting on the joint surface accentuates the effect of joint surface roughness on the shear strength. Higher shear stresses arenow required to initiate slippage along joints as the joint strength is significantly enhanced by a higher normal stress.

$$\tau_{strength}=C_o+Tan\Phi.\sigma_n$$

CONNECTION 38

"LEARNING OUTCOME 4.2.5" on page 133

Jointed rock masses under high confinement conditions are, at first glance, more stable as slip along the joint surface should not occur easily. However, if the normal stress becomes so high that asperities along the joint surfaces can be 'crushed', shear displacement can be allowed at much lower shear stresses than originally estimated. In this case, dilation on the joints will also be reduced as the overall roughness of the joint surface reduces when asperities are sheared or crushed.

The increase in normal stresses acting on joint surfaces can therefore be beneficial up to a certain level at which it might become detrimental to joint stability.

LEARNING OUTCOME 4.2.15

APPLY BARTON'S EQUATION TO ESTIMATE THE PEAK STRENGTH OF JOINTS

Barton indicated that joint shear strengths can be better described by

$$\tau_{strength}=\sigma_n \tan[JRC\log_{10}(JCS/\sigma_n)+\Phi_Y]$$

This joint strength relationship and the impact of each parameter within the equation on the joint shear strength are shown in the table below:

σ_n	Stress normal to the joint	MPa	Higher normal stress will increase joint strength as it clamps the joint together, but can reduce shear strength if the (JCS/σn) ratio approaches or reduces below 1 (Log of ratio becomes zero at a ratio of 1 and negative at below 1, indicating reduction in strength when σn exceeds JCS)
JRC	Joint roughness coefficient	Degrees, but effectively dimensionless	Higher JRC will increase joint strength as a rougher surface increases friction angle and therefore shear strength.

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JCS	Jointwall compressive strength,	MPa	Higher JCS will require a much higher $\sigma_n$to fail the asperities on the joint and allow slip, therefore higher JCS gives higher joint strength.
$ø_r$	Residual friction angle	Degrees	Joint angle after failure and suggests that higher residual friction angle would indicate higher final joint strength.

EXAMPLE

Applying Barton's relationship above, comment on the possibility for failure on a joint in the pillar given a shear stress of 28 MPa and normal stress of 48 MPa on the joint. The following information is also available: JRC=18, JCS=200MPa and a residual joint friction angle of 10°.

Shear strength $= \sigma_n. \operatorname{Tan}[JRC.\operatorname{Log}_{10}(JCS/\sigma_n)+ø_r]$

$= 48MPa. \operatorname{Tan}[18.\operatorname{Log}10(200/48) + 10°]$

$= 48MPa. \operatorname{Tan}[11.1562+10°]$

$= \underline{18.57}MPa$

NOTE

The JRC value can be indicated as degrees (°) to allow it to be added to the 10° residual friction angle. See the table above for comments on the appropriate units. The factor JRC.Log$_{10}$(JCS/σ_n)

Shear stress = 28Mpa

Shear stress – shear strength

= 28MPa – 18.57MPa

= $\underline{+\ 9.4}$MPa

<u>And therefore slip will occur.</u>

REFERENCE

- Page 97 - Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page
- Page 64 Chapter 4 - Practical rock engineering, 2006, E Hoek
- Page 127,131 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 4.2.15

DESCRIBE AND EXPLAIN THE DIFFERENCE BETWEEN JOINTS, FRACTURES AND CRACKS.

Joints

A joint is a natural discontinuity that is generally not parallel to lithological variations, and which shows no signs of shear displacement. Joints can be open, filled or healed. Such features are often not very persistent.

Fractures

A fracture is a separation of an object or material into two or more pieces under the action of stress. The difference between a joint and a fracture is that a joint is a natural discontinuity, whereas a fracture is stress induced. The orientation of stress fractures is aligned to the shape and

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direction of mining taking place in an excavation. A simplistic way of describing the orientation of the fractures is to compare them to the ripples created in water as a boat moves through it. Imagine the boat is the advancing excavation, and the ripples are the stress fractures. (Figure 40)

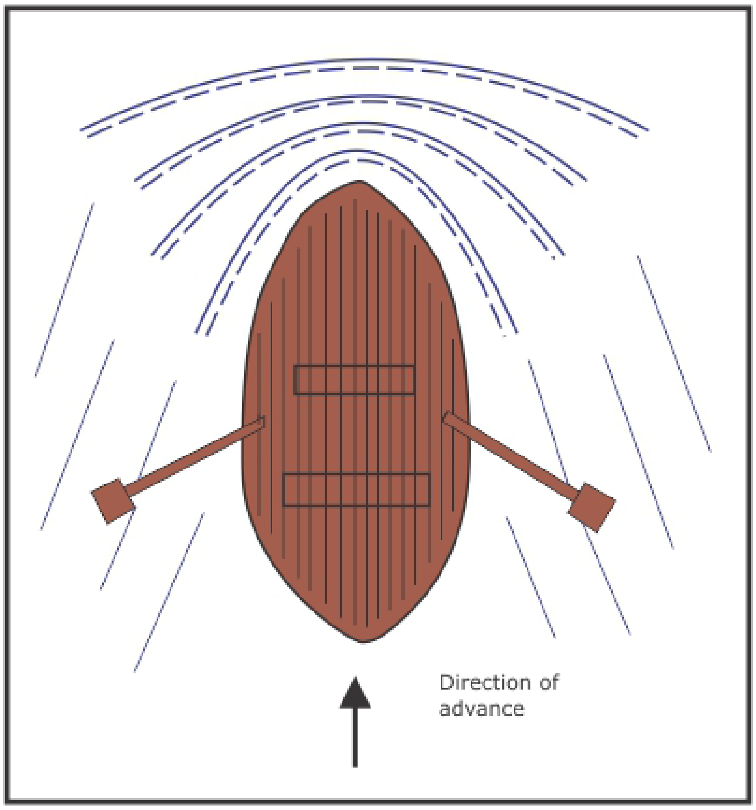

Figure 40: Comparison of stress fractures to ripples in water

Several types of stress fracturing occur in response to mining.

Shear fractures develop in a plane as a result of excessive shear stress exerted in the plane (Figure 40). Displacement occurs across these fractures, crushing the intact rock (comminution) resulting in finely crushed rock known as rock flour being found along the surfaces. These fractures tend to cut through existing discontinuities such as bedding planes or joints. Shear fractures are usually spaced 1-3m apart and can extend up to 20m into the hanging or foot wall.

Tensile fractures occur if the rock is subjected to tensile stress conditions. This is uncommon, but can occur in situations where the hanging wall beam is bent so that tensile stresses are created on the lower surface of the beam.

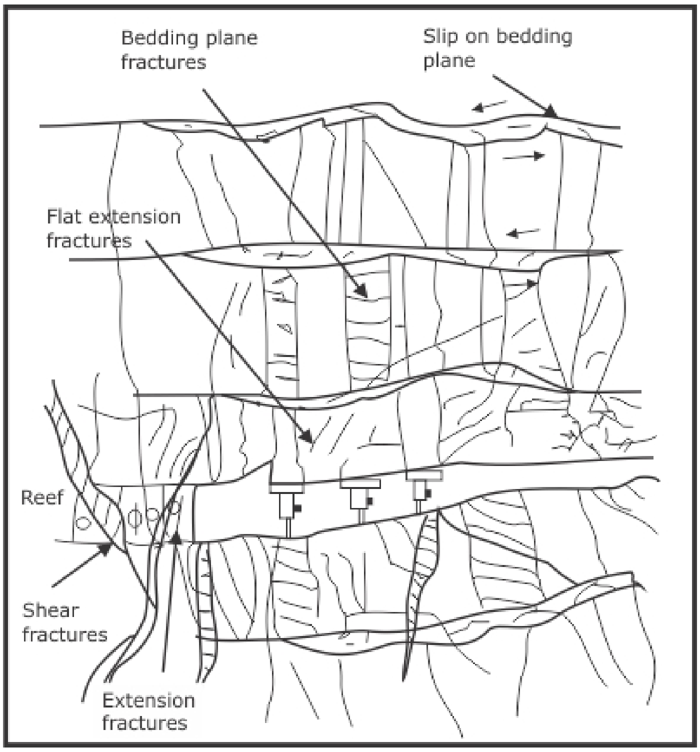

Figure 41: Typical layering and fracturing in an underground deep mine

Extension fractures are the most common type of stress fracture. They develop in the plane normal to the minor principal stress σ3 (or parallel to the direction of the major principal stress σ1) and therefore tend to follow the shape of the excavation outline. The intensity drops off rapidly after a few meters above and below the stope. These fractures give a good indication of the in-situ stress conditions and the shape of the excavation at the time they were formed.Extension fracturing tends to form in bands approximately 1m wide with zones of un-fractured rock between them.

Primary fractures consist of both shear and near vertical extension fractures that usually form beyond 2m into the face. Sometimes if the face advances into a zone of un-fractured rock, secondary fractures can form within the first 2m of the face. These are also vertical but tend to curve over toward the face in the hanging wall.Often this secondary fracturing does not take place, and the un-fractured zone remains intact.When this un-fractured 'hard patch' eventually fractures close to the face, the fractures are developed in the plane normal to σ3 (or parallel to the direction of the major principal stress σ1) direction at that time, and are therefore low angle (20° to 40°) and dip towards the face. These fractures cause considerable ground control problems. (Figure 42)

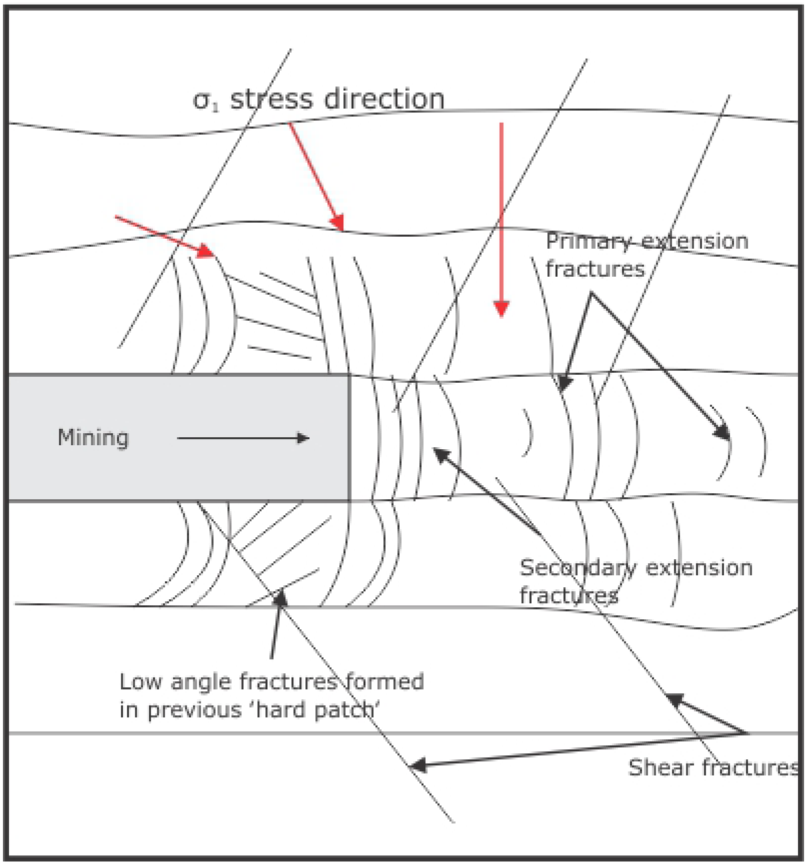

Figure 42 Stress fracturing around a stope

- Page 86,146 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown
- Page 5 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 16 - A handbook on rock engineering practice for tabular hard rock mines, 1999, JA Ryder and AJ Jager

CHAPTER 5

STRESS IN ROCK AND ROCK MASSES

INTRODUCTION

Chapter 4 allowed an understanding of the strength of the rock mass, with special reference to joint and jointed rock mass strengths. Since rock failure is critical to the rock engineer and failure is the combination of stress levels and rock (mass) strength, the stresses the rock mass is subjected to requires understanding.

This chapter leads the rock engineering practitioner to understand the origin of stress, determination or estimation of stress levels and basic calculations to estimate field strengths.

LEARNING OUTCOMES

STRESS ESTIMATION

The candidate must be able to demonstrate knowledge and understanding of the above subject area by being able to:

5.1 Explain Heim's Law;

5.2 Calculate the expected stress regime in rocks based on gravity and thePoisson effect only;

5.3 Explain and comment on the reasons why stresses may vary from those calculated above;

5.4 Describe and explain virgin stresses encountered in the different mining areas of South Africa;

5.5 Describe and explain the principles of stress measurement using strain cells;

5.6 Explain and comment on the reasons for the large variations in the results of the above stress measurements;

5.7 Describe and explain the principles of stress measurement based on AE principles and

5.8 Explain and discuss the use of Kirsch equations with regard to stress determinations.

REFERENCE

- Page 399 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 7,27,176 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 11 Chapter 1 - Practical rock engineering, 2006, E Hoek
- Page 138 Chapter 10 - Practical rock engineering, 2006, E Hoek
- Page 26 - Handbook on hard rock strata control, 1995, AJS Spearing

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LEARNING OUTCOME 5.1

EXPLAIN HEIM'S LAW

The state of stress in the Earth's crust before any mining has taken place is known as the "virgin" state of stress or the "primitive" state of stress. The virgin or primitive state of stress is derived from several components, namely:

- The weight of the overlying strata
- Superimposed tectonic stress, which is derived from regional forces that shape the crust and determine its structure
- Local stress variations imposed by individual geological structures such as faults, dykes, sills and joints
- Residual stresses, which endure in the rock long after the forces that caused them have dissipated and
- Pore water pressure.

In the stable cratonic environment, which exists across much of Southern Africa, the weight of the overburden is the most important component of the virgin stress tensor.

According to Heim's law (Heim's rule) all normal stress components are equal and depth dependant only when at depth and where the rock stress tends to be lithostatic (similar to hydrostatic where stresses in all directions are equal). Heim's law assumes that if a rockmass is left undisturbed for long enough shear stresses will disappear in time through a process of creep, making the normal stress components the principal stresses.

Vertical and horizontal stresses are therefore principal, equal and a function of density, gravitational acceleration and depth below surface ($\sigma_1=\rho gh$). Different rock types will undergo creep processes at different rates, for example halite (salt) will probably develop lithostatic stress conditions in as little as 10 000 years or less, while the same processes in brittle quartzite's may still be incomplete after 100's of millions of years.

Therefore according to the Heim's law $\sigma_1= \sigma_2= \sigma_3=\rho gh$.

A graphical representation of Heim's law is shown in Figure 1where the horizontal stress and vertical stress levels, as depth increases, are shown to be the same ($\sigma_h =\sigma_v$).

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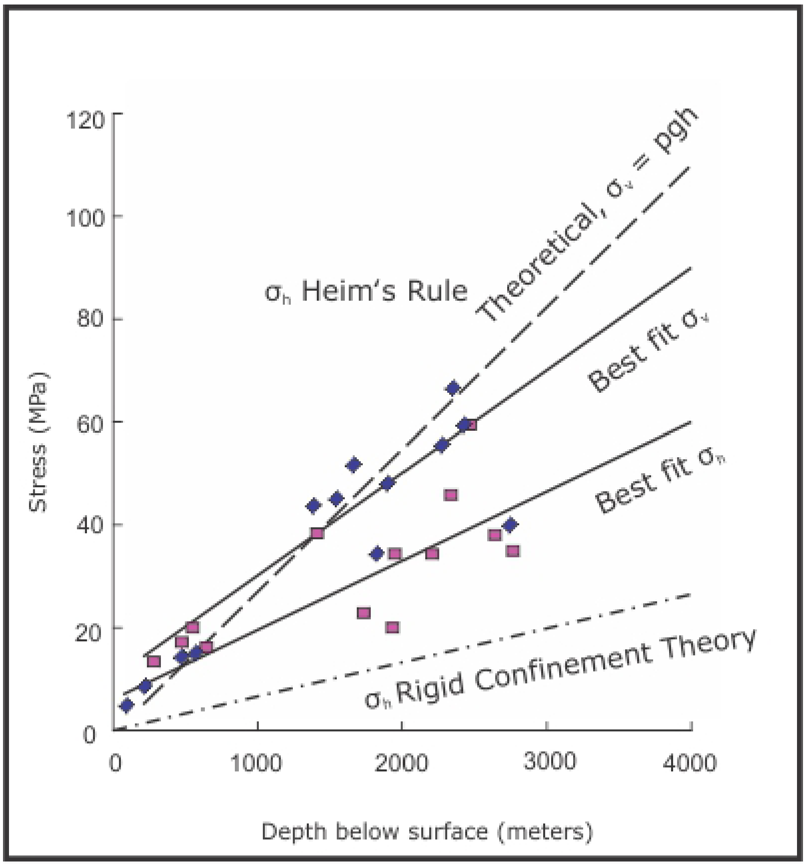

Figure 1: Horizontal stress according to the Heim's rule (Top line)

EXAMPLE

Example Using Heim's law, calculate the principal stresses at a depth of 1500m below surface in a rock mass with a density of 2700kg/m3.

According to Heim's law, the σ_v can be calculated from the density, depth of mining and gravitational acceleration:

σ_v = ρgh where

ρ = Rock density (kg/m3)

g = 9,81 m/s²

h = depth (m)

σ_v = 2700 kg/m³ x 9,81 m/s² x 1500 m

= 39,7305,000 Pa

= <u>39.73</u> MPa

Based on Heim's law, the σ_v calculated above is the principal virgin stress and is equal to the other principal stress components, yielding the following results:

$\sigma_1 = \sigma_2 = \sigma_3 = \underline{39.73}$ MPa.

REFERENCE

- Page 95 - Underground excavations in rock, 1980, E Hoek and ET Brown

LEARNING OUTCOME 5.2

CALCULATE THE EXPECTED STRESS REGIME IN ROCK BASED ON GRAVITY AND THEPOISSON EFFECT ONLY

The stress regime in rock based on gravity and the poisons ratio effect is also called the rigid confinement or lateral restraint theory. Terzaghi and

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Richart (1952) suggested that, for a gravitationally loaded rock mass in which no lateral (horizontal) strain was permitted during formation of the overlying strata, the value of k (ratio of the horizontal to vertical stress levels) is independent of depth and is given by k =v/(1-v) , where v is the Poisson's ratio of the rock mass.

$$\sigma_h=(v/(1-v))\sigma_v$$

In this theory, vertical stress is due to overburden weight and can be calculated using the well-known relationship $\sigma_v = \rho g h$. The horizontal stress is viewed to be caused by a build-up of stress due to the horizontal confinement within the rock mass. The presence of vertical stress strains the rock in the vertical direction and attempts to induce horizontal strain. However, the presence of rock around this particular point in the rock mass, horizontal strain is not allowed and a build-up of horizontal stress results. This horizontal stress is therefore a function of the vertical stress and Poisson's ratio that describes the ratio of vertical to horizontal strain. As the Poisson's ratio increases ($v= \varepsilon_h/\varepsilon_v$), so does the horizontal strain that is being inhibited, also resulting in a larger horizontal stress level due to the confinement.

EXAMPLE

Assuming stress build-up due to the Poisson's ratio effect only, calculate the principal stresses at a depth of 1500m below surface in a rock mass with a density of 2700kg/m^3 . Compare the results with that calculated using Heim's law example. Assume a Poisson's ratio of 0.2.

Horizontal stress is therefore deemed to be determined by the rock property (v) only.

Figure 1 indicates the horizontal stress determined according to the rigid confinement theory for a vertical stress that is depth dependant only ($\sigma_v=\rho g h$), as well as below that estimated using Heim's rule.

σ_v = ρgh where
ρ = Rock density (kg/m3)
g = 9,81 m/s²
h = depth (m)

σ_v = 2700 kg/m3 x 9,81 m/s² x 1500 m
= 39,7305,000 Pa
= <u>39.73</u> MPa

σ_h = v/(1-v). σ_v
= (0.2/(1-0.2))(39.73) MPa
= (0.25)(39.73) MPa
= <u>9.93</u>MPa

The portion of the equation v/(1-v) equates to 0.25 which means that the k ratio in this case is 0.25

REFERENCE

- Page 57 - Rock mechanics in mining practice, 1983, S Budavari et al
- Page 138 Chapter 10 - Practical rock engineering, 2006, E Hoek
- Page 26,75 - Handbook on hard rock strata control, 1995, AJS Spearing
- Page 143 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

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LEARNING OUTCOME 5.3

EXPLAIN AND COMMENT ON THE REASONS WHY STRESSES MAY VARY FROM THOSE CALCULATED ABOVE

Heim's rule assumes visco-plastic material flow to a state where shear stress levels become zero and stress components are equal and depth dependent only, such as in water. This rock mass behaviour is suggested to be prevalent 'at depth' and may indicate the stress state under conditions that are not appropriate for the rock mass in which mining currently occurs.

Rigid confinement suggests that horizontal stress build-up is due to the Poisson's effect only and therefore excludes the impact of residual stress, tectonics, topography, erosion, glaciation etc from this state of stress. It is known that these other conditions have a significant impact on the state of stress encountered during mining and should be catered for when estimating the state of stress.

Although other theories to estimate the vertical and horizontal stress states exist, none of these theories sufficiently describe the origins of the state of stress on its own. These theories may explain conditions that all play a role in the actual state of stress but all fall short in accurately estimating the state of stress. Stress measurements therefore remain the only possible method to determine the actual stress state accurately.

REFERENCE

- Page 58 - Rock mechanics in mining practice, 1983, S Budavari et al
- Page 8 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 143 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 5.4

DESCRIBE AND EXPLAIN VIRGIN STRESSES ENCOUNTERED IN THE DIFFERENT MINING AREASOF SOUTH AFRICA

Under high stress conditions it is possible to obtain an indication of in-situ stress magnitudes from observations of rock behaviour in boreholes and tunnels where spalling and slabbing on the surfaces of these openings can indicate the orientations of the prevalent principal stresses and may allow an estimation to be made of the major stress component orientations.

A database of in-situ stress measurements carried out in Southern Africa has been compiled by Stacey and Wesseloo (1998) and is available in several sources, but do not cover all the mining areas in detail. (Figure 2)Where in-situ stress measurements have not been made in a mining environment, it is recommended that an indication of the in-situ stress levels that might be applicable at the mine, is obtained from this database. However, should evidence indicate at a laterstage that the virgin stress levels are different to those indicated in the database, it may be necessary to carry out a programme of in-situ stress measurements at the mine.

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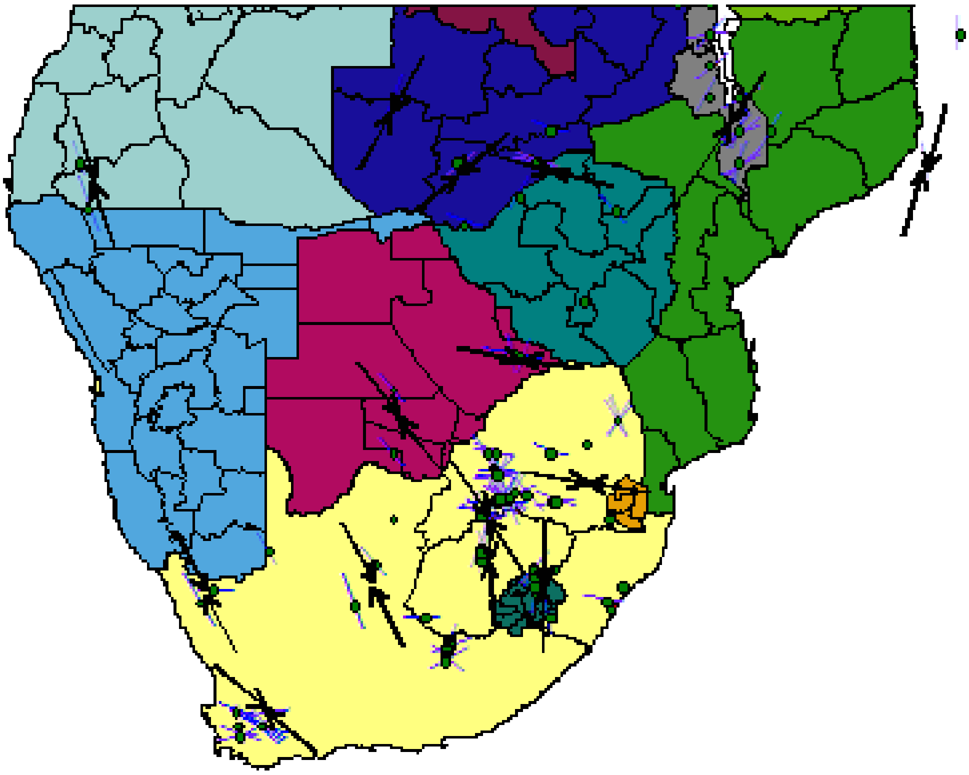

Figure 2: Orientation of horizontal stresses (Best practice handbook for other mines - TR Stacey - OTH 602)

The k-ratio is utilised to represent the ratio between stress components, with the best known ratio being that between the major horizontal to the vertical in-situ stresses (normally referred to as the ratio k_1). The distribution of this ratio for various mining areas and mining depths in South Africa is presented in Figure 3, showing a minimum cut-off value at a depth of 0.5 and ratios as large as 4 in shallow mining areas. The large variation in the k-ratio at all mining depths indicates the impact of local conditions or situations on the state of stress and the difficulty in explaining the existence of these levels with a single theory.

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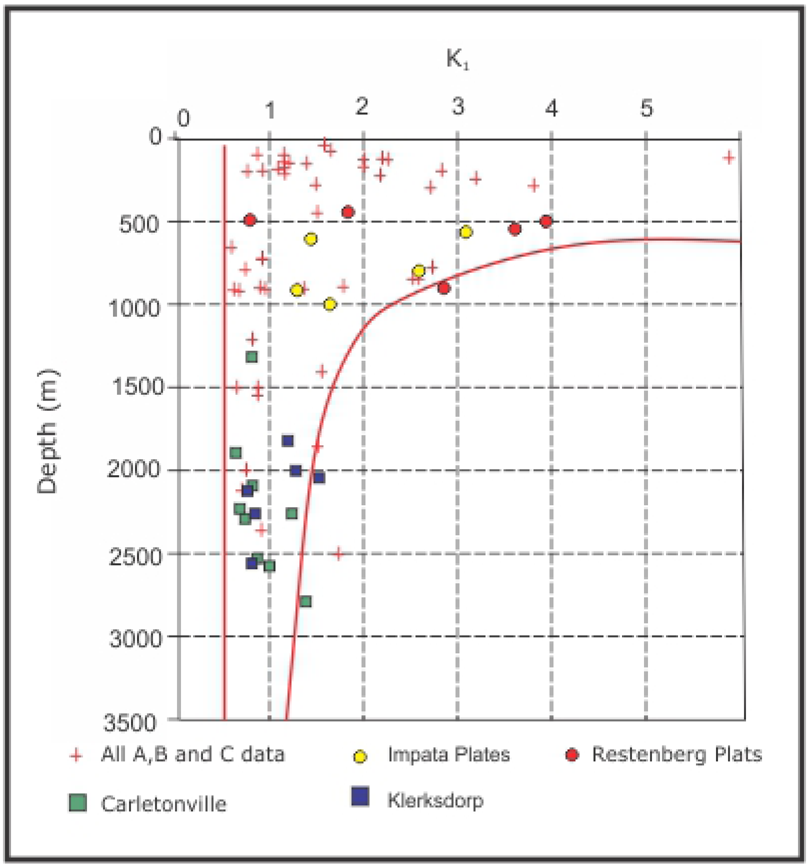

Figure 3: **Ratio k_1 of major horizontal in-situ stress to vertical stress (Best practice handbook for other mines - TR Stacey - OTH 602)**

Figure 4 below shows the ratio k_3 (minor horizontal to vertical in-situ stress)for various mining areas and depths in South Africa. Although the variation in the results appears lower than for k_1, values still vary between 0.2 (at depth) to approximately 2 in shallow areas.

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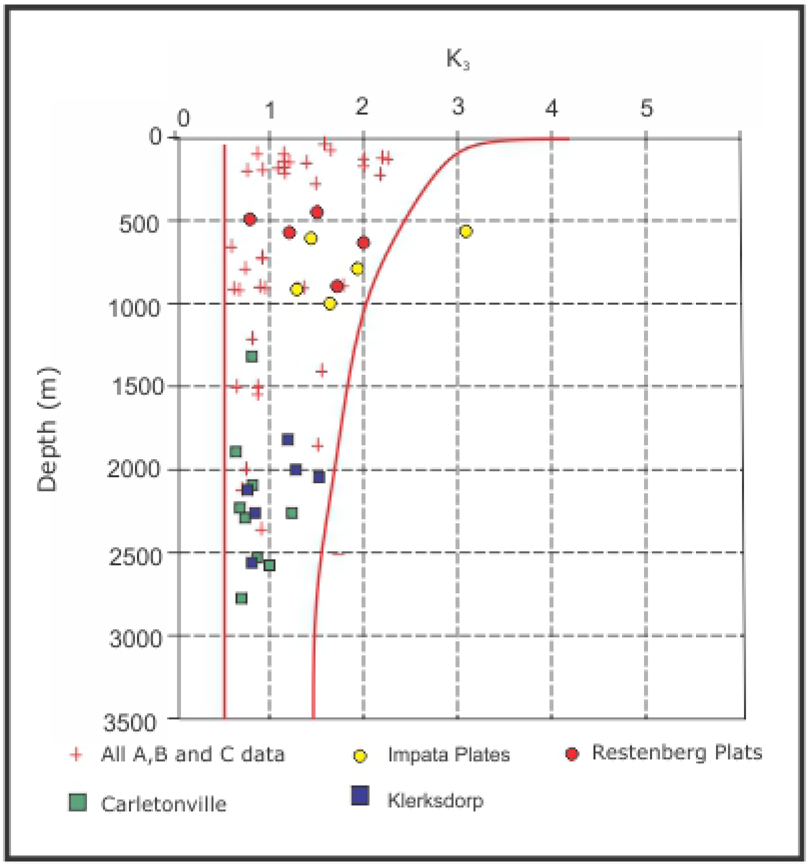

Figure 4 **Ratio k_3 of minor horizontal in-situ stress to vertical stress (Source: Best practice handbook for other mines - TR Stacey , OTH 602)**

- Page 59 - Rock mechanics in mining practice, 1983, S Budavari et al
- Page 403 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 115 - Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page
- Page 11 - A handbook on rock engineering practice for tabular hard rock mines, 1999, AJ Jager and JA Ryder
- Page 177 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden
- Page 159 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown
- Page 19,42 - Best practice rock engineering handbook for other mines, 2001, TR Stacey

LEARNING OUTCOME 5.5

DESCRIBE AND EXPLAIN THE PRINCIPLES OF STRESS MEASUREMENT USING STRAIN CELLS

The in-situ stress field in which mining excavations are made,is a very important input parameter for mine design to ensure safe mining operations. In deep mining situations, higher stress levels actually assist in confining the fractured rock mass. Excessively large stress levels may, however, induce failure of the rock massresulting in instabilities.

It is essential that the anticipated stress levels are correctly predicted by means of numerical modelling (the codes apply the original stress state

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as a boundary condition in the models) in the planning of mining operations while layouts and long term mining sequences are planned taking cognizance of the expected prevalent stress states (requires some knowledge of the stress state) as well as the expected stress state changes.

For this reason it is necessary to carry out in-situ stress measurements. Stress levels are measured using various methods, most actually measuring the strains the rock undergoes and then require calculations to estimate the stress levels that induced the measured strains.

Some in-situ stress measurement techniques are listed below:

- Borehole over-coring methods such as the doorstopper strain cell, the CSIR triaxial strain cell and the CSIRO Hi cell;
- Large diameter over-coring in a bored raise;
- Hydrofracturing;
- Back analysis from monitored deformations around excavations.

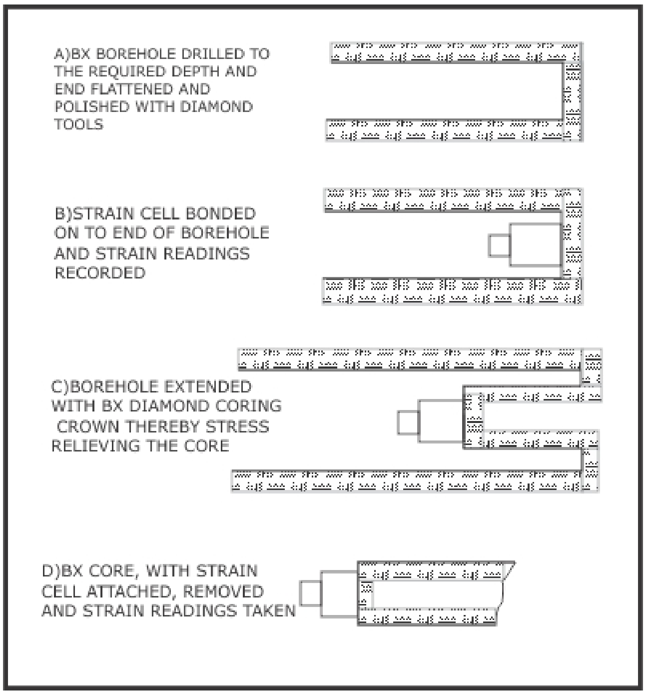

Figure 5:
Installation sequence of a CSIR doorstopper (Project report - GAP 220.)

The "doorstopper" is commonly used in the measurement of stresses around excavations or geological structures. With this technique, the state of stress is determined fromthree normal strain components measured in a particular orientation installed in a borehole drilled along the edge of the excavation. The doorstopper is then over-cored resulting in a stress/strain relief and strain gauges are used to measure this strain relief. These strains then have to be converted into stresses using the hostrocks elastic constants appropriate to small gauge lengths. It is necessary to have results of at least three orthogonal boreholes to determine the state of 3-D field stress in the rock mass.(Figure 5)

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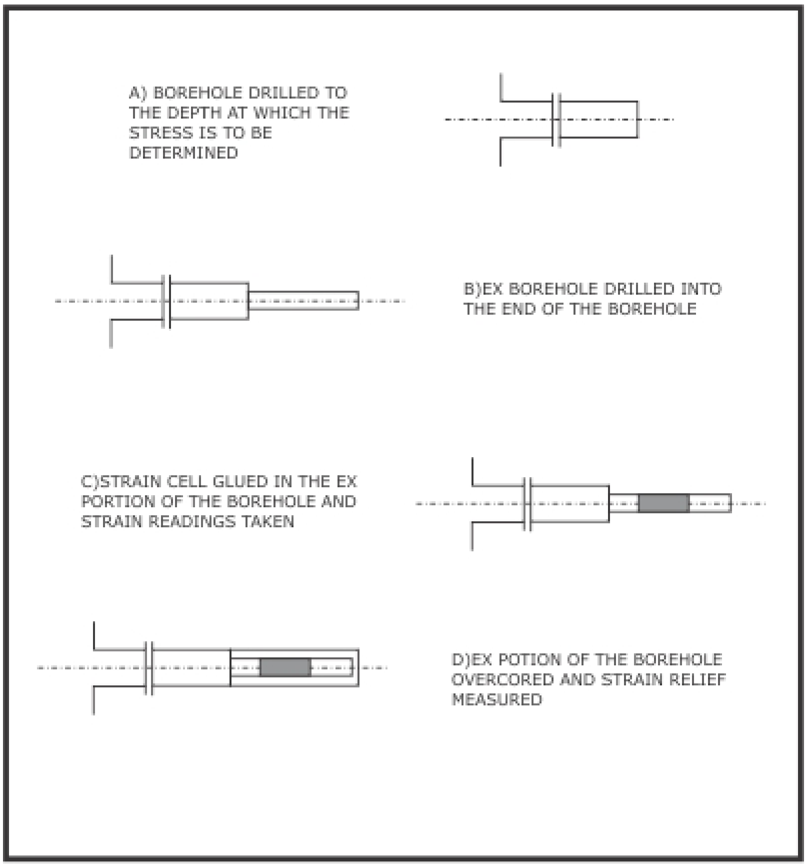

Figure 6 CSIR and CSRO triaxial system of stress measurements (Project report - GAP 220)

Two strain gauge rosette configurations of 0°-45°-90° or 0°-60°-120° rosettes are most commonly used in the measurement of strains. These strain gauge combinations are selected as they allow convenient trigonometric solutions to solve for the relevant strain components using the measured strain. (Figure 6)

For the different strain gauge rosettes,the magnitudes and directions of the principal strains can then be obtained using the equations given below:

0°-45°-90°rosette:

$$\varepsilon_{1,2} = \tfrac{1}{2}(\varepsilon_0 + \varepsilon_{90}) \pm \tfrac{1}{2}\sqrt{[(\varepsilon_0 - \varepsilon_{90})^2 + (2\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90})^2]}$$
$$\mathrm{Tan}2\Theta = (2\,\varepsilon_{45} - \varepsilon_0 - \varepsilon_{90}) / (\varepsilon_0 - \varepsilon_{90})$$

0°-60°-120°rosette:

$$\varepsilon_{1,2}=(\varepsilon_0+\varepsilon_{60}+\varepsilon_{120})/3\pm(2/3)\sqrt{[\varepsilon_0^2+\varepsilon_{60}^2+\varepsilon_{120}^2-(\varepsilon_0\varepsilon_{60}+\varepsilon_{60}\varepsilon_{120}+\varepsilon_{120}\varepsilon_0)]}$$
$$\mathrm{Tan}2\Theta = (\sqrt{3})\,(\varepsilon_{60}-\varepsilon_{120})/(2\varepsilon_0-\varepsilon_{60}-\varepsilon_{120})$$

NOTE

Examples in stress/strain transformations were dealt with in Section 2.7, Chapter 2 of this guide.

CONNECTION 39

Additional information on stress measurements is available in Project Reports GAP 220 and GAP 511b.

Other stress measurement methods and techniques

1. Strain measurements using the Borre probe equipment

The equipment for overcoring rock stress measurements using the Borre probe comprises:

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- pilot hole drilling equipment including both wire line and conventional pilot hole drilling equipment, planning tool and grinding bit;
- inspection tool (test probe) with built-in borehole cleaning brush;
- Borre probe with built-in data logger;
- set of strain gauges (to be mounted on the Borre probe);
- glue (for bonding strain gauges to the borehole wall);
- cell adapter (installation tool);
- glass fibre rods (for installation in sub-horizontal boreholes);
- bi-axial test equipment including load cell, pressure gauge, hydraulic pump and strain indicator; and
- portable computer.

The most vital part of the equipment is the probe itself, which is shown in Figure 7. The instrument carries nine electrical resistance strain gauges mounted in three rosettes. Each rosette comprises three strain gauges oriented

- parallel (axial or longitudinal gauges),
- perpendicular (circumferential or tangential gauges), and
- at a 45° angle, to the borehole axis, respectively, see Figure 8.

Therefore, the nine strain gauges of the Borre probe form an array representing seven spatially different directions. All strain gauges are mounted at a depth of 160mm in the pilot hole.

Figure 7: Borre probe

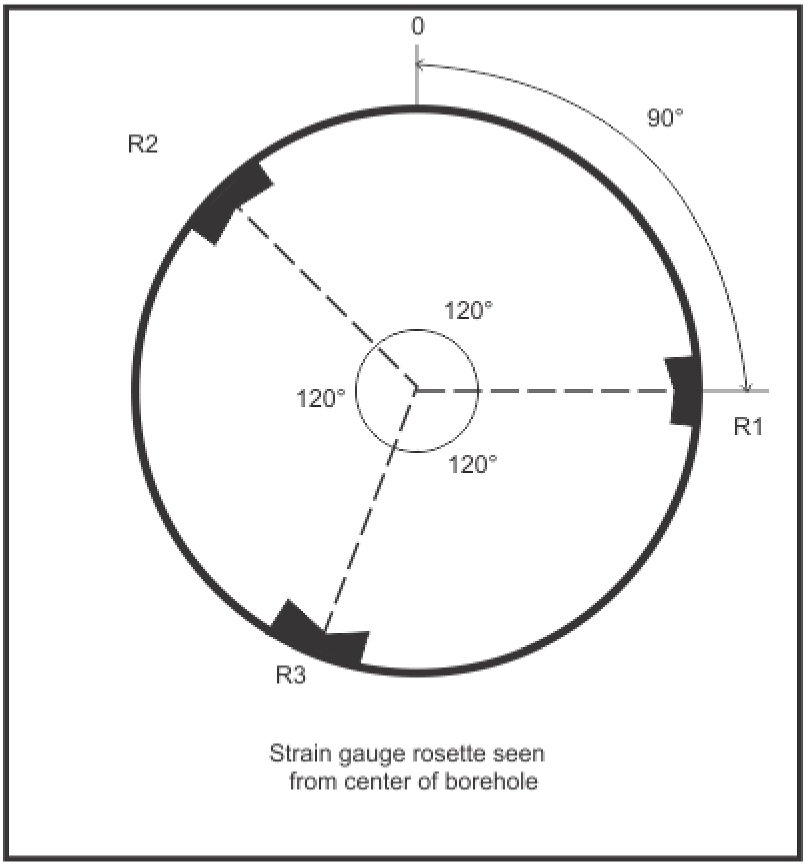

Figure 8: Strain gauge rosette

The strain gauges are connected to a data logger inside the probe. The probe also measures the temperature in the borehole to assess the temperature effects on the readings during the overcoring phase.

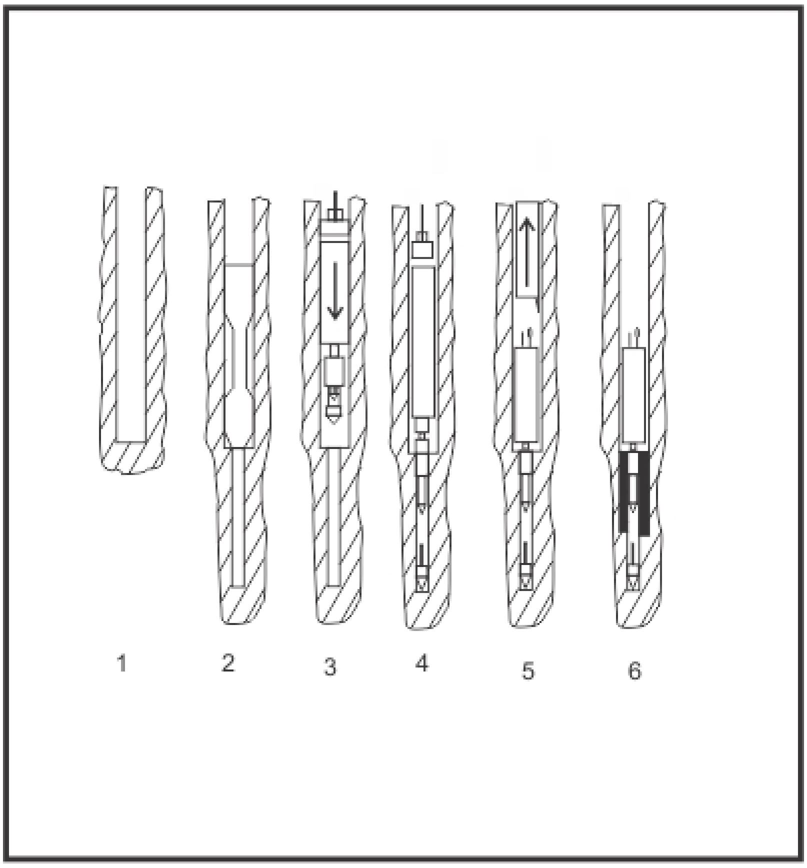

Figure 9: Installation and measurement procedure for overcoring with the Borre probe.

NOTE

Installation:(Figure 9)

i. Advance 76mm diameter main borehole to measurement depth. Grind the whole bottom using the planning tool.

ii. Drill 36mm diameter pilot hole and recover core for appraisal. Flush the borehole to remove drill cuttings.

iii. Prepare the probe for measurement and apply glue to strain gauges. Insert the probe in installation tool into hole.

iv. Tip of probe with strain gauges enters the pilot hole. Probe releases from installation tool through a latch, which also fixes the compass, thereby recording the installed probe orientation. Gauges bonded to pilot hole wall under pressure from the nose cone.

v. Pull out installation tool and retrieve to surface. The probe is bonded in place.

vi. Allow glue to harden overnight. Overcore the probe and record strain data using the built-in data logger. Break the core after completed overcoring and recover in core barrel to surface.

The probe is prepared and installed in the pilot borehole. After a predetermined time for glue hardening, over-coring of the probe is carried out. A detailed checklist is followed to control drilling rate, rotational speeds, flushing, etc. Flushing is recommended to start 10 to 15 minutes before the commencement of over-coring and continue for at least 5 to 10 minutes after completion of the over-coring. The borehole should then be left with no ongoing activity for at least another 5 to 10 minutes before the core is broken loose from the hole. This procedure ensures that sufficient strain data are recorded to assess temperature effects, possible non-ideal rock behaviour, etc. which may adversely affect strain readings and measurement results. After over-coring, the probe is recovered with the over-core sample inside the core barrel. Before removal of the sample and disconnection of the strain gauges, the data recorded are retrieved according to the Measurement Contractor’s procedures.

2. Core – discing

Core-discing is an indirect method to determine in-situ rock stress levels and is based on rock behaviour.Borehole cores often appear to have discs at certain intervals along the length of the drill core (Figure 10), formed as a result of stress relief after the core has been drilled.

The fracture trajectory experiments done by Haisom and Lee (1995: 19-24) showed that the concave axis gives the direction of the maximum principal stress, the intermediate principal stress is oriented in the convex axis and the minimum principal stress is normal to the core axis. (Figure 11)

Figure 10: Borehole core with discing (Matsuki et al, 2004).

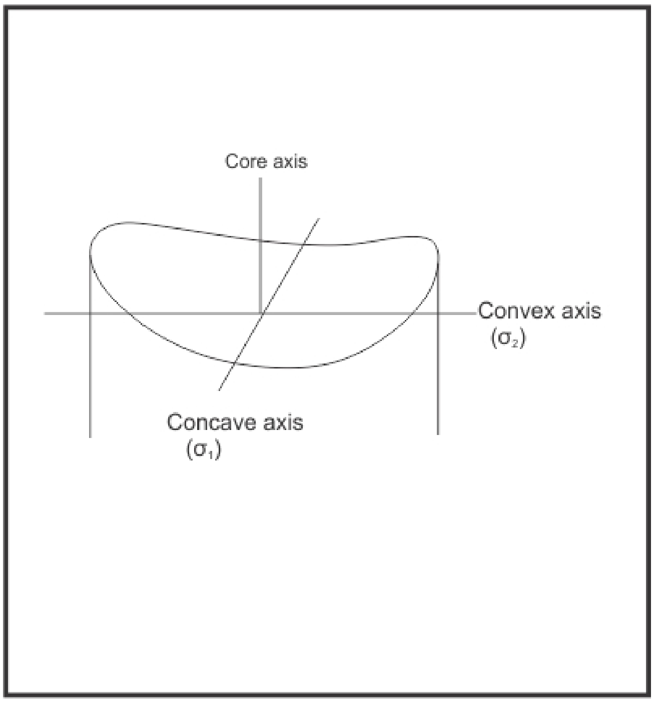

Figure 11: Disc end indicating direction of principal stresses (Matsuki et al, 2004).

According to Stacey and Wesseloo (2002), the stress orientations can also be approximated by observing the shape and symmetry of the discs:

i. When the core has symmetrical discs about the core axis, it is highly likely that drilling took place along the direction of the one of the principal stresses, as illustrated in Figure 12.

Figure 12: Symmetrical core discs (Stacey &Wesseloo, 2002).

ii. A principal stress axis that is at an incline to the borehole axis can be gauged from the relative asymmetry of the disc. When the stresses perpendicular to the core axis are not equal, then the core circumference will peak and trough, the line drawn between the peaks of the discs indicates the direction of the minor secondary principal stress.(Figure 13)

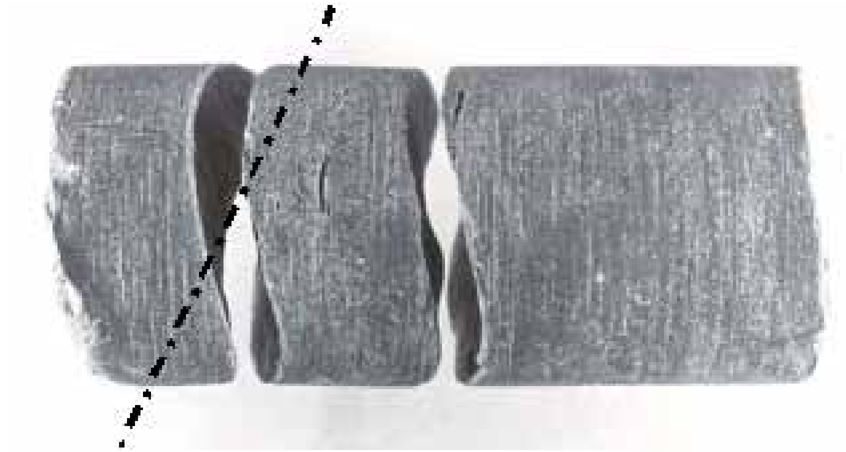

Figure 13: Core discs resulting from unequal stresses (Stacey & Wesseloo, 2002)

iii. Core discs that are not symmetrical indicate the presence of shear stress orientated across the borehole axis or the borehole axis is not in the same direction as the principal stress. (Figure 14)

Figure 14 Non-symmetrical core discing (Stacey &Wesseloo, 2002).

It has been indicated from experimental results that the disc thickness is a function of stress magnitude of the maximum principal stress (σ_1) which is the stress perpendicular to the disc thickness. The maximum principal stress is calculated using the formula

$$\sigma_1 = A + B/e^{(t/C)}$$

where A, B and C are curving fitting parameters which differs from rock to rock and t is the disc thickness in mm.

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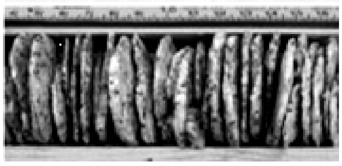
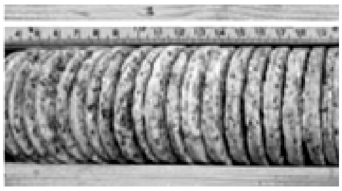
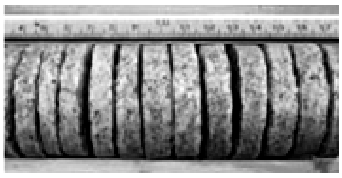
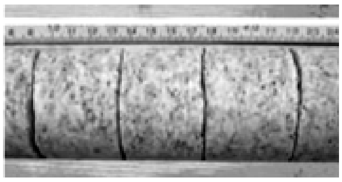
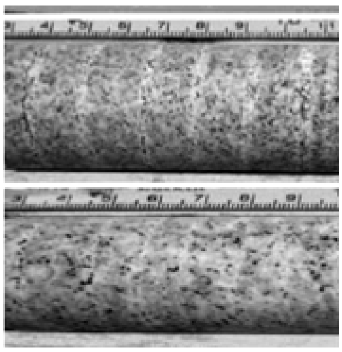
Term	Core photo	Description	Disk thickness t/D	Stress condition σ_1 / σ_t
Core disking		Crushed disking. The disks are cruished due to extremely high stress levels	< 0.5	>11
		Very thin disking. The majority of the disks are intact but a few are crushed	0.05 – 0.12	9 - 11
		Thin disking. The shape and spacing of the core disks are uniform	0.12 – 0.2	8 - 9
		Medium disking. The shape and spacing of the core disks are uniform.	0.2 – 0.4	7 - 8
		Thick disking. The shape of the core disks is uniform.	0.4 – 2.2	6.5 - 7
Partial disking		Partially disking. Distinct white lines appear at regular spacings similar to that observed when complete disks form, but the core remains intact. Spacings between the lines are similar to that of medium disking.	0.2 – 0.4	6 - 7

Figure 15: Disc thickness and fracturing variation with stress magnitude (Lim & Martin, 2010)

According to Chang et al (2003) this method is an indicator for qualitatively indicating 2D rock stresses, because it is assumed that the vertical stress is given by the overburden stress. Estimating stress using core discing method is the most appropriate when stress magnitudes at early stages are required.

This method is not limited to disc thickness or core length. However, type or technique of drilling influences the occurrence of discing, therefore observations and measurements of in-situ stresses using this method are likely to be inaccurate. Furthermore, the fact that it only indicates stresses during early stages implies that as mining progresses stresses induced cannot be determined using core discs (Chang et al, 2003).

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3. Jacking (The flat jack method)

The flat jack method is for the measurements of near surface rock stresses found in the exposed roof, walls, or floor of the excavated area, regardless of the surfaces being underground or on the surface.

This method is explained as a direct and non-destructive method whereby the test is performed on an in-situ material. The instrument is similar to an envelope with an inlet and outlet ports that would be pressurised with hydraulic oil. Dimensions of the jack instrument vary based on the function of the test. Based on the function and the area whereby the test would be utilised, the instrument's size and shape would be changed accordingly.

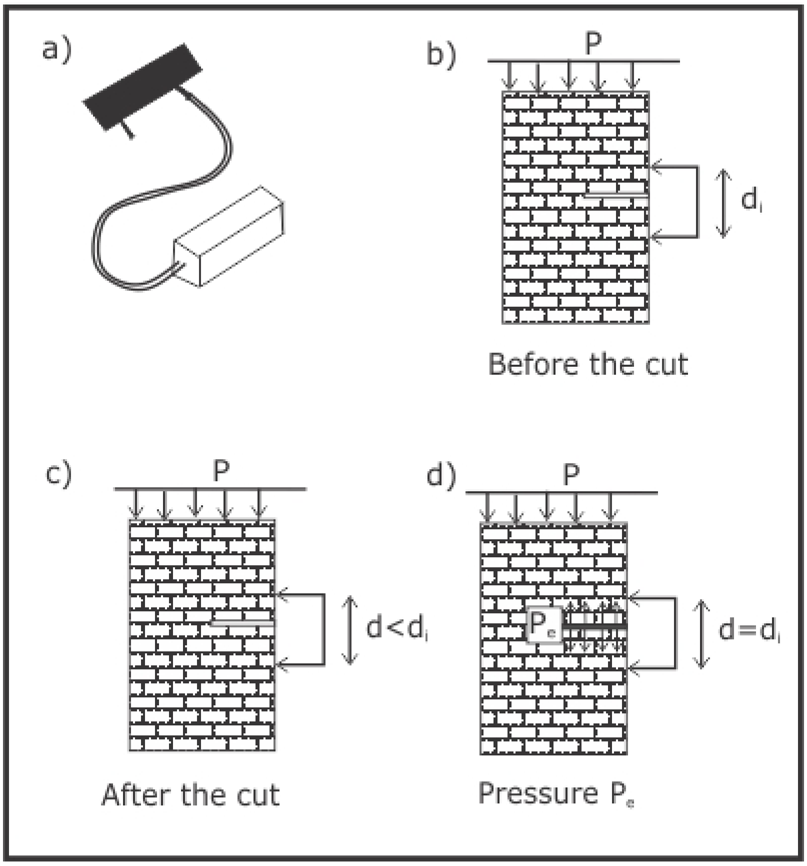

Figure 16: Demonstration of how a flat jack is used

Figure 16 demonstrates the process that should be followed when a flat jack test is applied:

- A field of displacement (labelled d_i in the Figure 16) is determined by selecting points of reference, these points should be fixed to the surface from which the test will be conducted;
- A slot is removed from the surface on which the stress would be measured. The slot should be perpendicular to the initially measured field of displacement (d_i);
- Removal of a slot from the surface whereby stresses will be measured results in stress relief and therefore deformation of the area around the slot. This will result in a decrease in the field of displacement. The decrease can be measured by determining the new "displacement zone" (denoted as d in the Figure 16 below), therefore $d<d_i$;
- A thin jack is inserted into the slot and compressive stress is applied to the walls around the jack. The compressive stress causes the new displacement field (d) to move towards the initial displacement (d_i)

until a point at which $d \approx d_i$ or $d = d_i$. When interpreting the results it is important to take note of the following factors:

- The stress that is needed to return the displacements to the state d=di is taken from the jack, but the jack indicates higher stress levels due to the tendency of the flat jack to resist expansion when pressurised. The rock stress will therefore be less than the compressive stress from the hydraulic flat-jack;
- The area of the slot is greater than that of the flat jack.

The jacking method is preferred since it is a non-destructive test. Samples that are not destructed can be used to analyse the mechanical behaviour of rocks sampled. The test yields the following results:

- Compressive stress
- Deformability properties, and
- Load applied to the sample.

In-situ stresses are determined by measurements from the manner in which the rock responds to the pressure. During the test, the rock is assumed to be in the elastic phase. Each test that is performed is a single component of the in-situ stress field in the rock being tested, therefore the acceptable number of tests is six, so that a complete set of stress components can be obtained.

Assumptions made when using the flat jack method:

- The stress experienced in this test is compressive;
- The rock mass around the wall on which the test is being conducted is homogenous, the rocks around the slot deform symmetrically;
- There are uniform stresses at the point where the test is conducted;
- The flat jack applies uniform stress to the rock around it;
- The mass of rock where the test is conducted experiences elastic deformation.

The advantages of jacking stress method:

- Flat jack does not require knowledge of elastic stresses and the rock's reaction to stresses;
- It is a direct stress measurementat the point at which the jack is located in the strata;
- The equipment is stable, therefore fewer errors are expected;
- The test can be performed over large areas;
- This method is less expensive.

Stress levels and orientations

Stresses measured by the flat jack are in the regions from around 0.3 MPa to 3.5 MPa. The flat jack applies compression stresses to the rock around it in a direction perpendicular to the jack.

Application and impact on the mining environment

Figure 17 to Figure 20 illustrate how the method can be applied.Figure 17suggests how a flat jack plate is being inserted into the slot that would have been cut out from the face of the rock strata where tests will be conducted. Once the jack has been placed into the slot, the rest of the equipment is connected and the test can commence.

Figure 17: Flat jack being inserted into the slot

Figure 18: Equipment used to test stress

Data can continuouslybe collected as the test is being conducted if an electronic system, is used, but can also be recorded manually.

Figure 19: Data being captured during testing

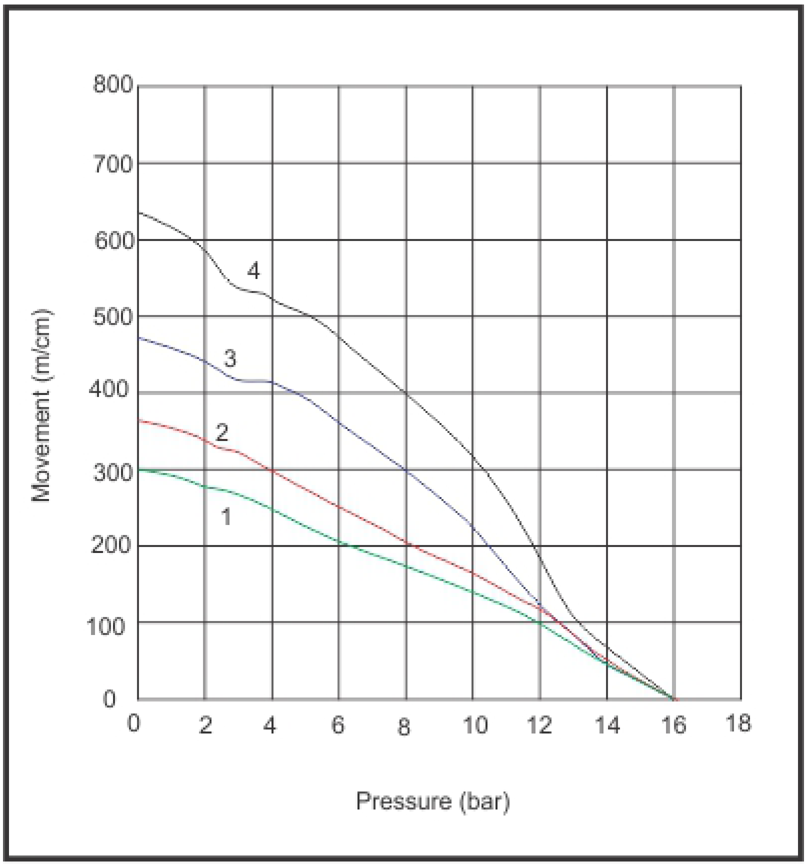

Figure 20: Results from a test

Figure 20(a) shows points at which displacement was measured. In Figure 20(b), the results are provided for analysis. The figure shows that as the pressure is applied to the displaced rock by the flat jack, a pressure is reached at which the displacement returns to zero. The pressure applied by the jack at this point suggests the rock stress level at that point.

4. Hydraulic Methods

There are two hydraulic methods that can be used to measure the stress of rock in situ; the Hydraulic Fracturing method (HF) and the Hydraulic Test on Pre-existing Fractures (HTPF).

These methods use the same mechanism on determining the rock stress, the only difference is that with the HF method, a fracture has to be initiated whereas with the HTPF method, a pre-existing fracture is re-opened or extended.

4.1 Hydraulic Fracturing method (HF)

In order to measure the rock stress, a borehole has to be drilled and then sealed off with inflatable rubber packers. The borehole has to be drilled in an area that is free from any geological disturbances. Typical diameters of the borehole range between 76 to 96mm. The area that needs to be sealed off along the borehole,is normally 1m. The rubber packers have to be pressurised to stick to the sidewall of the borehole. The packers are normally pressurised to 2-4MPa.

Once that is achieved, a hydraulic fluid has to be pumped into the borehole at a constant flow rate; water is normally used as a hydraulic fluid but alcohol is also an option. Pumping a hydraulic fluid into the borehole increases the pressure, up to peak pressure indicating tensile stress at the sidewall of the borehole.

The pressure that initiates a fracture (peak pressure) is known as the breakdown pressure. As soon as a fracture is initiated, the pumping is stopped and the pressure gradually decreases until it is equal to the surrounding pressure. As the pressure decreases, the fracture closes where the pressure at which the fracture is closed is known as the shut-in pressure. In order to obtain accurate results, this procedure has to be repeated a number of times at a constant flow rate of the hydraulic fluid.

According to Ljunggren et al (2003), during the fracture initiation process, two fractures are created at the same time orientated in opposite directions and the fracture plane created lies parallel to the borehole axis. The fracture developed is orientated in the same direction as the major horizontal stress and lies perpendicular to the minor principal stress.

The equipment used for HF is the same as the equipment used for HTPF. Figure 21 illustrates the equipment set up when conducting the test and

Figure 22 gives a summary of the equipment used and the function of each piece of equipment.

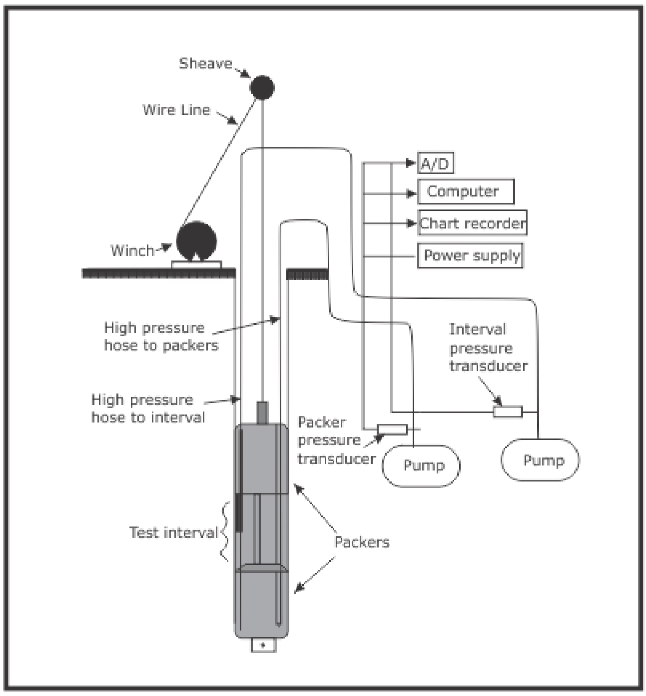

Figure 21: HF equipment set up (Haimson and Cornet, 2003; 1013)

Equipment	Function
Inflatable rubber packers	Seal off the borehole test interval, these rubbers should be placed at least 6×diameter of the borehole.
High-pressure tubing/hose	Transportation of hydraulic fluid to the zone of interest.
Pumps	Generated hydrau-lic fluid pressure.
Pressure gauges	Situated on surface Give information regard-ing hydraulic fluid pressure.
Pressure transducers	Monitor and transmit pressure data to a recording device.
Flow meter	Situated on surface Monitors the rate at which the hydraulic fluid flows.
Recording equipment	Records information from the transducers and the flow meter. The information is transferred to the computer via a digital board.

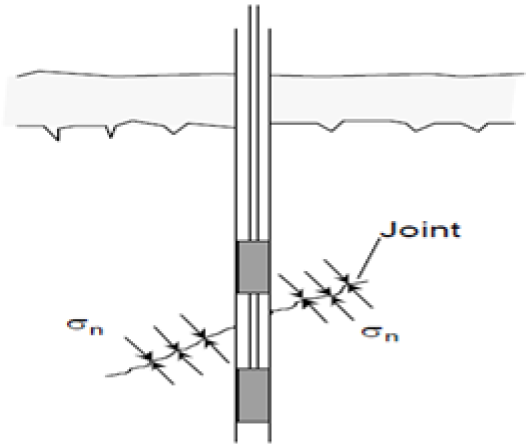

Figure 22 Equipment used and functions (Haimson and Cornet, 2003)

The calculations given by Haimson and Cornet (2003) only apply to vertical holes and tests that will produce vertical fractures, the inclination for the fractures must lie within a range of ±15% or so.The major and minor principal stresses have to be calculated.

Minor principal stress calculation (σ_h)
The fractures formed during the HF technique form perpendicular to the minimum horizontal principal stress. According to Haimson and Cornet (2003) ”the shut-in pressure (Ps) is the pressure needed to equilibrate the fracture-normal stress, which in this case is (σ_H)”. In other words, the shut-in pressure is the normal stress that acts on the fracture plane. Therefore the following equation is obtained
σ_h= Ps in the direction of the minor principal stress, perpendicular to the fracture

Major principal stress calculation (σ_H)
The maximum principal stress is calculated with an assumption that the rock behaves in a linear elastic manner and there is no pore fluid in the zone of interest. With these assumptions, the equation below can be used to calculate the magnitude of the maximum principal stress (Haimson and Cornet , 2003).

$$\sigma_H = T + 3\,\sigma_h - P_b$$

where T = tensile strength of the tested rock, P_b= breakdown pressure, σ_h= minor principal stress, σH direction = direction of vertical hydraulic fracture strike.

The major principal stress is perpendicular to the minor principal stress orientations.

Assumptions and limitations

Haimson and Cornet (2003) stated the assumptions and limitations of utilising the HF method, and they are as follow:

- The method has no limit to the test depth but it is important that the rock within the borehole is elastic and brittle and the borehole must be stable enough in order to access the area of interest;
- The rock mass is assumed to behave in an elastic manner, homogeneous and isotropic;
- Interpretation of the results is possible if the borehole axis is parallel to one of the principal stresses and is contained in the induced fracture plane;
- The borehole must either be diamond drilled or have a smooth surface.

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4.2 Hydraulic Test on Pre-existing Fractures (HTPF)

The HTPF method was developed from the HF method. The same measuring equipment used in the HF method is also used in the HTPF method. The same method of initiating pressure and stress on the borehole sidewall is used, the only difference is that there has to be a pre-existing fracture which is re-opened/extended. With this method tensile stress is not taken into consideration, instead, the normal stress that supports the fracture plane is evaluated.

According to Ljunggren et al (2003), there are four important factors(field parameters) that the HTPF method is dependent on and they are as follows:

- test depth;
- shut-in pressure. The shut-in pressure is the normal stress that acts on the fracture plane;
- dip of the tested fracture;
- strike of the tested fracture.

If the above-mentioned parameters are known, the rock stress can be determined in 2D and in 3D. In order to obtain good results in 2D, at least six pre-existing fractures with different dips and strikes should be analysed. To increase the accuracy of the results, it is suggested to work on 10 to 12 pre-existing fractures with different dips and strikes but the fractures must have the same depth interval (test depth).

For stress measurements in 3D, more pre-existing fractures need to be analysed, at least 18 to 20 fractures. Figure 23 is a diagram illustrating the concept of HTPF

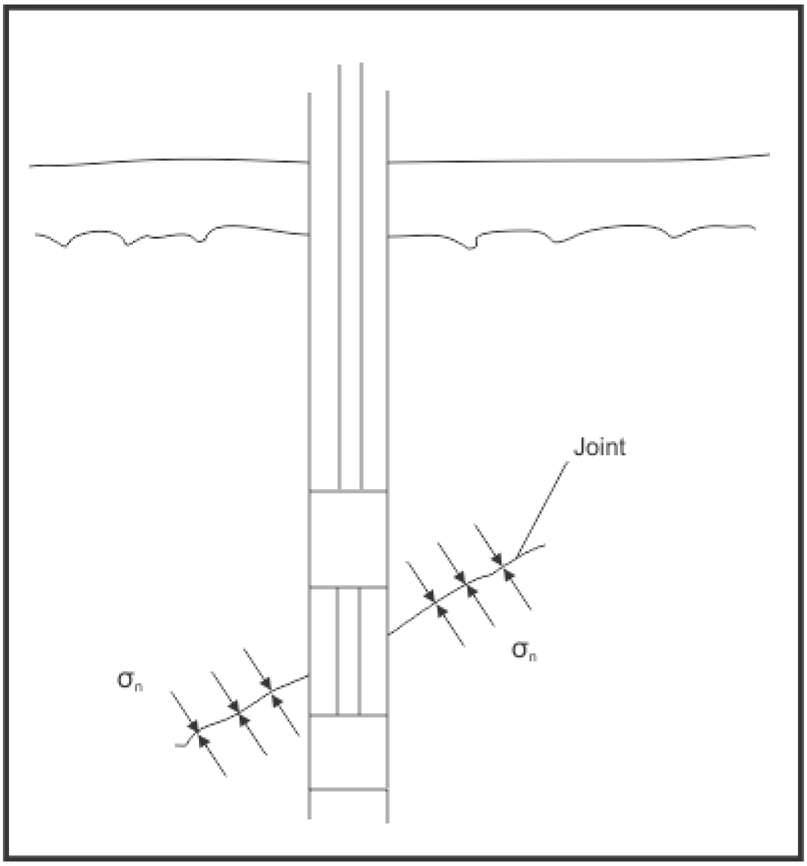

Figure 23: The concept of HTPF

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Assumptions and limitations

- The assumptions and limitations of the HTPF as stated by Haimson and Cornet (2003) are as follows
- The method has no limit to the test depth but it is important that the rock within the borehole is elastic and brittle and the borehole must be stable enough in order to access the area of interest;
- An assumption that there is a pre-existing fracture is made and it is also assumed that the fracture can be re-opened/extended by means of mechanical action while conducting the test;
- There are no limits regarding the orientation of the borehole, and the HTPF is independent of the pore fracture and material property;
- "For a complete stress tensor determination, the method requires a theoretical minimum of six tests, each conducted on pre-existing non-parallel fractures;
- Additional tests are recommended in order to correct for uncertainties. Figure 24 illustrates the advantages, disadvantages and applications of the HF and HTPR methods.

Method	**Advantages**	**Disadvantages**	**Application**
HF	• Measurements can be performed in a pre-existing hole • Large rock volume is used to conduct the tests • Quick method to test for rock stress	• Stress measurements can only be done in 2D • Disturbs water chemistry	• Can be applied to shallow to deep measurements • Applied to rocks with a volume ranging between 0.5 and 50m^3
HTPF	• Measurements can be performed in a pre-existing hole • Can be applied in an area where HF and overcoring have failed	• Test cannot be conducted without the existence of fractures • Time consuming method	• The method is conducted where HF and overcoring have failed since it is time consuming. It is never considered as a first option for stress measurements • Applied in rocks with a volume ranging between 1and10m^3

Figure 24 Hydraulic methods and their application. (Ljunggren et al, 2003; 987).

- Page 48 - Rock mechanics in mining practice, 1983, S Budavari et al
- Page 95 - Underground excavations in rock, 1980, E Hoek and ET Brown
- Page 143 Chapter 10 - Practical rock engineering, 2006, E Hoek
- Page 55,411,415 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page137 - A handbook on rock engineering practice for tabular hard rock mines, 1999, AJ Jager and JA Ryder
- Page 147 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

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LEARNING OUTCOME 5.6

EXPLAIN AND COMMENT ON THE REASONS FOR THE LARGE VARIATIONS IN THE RESULTS OFTHE ABOVE STRESS MEASUREMENTS

Borehole over-coring methods rely on small changes in strains on the surface of the borehole. As the volume of rock surface involved is relatively small, the actual behaviour of the rock may be influenced by the local behaviour of the rock at the measurement location.Errors in stress measurements may also occur when the incorrect rock properties are used in the strain/stress transformation calculations.

Strain gauges may be the same as the grain sizes in which case the measurement will be unreliable. Minor micro-fracturing beneath a strain gauge can affect the strain measurement drastically.Martin and Christiansson 1991 have found variations of more than 50% in apparently good quality granite due to micro-fracturing in the vicinity of strain gauges.
In very high stress conditions, large variations in stress measurements

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can be expected. In such cases a large amount of measurement data needs to be collected and a statistical technique should be applied to have the best results.

The execution of the measurement, such as with any instrumentation, also heavily relies on the user of the equipment that should be considered when variable results are reported.

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- Project report GAP 220 Reliable cost effective technique for in-site ground stress measurements in deep gold mines – TR Stacey.

INTERESTING INFO

Stress measurement database

A database system called ArcExplorer where records of reliable stress measurements are recorded has been developed by the Environmental Systems Research Institute (ESRI), the organisation responsible for the development of the ARC/INFO and ArcView systems. An ArcExplorerdownload is available on http://www.esri.com/.

It is recommended that the download and installation procedures described in Appendix 3 of the GAP 511b project report be followed.

ArcExplorer uses shape files containing the spatial information of each data point in the database and a DBASE file containing the alphanumeric data of each data point.

Where available, the data for each measurement entered into the database include:

- Identification number in the database (sequential number for each measurement as the data are captured);
- Country in which the record, observation, measurement was made (for example, Zimbabwe);
- Province in the country (for example, Northern Cape);
- Locality of the record (for example, Kloof Mine);
- Site of the record (for example, Shaft 6);
- Location of the record (for example, 63 level, 19 west crosscut);
- Longitude and latitude of the measurement;
- Date of the measurement, observation, record;
- Method of in-situ stress measurement or observation (for example, Doorstopper strain cell; flat jack; borehole breakout etc);
- Depth of the record below surface in metres;
- Three principal stresses (S1, S2, S3 in MPa), their bearings and dip angles;
- Estimated overburden stress (SOB);
- Normal stress components in the east, north and vertical up axes (SX, SY, SZ, SXY, SYZ, SXZ stress components);
- Two horizontal secondary principal stresses (SH1, SH3) and the bearing (BSH1);
- Ratios between these horizontal secondary principal stresses and the estimated overburden stress (K1, K3);
- Ratios between the SX and SY horizontal stresses and the estimated overburden stress (KX, KY);
- Quality grading for the individual record (see below);
- Quality grading for the combined record resulting from a group of individual records;
- Number of measurement points contributing to a record;
- Modulus of elasticity of the rock material at the measurement point (E in GPa);
- Poisson's ratio of the rock material at the measurement point (pr);
- Rock type in which the record originated;
- Geological sequence relevant to the record;
- Reference to the source of the record;
- Comments regarding the record (for example, measurement point close to a fault contact; measurement could be influenced by variable topography; measurement in valley bottom; measurement within the influence of excavations, etc)

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REFERENCE

- Project report GAP 511b - Evaluation and upgrading of records of stress measurement data in the Mining industry TR Stacey and J Wesseloo.

LEARNING OUTCOME 5.7

DESCRIBE AND EXPLAIN THE PRINCIPLES OF STRESS MEASUREMENT BASED ON AE PRINCIPLES

Acoustic emission (AE) is defined as "a naturally occurring phenomenon whereby external stimuli, such as mechanical loading, generate sources of elastic waves. AE occurs when there is a rapid release of energy in a material due tostress waves generated when displacements occur within or on its surface.

This method is based on the ability of in-situ rock to develop a memory of the stress field it was subjected to, in its confining environment. A rock that was originally confined and then relaxed will have a significant increase in the acoustic emission rate when the new reapplied stress exceeds the original confining stress. This behaviour results in a phenomenon called the Kaiser Effect which was originally observed in polycrystalline metal. (Stacey &Wesseloo, 2002).

This method involves drilling smaller cores from the core that was removed from the stressed environment, as illustrated in Figure 25. The directions of the secondary cores relative to the original core areimportant.The smaller secondary cores are flattened at the ends and subjected to uni-axial tests in a laboratory environment with acoustic emission sensors attached for monitoring, as illustrated in Figure 26.

Figure 25: Acoustic emission sampling from an orientated 63mm

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diameter core (Golder Associates)

Figure 26: Test sample with acoustic emission sensors attached (Golder Associates)

Using the data obtained during the laboratory experiment, an acoustic emission versus stress graph can be plotted from which the Kaiser Effect change point, shown in Figure 27, where there is a change in the slope of the graph. The change point is associated with the maximum stress the rock was subjected to underground. A sufficient number of secondary core tests should be conducted to be able to determine the three dimensional states of stress.

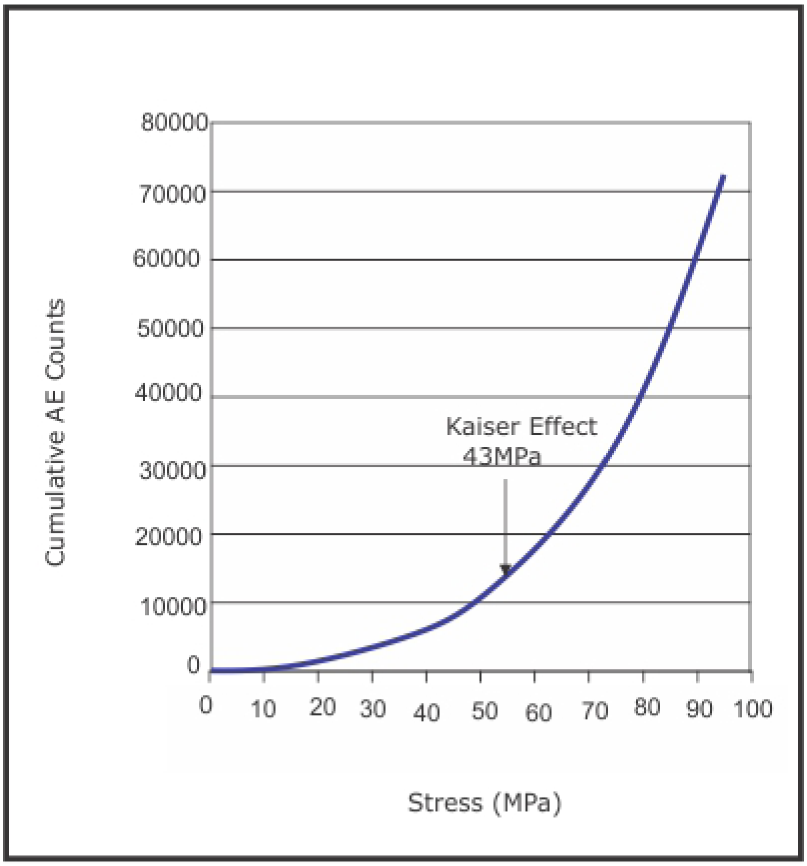

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Figure 27: Graph of AE vs stress (Golder Associates)

Compared to strain and deformation dependent methods, this method is advantageous since it gives a direct measure of stress and reduces experimental errors. Testing activities are carried out in the laboratory, therefore interference with mining activities is at a minimum or even eliminated in cases whereby cores that were initially obtained for other purposes such as exploration are used.

The AE method can be used ingreenfield sites andon both shallow and deep mining operations. According to Stacey and Wesseloo, the effect of core fractures on the application of this stress measuring technique has not been established.Success has been achieved in the following mines, but it is assumed that application in deep gold mines might be problematic:

- Coal measures in European mines, siltstone/sandstones, 400 to 1000m deep;
- Coal measures in South African mines, siltstone/sandstones 50 to 150m deep;
- Deep level South African gold/platinum mines, quartzite/norites, 1100 to 2000m deep; and
- European hard rock mines, silicified tuffs/calcareous siltstones, 500 to 900m deep.

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LEARNING OUTCOME 5.8

EXPLAIN AND DISCUSS THE USE OF KIRSCH EQUATIONS IN RESPECT OF STRESSDETERMINATIONS.

According to Jaeger and Cook (1979) most tunnels ,shafts, orepasses, boxholes and drillholes can be approximated by a tube-shaped excavation where its cross section is circular and its length is large in relation to the

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cross section diameter. Provided the applied stress is sufficiently low and the rock remains elastic, the stress field around the excavation can be analysed in 2D. Figure 28 provides an example.

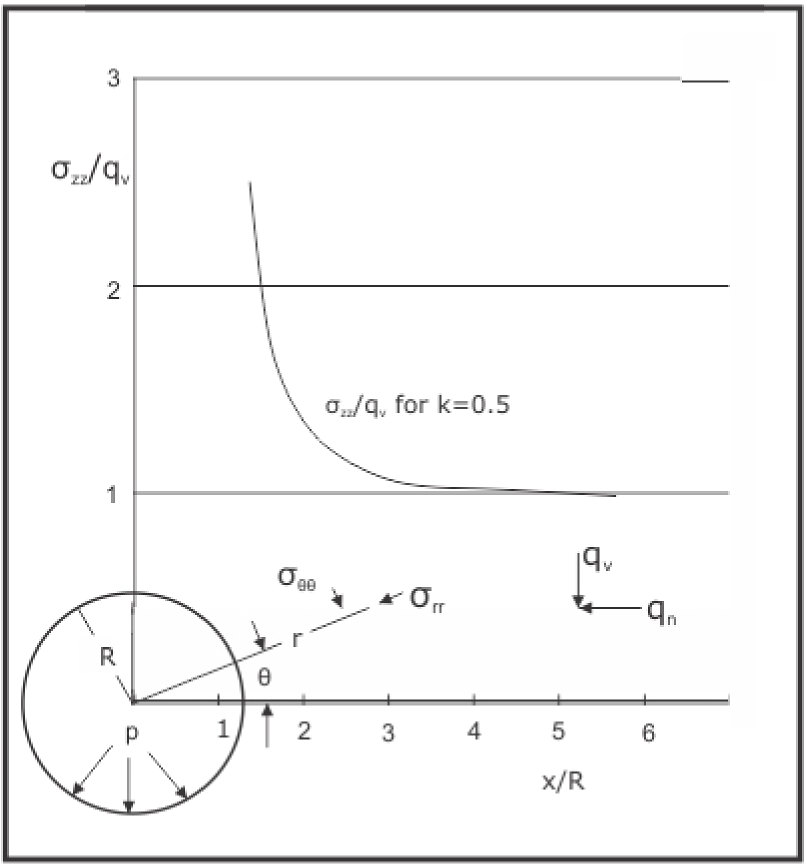

Figure 28: Tangential stress distribution around a circular opening using a polar coordinate system (Ryder &Jager)

The application of the Kirsch equations is limited to the following assumptions:

- Applicable to excavations that can be approximated by circular shaped excavations only;
- Elastic rock mass behaviour;
- Applied stress components must be perpendicular to the circle axis;

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"Polarco-ordinates"onpage228

The Kirsch equations are applied in a polar coordinate systemwhere the radial axis is orientated from the centre of the circle radially outward and the tangential axis orientated perpendicular to the radial axis.

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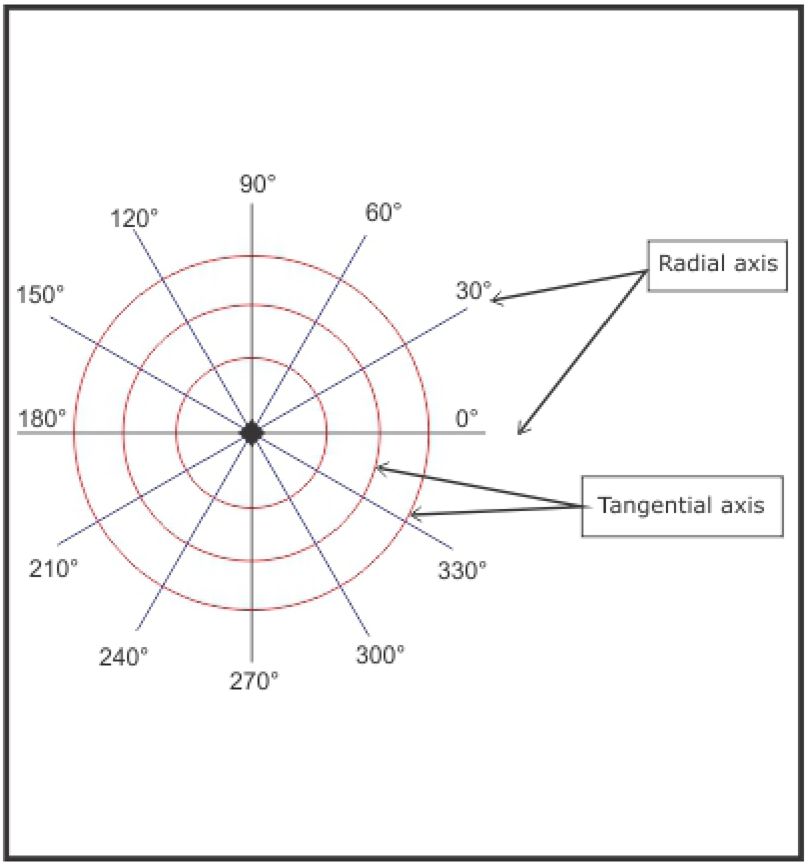

Figure 29 Polar coordinate system

The stress distribution around the circular opening can be calculated in terms of radial stress (σ_{rr}), orientated towards the centre of the opening and tangential stress ($\sigma_{\Theta\Theta}$), orientated perpendicular to the radial stress or parallel to the opening circumference, using the Kirsch equations (named after its originator). The shear stress in polar coordinates ($\tau_{\Theta\Theta}$) can also be determined.

The Kirch equations are shown below:

- Radial stress

$$\sigma_{rr} = ½ (q_h+q_v) (1-t^2) + pt^2 + ½ (q_h-q_v) (1- 4t^2 +3t^4) \cos2\Theta$$

- Tangential stress

$$\sigma_{\Theta\Theta} = ½ (q_h+q_v) (1+t^2) – pt^2 - ½ (q_h-q_v) (1 + 3t^4) \cos2\Theta$$

- Shear stress:

$$\tau_{r\Theta} = - ½ (q_h-q_v) (1+ 2t^2 -3t^4) \sin2\Theta$$

where

- *$t = R/r$,*
- *R = radius of the circular opening,*
- *r is the distance from the centre of the circular opening to the point*
- *where stress levels will be calculated,*
- *the stress field is given by q_h and q_v that represent the applied horizontal and vertical stresses respectively,*

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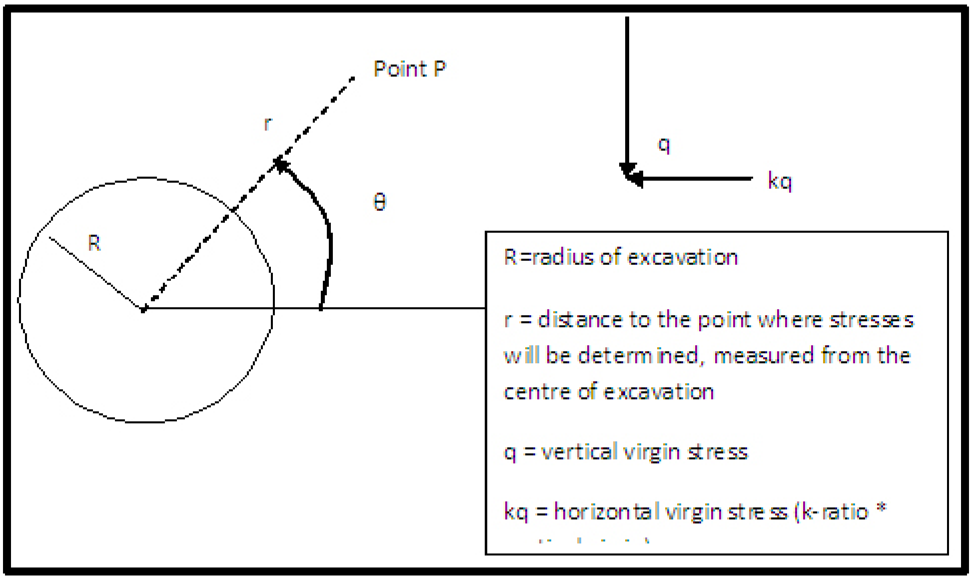
- *where appropriate P is the internal pressure on the inside of the opening and*
- *Θ indicates the angle for the positive x-axis to the line along which the radial and tangential stresses are calculated.*



In applying the Kirsch equations, some important solutions can easily be found for specific conditions:

The stresses around the periphery of the opening can be found by setting r = R (t=1). The resulting equations along the periphery of a circular opening are therefore:

$$\sigma_{rr} = p$$
$$\sigma_{\Theta\Theta} = qh\,[1- 2\cos 2\Theta]=q_v[1+ 2\cos 2\Theta] - p$$
$$\tau_{r\Theta} = 0$$

The maximum tangential stress $\sigma_{\Theta\Theta}$ occurs on the periphery of the opening and where $\Theta = 0^0$ (if the major applied stress component is vertical) and can be determined from the equation

$$\sigma_{\Theta\Theta}\ \max = 3q_v - q_h - p$$

The minimum tangential stress $\sigma_{\Theta\Theta}$ occurs where $\Theta = 90^0$ (if the major applied stress is vertical) and can be determined from the equation

$$\sigma_{\Theta\Theta}\ \min = 3q_h - q_v - p$$

Note that the minimum tangential stress can be negative in value if the internal pressure $p > 3q_h - q_v$. In mining situations there are no internal pressures and therefore p=0.

EXAMPLE

Two tunnels are to be bored 12m apart (skin-to-skin) at a mining depth of 3000m where the k-ratio is 0.5. Both have diameters of 10m. Will stress interaction be expected? Density of the overburden is 2.8 tonnes/m3.(No internal pressure is present in the excavation.)

Process:

- Draw the layout.
- Assume and draw the stress field.
- Calculate vertical virgin stress and assume a k-ratio for this depth, or calculate vertical virgin stress and assume a theory to get to horizontal stress that is appropriate to this depth.
- Calculate the Tangential stress only as this is the stress that will affect each other – state this clearly.
- Calculate the Tangential stress at points from the first excavation edge towards the 2nd excavation that is 12m away. Choose to

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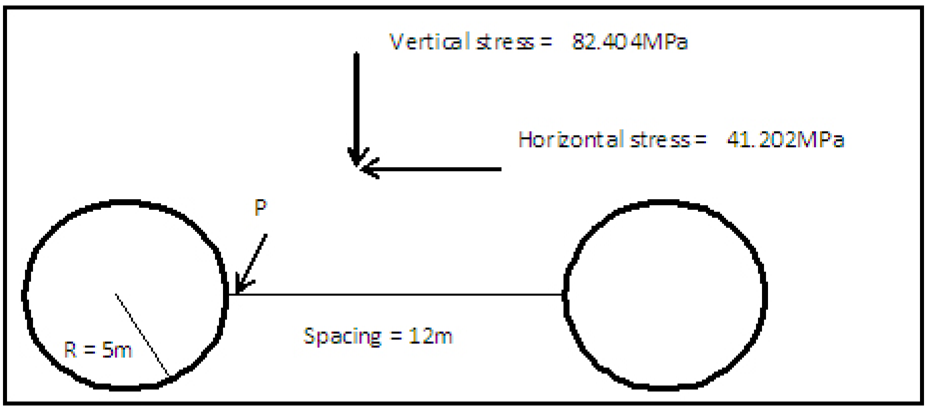
calculate stress every 2m, leaving you with seven points at which to perform the calculations.

- Plot the results (tangential stress) on a graph showing the excavations and the middling between them.
- Determine the Tangential stress caused by the 2nd excavation in the middling between the excavations (θ = 180).
- Plot the results on the same graph as above.
- Determine the total field stress between the two excavations taking into account that both calculations performed in (3) and (6) are 'field stress' levels (i.e. virgin stress and induced stress) and both thereforeinclude the virgin vertical stress components. You should add the field stresses calculated at each point due to the individual excavations and then subtract onecomponent of the virgin stress from this total to realise the final total field stress.
- Plot the total field stress on the same graph between the excavations.
- Two issues should be taken into account:
  - Is excavation 1 inducing stress on excavation 2 or vice versa?
  - Look at individual tangential field stress plots,
  - If the stress levels caused by excavation 1 have not returned to virgin stress on the edge of excavation 2, then excavation 1 is inducing stress onto excavation 2, and vice versa.
- Are the excavations inducing stress on the pillar between them?
  - Compare the total field stress values with the virgin stress level in the area between the excavations (i.e. pillar between excavations);
  - If field stress is larger than virgin stress, stress is induced at that point and the pillar between the excavations is affected;
- Keep in mind that the level of induced stress must be related to rock or rock mass strength before you can say whether an effect will be detrimental or not.



Vertical stress

ρgh = (2.8tonnes/m3). (9.81 m.s-2).(3km) = 82.404 MPa

Horizontal stress

k.vertical stress = (0.5). 83.404 MPa = 41.202 MPa

At point P:

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r = 5 m (i.e. on the edge of the excavation):

t = R/r = 5/5 = 1 , p = 0 and θ = 0°.

$\sigma_{\Theta\Theta}$ = $\frac{1}{2}(q_h+q_v)(1+t^2) - pt^2 - \frac{1}{2}(q_h-q_v)(1+3t^4)\cos 2\Theta$

= ½ (41.202+82.404MPa)(1+12)- ½ (41.202-82.404)[1+3.(14)]Cos2(0)MPa
= ½ (123.606MPa)(2) – ½ (-41.202)(4)(1)MPa
= 123.606MPa + 82.404MPa
= <u>206.01 MPa</u>

These calculations must now be completed for different 't' values as the distance 'r' towards the 2nd excavation increases from 0 to 12m (in 2.0m steps) – See results table.

Distance from excavation centre (m)	Distance from excavation edge (m)	Radial stress (MPa)	Tangential stress (MPa)
5	0	0.00	206.10
7	2	35.62	130.02
9	4	41.67	107.37
11	6	42.82	97.81
13	8	42.90	92.90
15	10	42.73	90.03
17	12	42.52	88..21

NOTE

For completeness sake and not to answer this question:

σ_{rr} = $\frac{1}{2}(q_h+q_v)(1-t^2)+pt^2+\frac{1}{2}(q_h-q_v)(1-4t^2+3t^4)\cos 2\Theta$
= ½ (41.202+82.404)(1-(1)2)+½(41.202-82.404) (1- 4(1)2+3(1)4)cos2(0)
= ½ (123.606)(0) + ½ (-41.202) (0) .1
= <u>0 MPa</u>

and

$\tau_{r\Theta}$ = $-\frac{1}{2}(q_h-q_v)(1+2t^2-3t^4)\sin 2\Theta$
= - ½ (41.202-82.404)(1+ 2(1)2 -3(1)4)sin2(0)
= - ½ (-41.202)(0)(0)
= <u>0 MPa</u>

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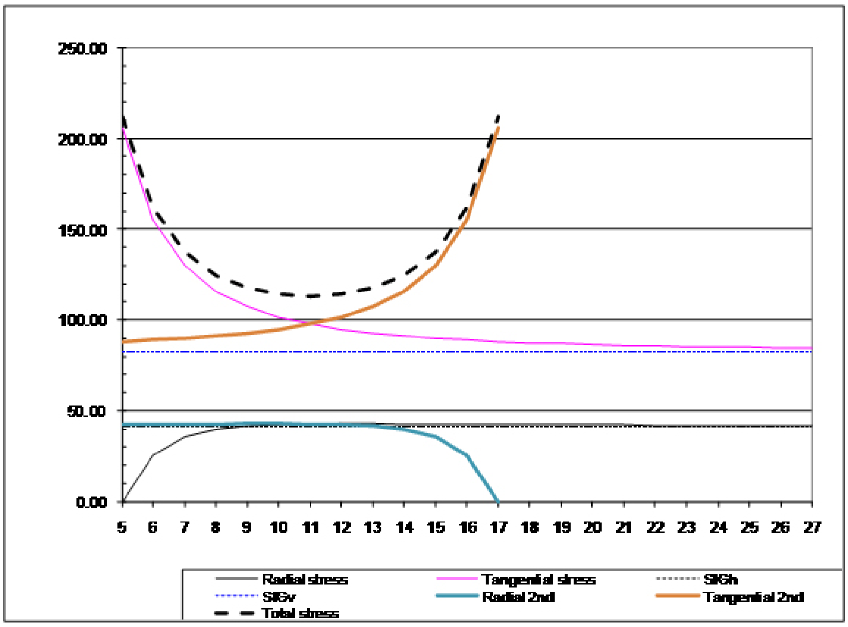

Final answer:

- Both excavations are affecting the stress levels at the position of the other excavation.
- Total field stress is higher than virgin stress in pillar. Excavations are therefore inducing higher stress levels in the pillar.

REFERENCE

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- Page 103 - Underground excavations in rock, 1980, E Hoek and ET Brown
- Page 41,173 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

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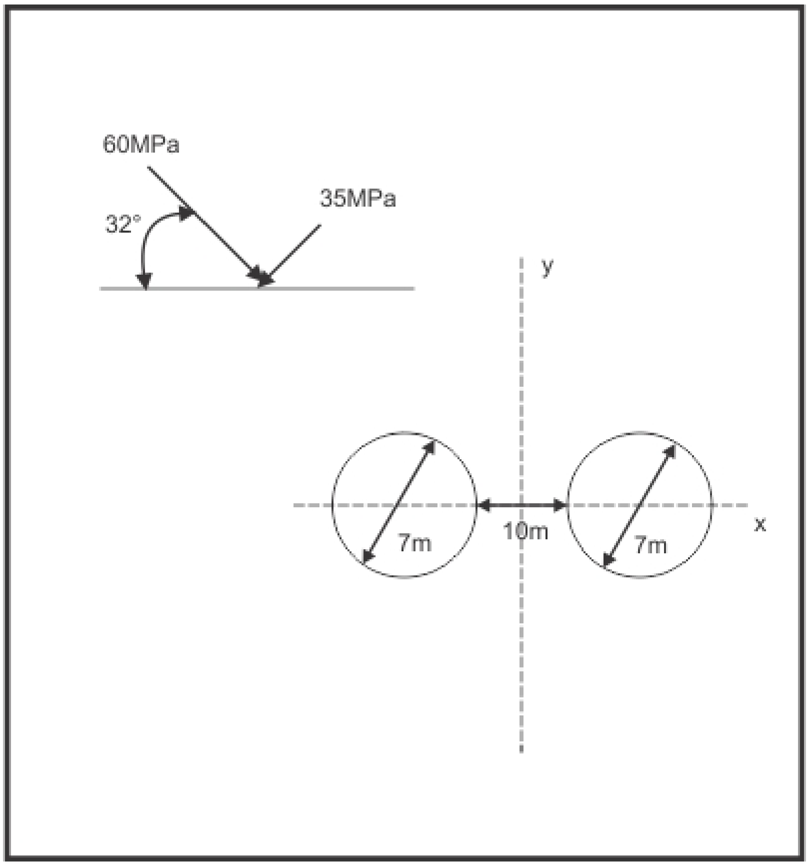
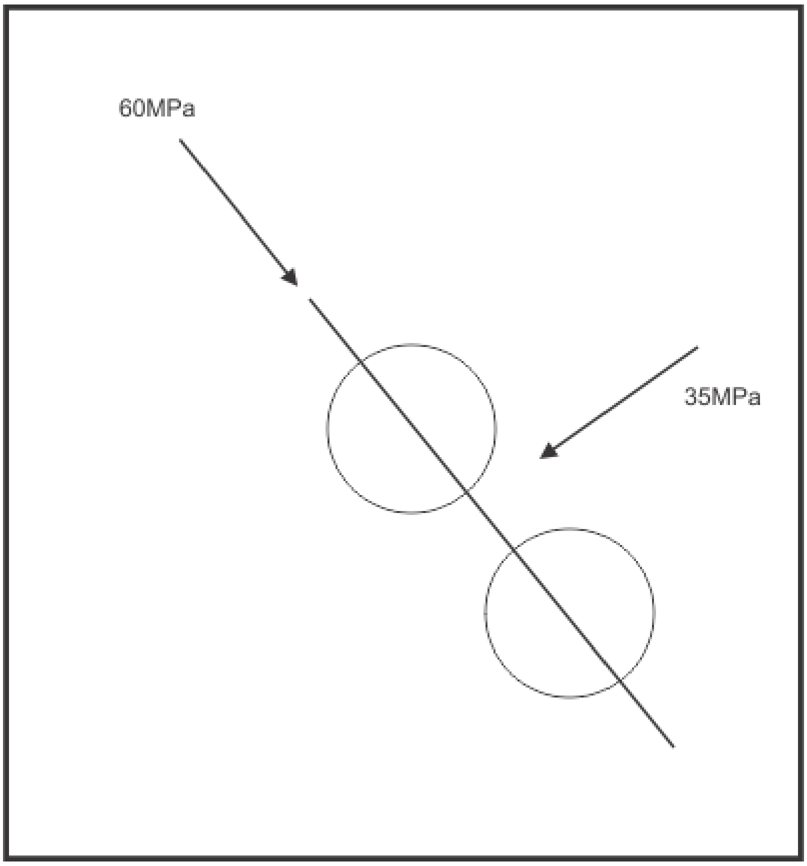
EXAMPLE

Given the planned layout of two vertical orepasses that have been orientated and spaced (on plan) in a state of horizontal principal stress as shown in the figure, answer the following questions. Suggest from rule-of-thumb the optimum orientation of the two orepasses by re-drawing the orepass layouts in the figure, maintaining the given orepass dimensions and spacing and showing the stress state. Explain your decision.

The orientation of the orepasses must be such that the line connecting the centres of the orepasses is parallel to the major stress direction as given in the state of stress in the figure. This will prevent the highest stress concentrations from forming between the ore passes, increasing the tangential stress levels, causing stress interaction and possibly fracturing and scaling.





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ANIMATION

EXAMPLE

Using the optimum orientation above, calculate the maximum field stress levels between the orepasses and along a line connecting the centre's of the two orepasses, at points spaced 2.0m apart along this line. Present your results in graphical form.

Answer 5.2.

Using orientation above, horizontal stress is 60MPa and vertical 35Mpa, resulting in a k - ratio of k=1.71. Calculate sig r and sig θ using Kirsch equations for orepass1 at 0° and orepass two at 180°. Since the two orepasses are of same dimension with same stress levels to which it is being subjected, the stress analysis for the 2nd orepass would be exactly the same as for the 1st orepass.

R (1^{st} / 2^{nd})	3.5 / 13.5	5.5 / 11.5	7.5 / 9.5	9.5 / 7.5	11.5 / 5.5	13.5 / 3.5
Sig r	0	26.69	40.55	47.45	51.28	53.60
Sig θ	45.24	48.19	43.65	40.83	39.15	38.10

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CHAPTER

6

ROCK MASS PROPERTIES

INTRODUCTION

This chapter provides the rock engineering practitioner with more detail on the estimation of rock mass property and classification systems, all in an effort to adequately determine the rock mass strengths.

6.1 ROCK MASSPROPERTIES.

The candidate must be able to demonstrate knowledge and understanding of the above subject area by being able to:

6.1.1 Describe, explain and illustrate the volume effect on the strength of intact rock;

6.1.2 Describe, explain and illustrate the role of joint strength on the strength of a rock mass;

6.1.3 Describe, explain and illustrate, the role of joint frequency on the strength of a rock mass;

6.1.4 Describe, explain and illustrate the role of joint length on the strength of a rock mass;

6.1.5 Describe, explain and illustrate the role of intact rock strength on the strength of a rock mass

6.1.6 Describe, explain and illustrate the role of groundwater on the strength of arock mass and

6.1.7 Describe and explain the general relationship between strength and seismicwave velocity in a rock mass.

6.2 ROCK MASS CLASSIFICATION.

- Apply Barton, Bieniawski and Laubscher's geomechanics classificationsystems to classify rock masses;

6.2.1 Determine rock mass 'm' and 's' parameters for the Hoek and Brown criterion,based upon rock classification;

6.2.2 Apply the Hoek and Brown criterion to estimate the strength of a rock mass;

6.2.3 Identify, describe and explain the limitations of the Hoek and Brown criterion;

6.2.4 Estimate rock mass deformability from joint stiffness and rock massclassification results.

REFERENCE

- Page 165,166 - Underground excavations in rock, 1980, E Hoek and ET Brown

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ROCK MASS PROPERTIES

LEARNING OUTCOME 6.1.1

DESCRIBE, EXPLAIN AND ILLUSTRATE THE VOLUME EFFECT ON THE STRENGTH OF INTACT ROCK

CONNECTION 40

This outcome is similar to Chapter 4, "LEARNING OUTCOME 4.1.16" on page 124and has been discussed in that section.

It is generally assumed that there is a significant reduction in strength with increasing sample size. Hoek and Brown (1980a) have suggested that the uni-axial compressive strength σ_{cd} of a rock specimen with a diameter of d mm can be related to the uni-axial compressive strength σ_{c50} of a 50 mm diameter sample by the following relationship:

$$\sigma_{cd}=\sigma_{c50}.(50/d)^{0.18}$$

This relationship, together with the data upon which it was based, is shown in Figure 1.

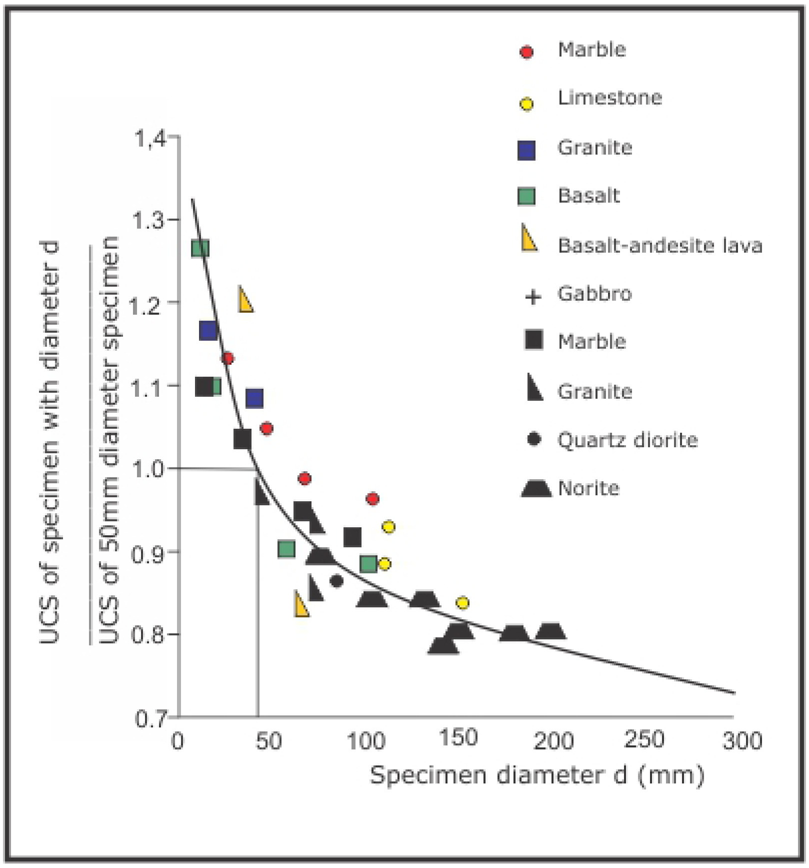

Figure 1: Influence of specimen size on the strength of intact rock (Hoek&Brown)

REFERENCE

- Page 68 Chapter 4, Practical rock engineering, 2006, E Hoek

INTRODUCTION TO OUTCOME 6.1.2 - 6.1.6

NOTE

As an introduction to Outcomes 6.1.2 through 6.1.6, the concept of rock mass strength is described below. Reference will be made to this section in the following outcomes and it is suggested that this portion is studied in detail before the following outcomes are attempted.

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The strength of a jointed rock mass can be estimated through the following parameters:

- Hoek-Brown failure criterion;
- Global Rock Mass Strength;
- Rock Mass Strength (RMS);
- Design Rock Mass Strength (DRMS) and
- Other rock mass rating systems.

Hoek-Brown failure criterion

The Geological Strength Index (GSI) (introduced by Hoek in 1994) provides a 'number' that, when combined with rock properties and the state of stress can be used to estimate the strength of a jointed rock mass. It has been shown previously that the Hoek-Brown failure criterion is appropriate to evaluate rock mass failure when the rock mass is significantly jointed (Figure 2).

The GSI is used to determine the Hoek-Brown quality parameter 's' within the Hoek-Brown failure criterion. This criterion suggests that a rock mass of specific material type and quality will be able to sustain certain stress levels. This rock mass strength is given by the Hoek-Brown failure criterion that can be written in the following form:

$$\sigma_1 = \sigma_3 + \sigma_{ci} (m_b . \sigma_3/\sigma_{ci} + s)^a$$

By setting σ_3 = 0, the uni-axial compressive strength of a rock mass of quality 's' can be given by

$$\sigma_c = \sigma_{ci}.s^a$$

where σ_{ci} is the UCS of an intact rock sample (also see RocLab software output in Figure 5).
The presence of jointing will affect the value of the 's-value' through the following relationship

$$s = \exp((GSI-100)/(9-3D))$$

where D is a blasting damage factor (Figure 4) and the GSI represents the Geological Strength Index (Figure 3).

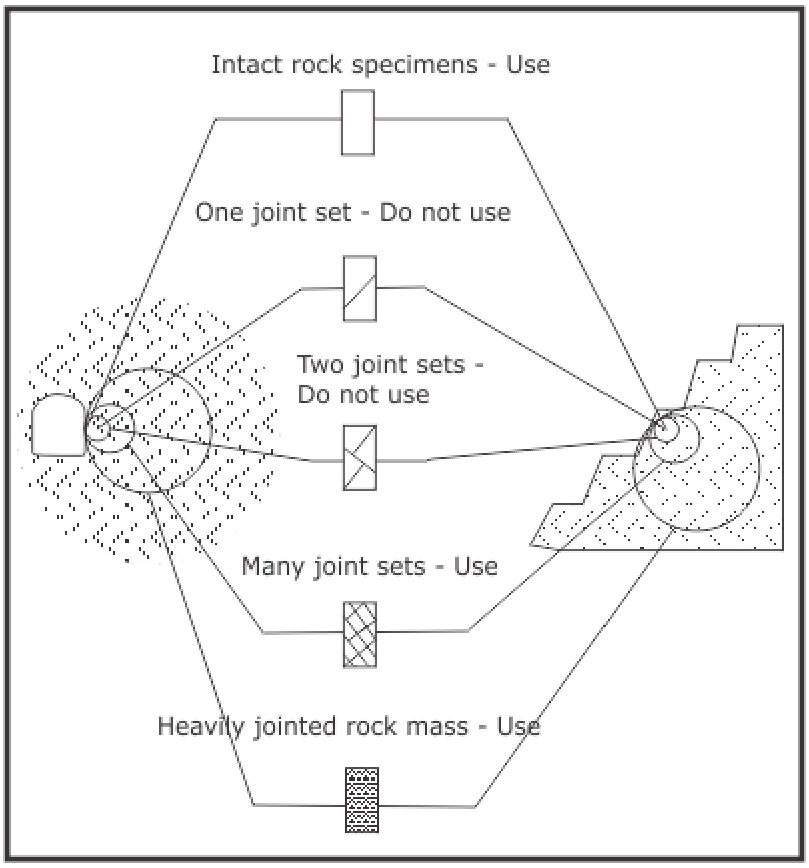

Figure 2: Applicability of the Hoek-Brown failure criterion to different rock mass conditions

If the discontinuity spacing is large compared to the dimensions of the tunnel or slope under consideration, the GSI tables and the Hoek-Brown criterion should not be used (see notes at different rock samples in Figure 2) and the discontinuities should be treated individually.

The Geological Strength Index suggests that the strength of a jointed rock mass depends on the properties of the intact rock pieces and also upon the freedom of these pieces to slide and rotate under different stress conditions. This freedom is controlled by the geometrical shape of the intact rock pieces as well as the condition of the surfaces separating the pieces.

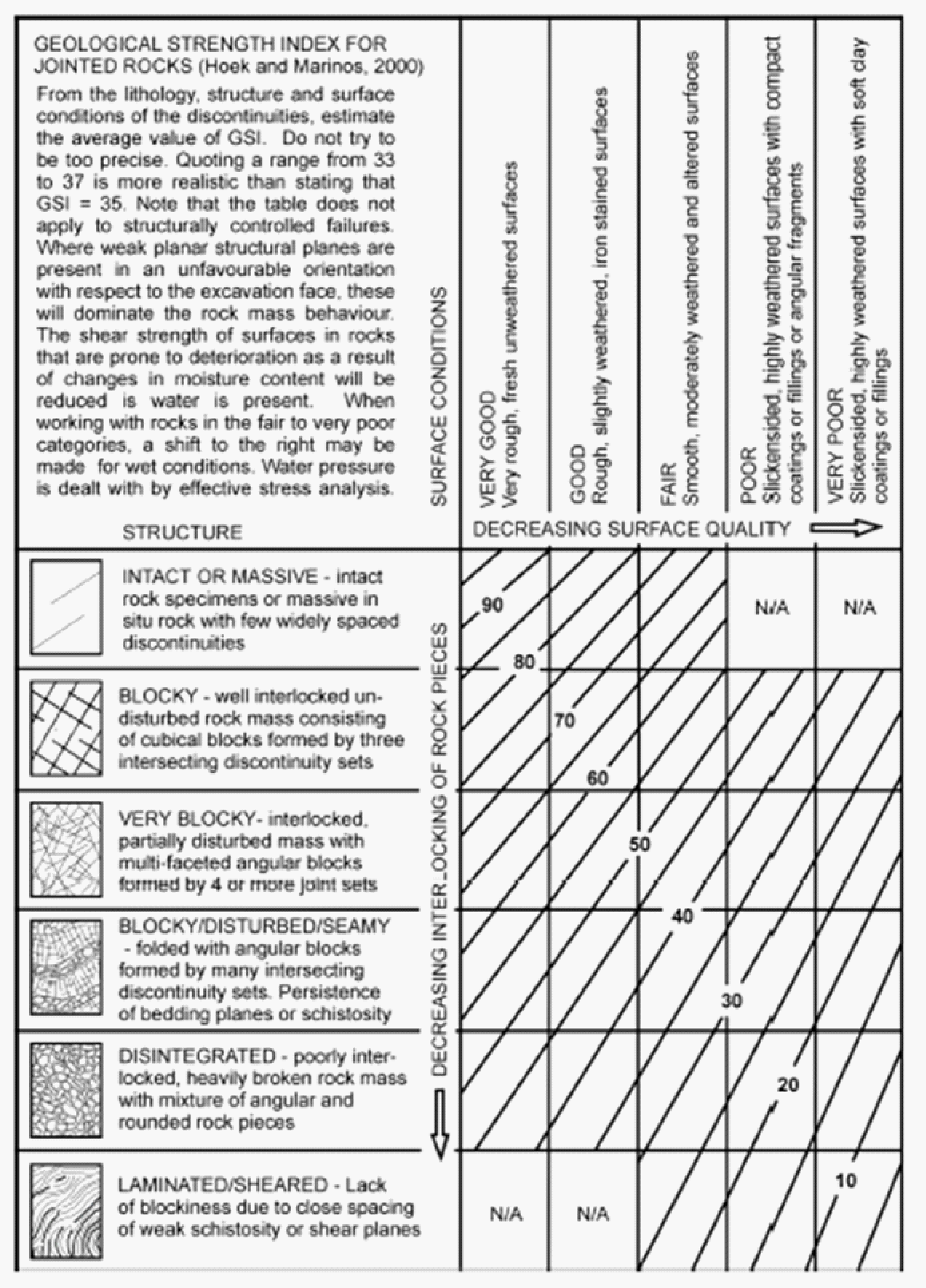

Figure 3: GSI selection

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Appearance of rock mass	Description of rock mass	Suggested value of D
	Excellent quality controlled blasting or excavation by Tunnel Boring Machine results in minimal disturbance to the confined rock mass surrounding a tunnel.	D = 0
	Mechanical or hand excavation in poor quality rock masses (no blasting) results in minimal disturbance to the surrounding rock mass. Where squeezing problems result in significant floor heave, disturbance can be severe unless a temporary invert, as shown in the photograph, is placed.	D = 0 D = 0.5 No invert
	Very poor quality blasting in a hard rock tunnel results in severe local damage, extending 2 or 3 m, in the surrounding rock mass.	D = 0.8
	Small scale blasting in civil engineering slopes results in modest rock mass damage, particularly if controlled blasting is used as shown on the left hand side of the photograph. However, stress relief results in some disturbance.	D = 0.7 Good blasting D = 1.0 Poor blasting
	Very large open pit mine slopes suffer significant disturbance due to heavy production blasting and also due to stress relief from overburden removal. In some softer rocks excavation can be carried out by ripping and dozing and the degree of damage to the slopes is less.	D = 1.0 Production blasting D = 0.7 Mechanical excavation

Figure 4: Blasting D-factor selection

Global rock mass strength

Another option to estimate the strength of a rock mass is to consider the overall behaviour of the rock mass rather than detailed failure within the rock mass. A concept called 'Global Rock Mass Strength' was derived by Hoek and Brown (1997) which suggests that the global rock mass strength can be determined from

$$\sigma_{cm} = \frac{2c\,Cos\phi}{1 - Sin\phi} \text{ or}$$

$$\sigma_{cm} = \sigma_{ci}\,\frac{(m_b + 4s - a\,(m_b - 8s))\left(\frac{m_b}{4} + s\right)^{a-1}}{2\,(1 + a)\,(2 + a)}$$

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where the Hoek-Brown parameters

$$m_b = m_i.\exp((GSI-100)/(28-14D))$$

$$a = \frac{1}{2} + \frac{1}{6}\left(e^{\frac{-GSI}{15}} - e^{\frac{-20}{s}}\right) \text{ and}$$

$$s = \exp((GSI-100)/(9-3D))$$

are all functions of discontinuities present within the rock mass (as indicated by the GSI), D is a blast damage factor (Figure 4), C is the rock mass cohesion and ø is the rock mass angle of friction (Mohr-Coulomb parameters).

This strength can be obtained by applying the relevant parameters to the Rocscience software (RocLab, www.rocscience.com) as shown in Figure 5.

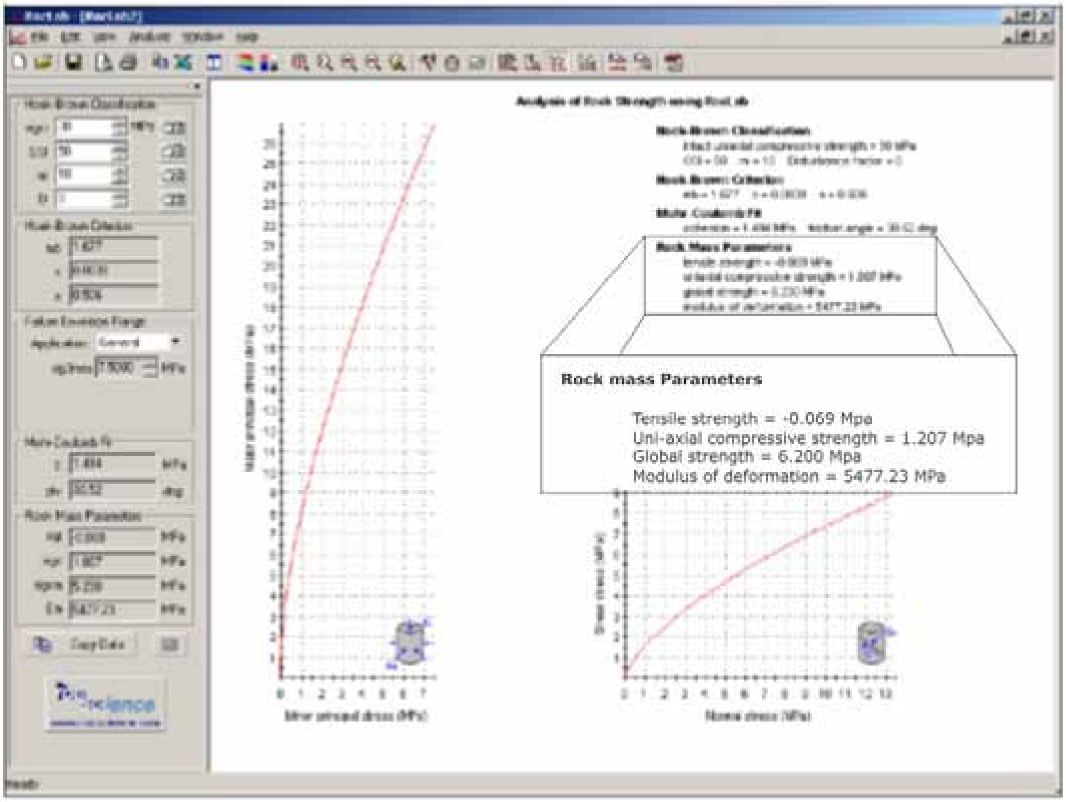

Figure 5: RocLab : Uni-axial Compressive and Global Rock Mass Strengths

Rock Mass Strength (RMS)

The RMS concept is an empirical approach to determine the strength of a rock mass and is based on the Mining Rock Mass Rating (MRMR) Geomechanicsclassification system.

CONNECTION 41

"Learning Outcome 6.2.1" on page 200

The RMS can be determined from

$$RMS = \sigma_c\left(\frac{MRMR - rating_{UCS}}{100}\right)$$

where MRMR is the Mining Rock Mass Rating, RatingUCS is the rating allocated to the UCS of the intact rock material as given in the table below and σ_c is the material uni-axial compressive strength.

UCS (MPa)	185	184-165	184-165	164-145	144-125	124-105	104-85	84-65	44-25	24-5	4-0
Rating	20	18	16	14	12	10	8	6	4	2	0

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Design Rock Mass Strength (DRMS)

CONNECTION 42

"Learning Outcome 6.2.1" on page 200

A series of downgrading adjustments are made to the RMS to take into account the effects of weathering, joint orientation and method of excavation. These downgrading adjustments are applied by adjusting the MRMR ratings in the calculation of the rock mass strength using the following:

$$RMS = \sigma_c \left(\frac{MRMR_{adjusted} - rating_{UCS}}{100}\right)$$

Other rock mass rating systems

Although none of the other rock mass rating systems, including Bieniawski's rock mass rating (RMR), Barton's Q system and Q' (leading into the Modified Stability Number N') are directly linked to the strength of a rock mass, there is common acceptance that lower rock mass qualities will yield poorer rock mass strengths. For this reason it is found that poorer rock mass ratings will lead to more stringent support requirements, smaller stable mining spans, shorter stand-up times, etc.

LEARNING OUTCOME 6.1.2

DESCRIBE, EXPLAIN AND ILLUSTRATE THE ROLE OF JOINT STRENGTH ON THE STRENGTH OF A ROCK MASS

CONNECTION 43

Chapter 4, "LEARNING OUTCOME 4.2.2" on page 131, "LEARNING OUTCOME 4.2.3" on page 132 "LEARNING OUTCOME 4.2.4" on page 133, "LEARNING OUTCOME 4.2.7" on page 138 and "LEARNING OUTCOME 4.2.9" on page 140 describe factors that affect joint strengths in detail.

These factors include: friction angles, joint infilling, presence of water, joint surface roughness and the normal stress acting on the joint surface. A summary of the effect of joint strength on the strength of the rock mass is listed below but a study of the Chapter 4 sections is suggested.

Summary of the effect of joint strength on the rock mass:

- Friction has an effect on small-scale (between the surfaces of minute cracks) as well as on a large-scale(between the surfaces of joints) discontinuity strengths and will therefore affect the strengths of each of the large number of discontinuities making up the rock mass;
- The shear strength of a discontinuity can be calculated from $\tau = C_o + \sigma_{normal}.\mathrm{Tan}(\Phi_r + i)$.
  - Friction angles (Φ_r) associated with discontinuity surfaces for each of the large number of discontinuities within a rock mass may be different;
  - Filling material on discontinuity surfaces can have a huge influence on the shear strength of thediscontinuity plane as it affects the friction angle (Φ_r) of, as well as cohesion (C_o) on the plane;
  - Changes to the normal stress (σ_{normal}) acting on discontinuity surfaces can be reduced by pore pressure on the surfaces resulting in the reduction of the plane's shear strength.When the normal stress (clamping stress) on the discontinuity surface is increased, a larger shear stress is required to cause slip or shear displacement along the joint surface. However, if this normal stress becomes excessively large, the asperities on the surface can be crushed, reducing the 'roughness angle' and reducing the shear strength of the discontinuity;
  - An increase in the (Φ_r + i) angle will result inan increase in the discontinuity shear strength. This means that with an increase in surface roughness (since irepresent the 'roughness angle' of the discontinuity surface), the shear strength of the joint surface will also increase (Figure 6).

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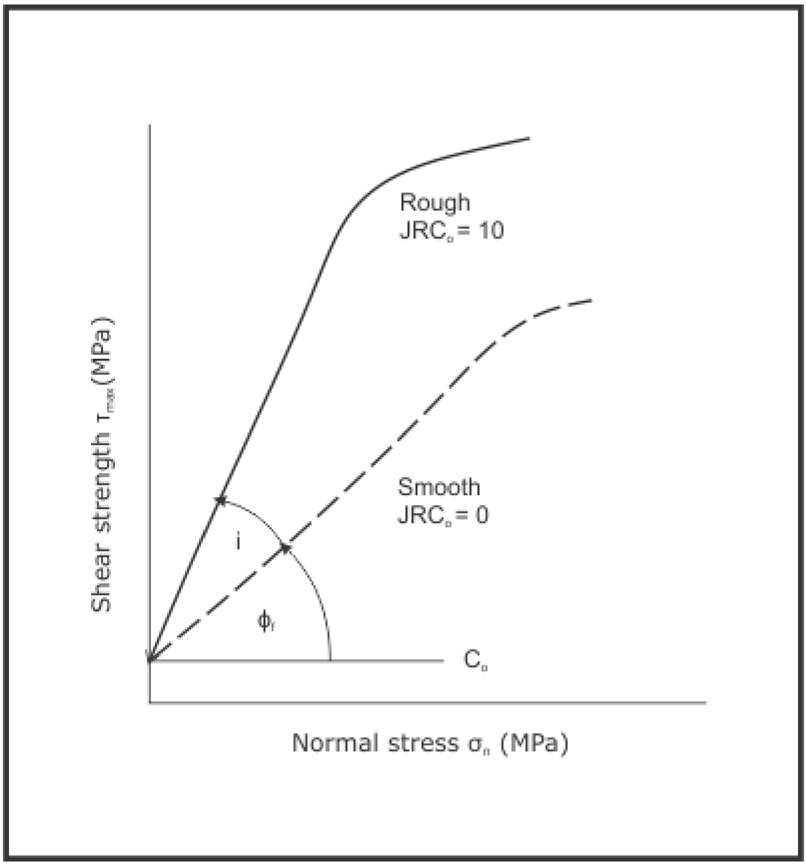

Figure 6: Peak shear strength as a function of normal stress graph showing smooth and rough joints (Ryder&Jager)

Since a rock mass usually includes a large number of discontinuities, all with possible different orientations, surface roughness, infilling, etc, the strengths of these discontinuities will largely determine the strength of the rock mass.As the sample increases in dimension, more discontinuities are included within the sample where each discontinuity will impact on the failure and the strength of the rock mass (Figure 7) in a manner that is determined by its own shear strength. It is therefore logical to conclude that factors that affect individual joint strengths will also impact on the rock mass strength within which the joints exist.

In applying the rock mass strength methods explained earlier:

- the ratings that combine into the MRMR (ultimately used to determine the rock mass strength) include ratings for 'joint condition' and refer to issues that affect the strength of the discontinuity, as explained in the paragraphs above. These issues include:
    - Joint expression: The shape of the discontinuity on a larger scale affecting shear strength due to ability to allow shear on a wavy, curved or straight discontinuity surface;
    - Surface roughness: Roughness has been shown to directly affect the strength of a discontinuity and selection between 'Very rough' and 'Polished' states allows an indication of the joint strengths within the rock mass,
    - Infilling or surface alteration: The presence of different types and strengths of infilling affects the cohesion and friction angle of the discontinuities. Selection of 'stronger than wall rock' to fine, sheared, soft material indicates the impact on these two parameters and therefore on joint and rock mass strengths.

CONNECTION 43

"Learning Outcome 6.2.1" on page 200

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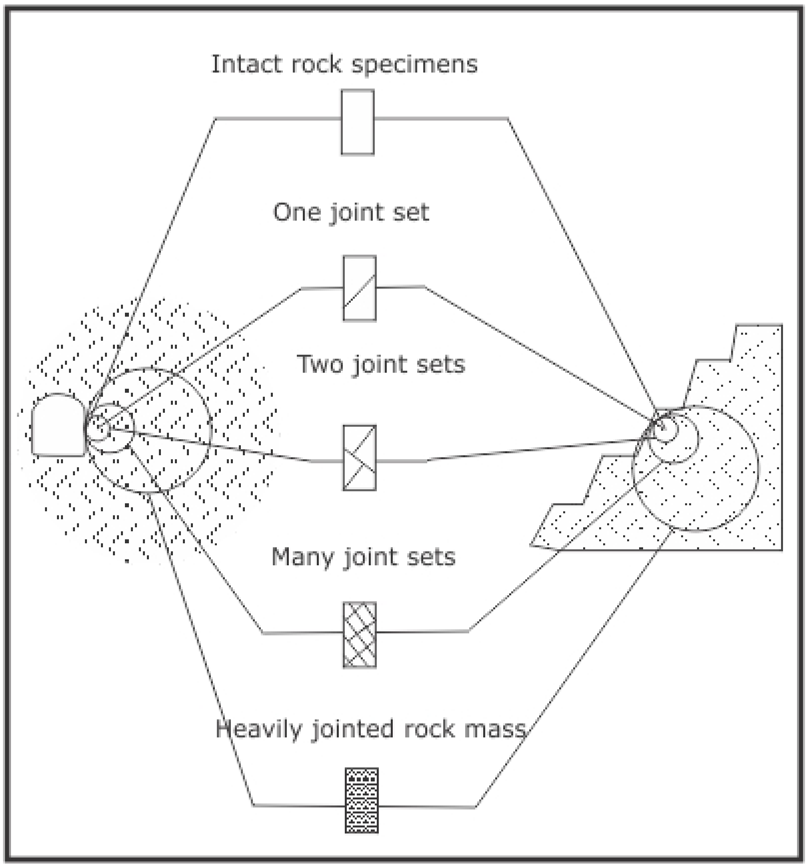

Figure 7: Transfer from intact to jointed rock mass strength sample scale

REFERENCE

- Page 129 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

LEARNING OUTCOME 6.1.3

DESCRIBE, EXPLAIN AND ILLUSTRATE, THE ROLE OF JOINT FREQUENCY ON THE STRENGTH OF A ROCK MASS

CONNECTION 44

Joint frequency has been addressed in Chapter 4, "LEARNING OUTCOME 4.2.1" on page 127

The following summary is provided:

- Joint frequency is determined by measuring all joints that intersect a scan line used during the mapping of joints or along the core during the logging of core.
- When mapping / logging discontinuities such as joints it is important to:
  - Record the number of joints per run or per unit length.
  - The fracture frequency per meter is determined by counting the fractures per meter or by dividing the joint count by the length of the core / scan line.
  - Do not include drilling-induced fractures, stress fractures, healed joints or veins in the count.

The more closely spaced joints within joint sets that exist within a rock mass are, the more weakness planes exist within the rock mass and the potentially weaker the rock mass is.

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In applying the rock mass strength methods explained earlier:

- the ratings that combine into the MRMR (ultimately used to determine the rock mass strength) include ratings for 'joint spacings') and refer to issues that affect the strength of the discontinuity, as explained in the paragraphs above. The number of joint sets and the spacing of joints in the sets are selected to determine the appropriate joint spacing rating into the resulting RMS and DRMS values;
- the GSI value utilised in the Hoek-Brown and Global Rock Mass strength methods intrinsically includes joint spacings through the use of the GSI value that reduces significantly as the structure changes (Figure 3).

CONNECTION 45

"Learning Outcome 6.2.1" on page 200

LEARNING OUTCOME 6.1.4

DESCRIBE, EXPLAIN AND ILLUSTRATE THE ROLE OF JOINT LENGTH ON THE STRENGTH OF A ROCK MASS

Joint lengths are referred to as the persistence of the joint in rock mass rating systems.Although the rock mass strength methods indicated in the previous sections do not directly include the joint persistence in the determination of the rock mass strength, it has the following impact on the rock mass:

- Persistent joints create weakness planes that extend throughout the rock mass and provide definite planes on which shear could occur;
- Larger joint lengths, i.e. more persistent joint sets, weaken the rock mass strength as they reduce the jointwall compressive strength and joint roughness, reducing the roughness angle of the plane and therefore also the strength of the rock mass.

CONNECTION

"LEARNING OUTCOME 4.2.6" on page 135

Figure 8 is an example of a spacing-persistence rating used in the Coal Mine Roof Rating(CMMR) system (Molinda & Mark, 1994) in an attempt to determine the strength of the coal roof, clearly indicating that more persistent joints (longer) receive lower ratings.

Increasing persistence

Persistence m	(1) > 1.8 m	(2) 0.6–1.8 m	(3) 20–61 cm	(4) 6–20 cm	(5) < 6 cm
(1) 0–0.9	35	30	24	17	9
(2) 0.9–3	32	27	21	15	9
(3) 3 to 9	30	25	20	13	9
(4) > 9	30	25	20	13	9

Figure 8: Spacing-persistence rating used in the CMRR, or Coal Mine Roof Rating

REFERENCE

- Page 19,23 - Rock engineering for underground coal mining, 2002, J Van Der Merwe and BJ Madden

LEARNING OUTCOME 6.1.5

DESCRIBE, EXPLAIN AND ILLUSTRATE THE ROLE OF INTACT ROCK STRENGTH ON THE STRENGTH OF A ROCK MASS

Intact rock strengths are obtained from testing small specimens in one of several testing methods including, uni-axial, tri-axial, Brazilian disk tests, etc.

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CONNECTION 46

See paper 2 Guide, Outcome 6.1.1

In the field, rock strengths (small samples) can also be obtained from:

- Inexpensive point load testing methods (after correlation of point load index tests results with laboratory tested results);
- Field strength estimates(Figure 9) based on application of a geological hammer (correlation with this method is also suggested).

Grade*	Term	Uniaxial Comp. Strength (MPa)	Point Load Index (MPa)	Field estimate of strength	Examples
R6	Extremely Strong	> 250	>10	Specimen can only be chipped with a geological hammer	Fresh basalt, chert, diabase, gneiss, granite, quartzite
R5	Very strong	100 - 250	4 - 10	Specimen requires many blows of a geological hammer to fracture it	Amphibolite, sandstone, basalt, gabbro, gneiss, granodiorite, limestone, marble, rhyolite, tuff
R4	Strong	50 - 100	2 - 4	Specimen requires more than one blow of a geological hammer to fracture it	Limestone, marble, phyllite, sandstone, schist, shale
R3	Medium strong	25 - 50	1 - 2	Cannot be scraped or peeled with a pocket knife, specimen can be fractured with a single blow from a geological hammer	Claystone, coal, concrete, schist, shale, siltstone
R2	Weak	5 - 25	**	Can be peeled with a pocket knife with difficulty, shallow indentation made by firm blow with point of a geological hammer	Chalk, rocksalt, potash
R1	Very weak	1 - 5	**	Crumbles under firm blows with point of a geological hammer, can be peeled by a pocket knife	Highly weathered or altered rock
R0	Extremely weak	0.25 - 1	**	Indented by thumbnail	Stiff fault gouge

* Grade according to Brown (1981).
** Point load tests on rocks with a uniaxial compressive strength below 25 MPa are likely to yield highly ambiguous results.

Figure 9: Field estimates of intact rock strengths, Brown (1981)

The intact rock strength of a defined rock mass zone is affected by the presenceof alternating weak and strong intact rock within this rock mass. This occurs especially in bedded deposits where an average value needs to be assigned to the rock mass on the basis that the weaker rock will have a greater influence on the average value. The empirical chart shown in Figure 10 below (Stacey et al, 1986) can be used to establish the reduced values.

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EXAMPLE

If 40% of a certain rock mass consists of a material with UCS 20MPa (20% of strong material) while the rest of the rock mass consist of a material with UCS 100MPa, the representative average rock mass strength is 48% of the strong rock material, i.e. 48MPa.

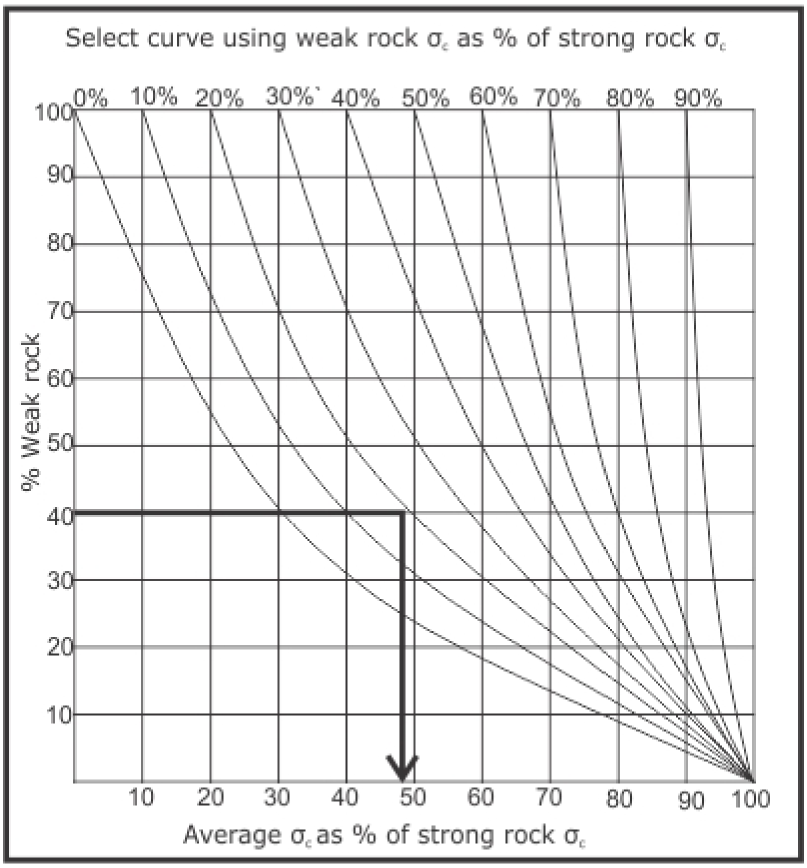

Figure 10: Determination of average IRS where the rock mass contains weak and strong zones

Most of the methods to determine rock mass strengths discussed in the introduction section, include an estimation of the UCS of the intact rock:

CONNECTION 47

Introduction to "Learning Outcome 6.1.2" on page 193

- Rock mass rating systems (MRMR, RMR);
- Global and uni-axial rock mass strengths (RocLab).

REFERENCE

- Page 70 Chapter 11 - Practical rock engineering, 2006, E Hoek

LEARNING OUTCOME 6.1.6

DESCRIBE, EXPLAIN AND ILLUSTRATE THE ROLE OF GROUNDWATER ON THE STRENGTH OF AROCK MASS

Due to the presence of groundwater in a rock mass, water pressure is present on the surfaces of the discontinuity surfaces, forcing the discontinuity contacts apart. This reduces the normal stress σ_n acting on the discontinuity plane.When stress in the rock mass reaches a state of equilibrium, the reduced normal stress on discontinuity planes can be defined by $\sigma_n' = (\sigma_n - u)$, where u is the pore pressure. The reduced normal stress σ_n' is called the 'effective normal stress' and it can be used in place of the normal stress term σ_n in the relevant joint strength equations, indicating that groundwater will reduce individual joint strengths and therefore also

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CONNECTION 48

See pore pressure – "LEARNING OUTCOME 4.1.16" on page 124 and "LEARNING OUTCOME 4.2.4" on page 133

CONNECTION 49

"Learning Outcome 6.2.1" on page 200

the overall rock mass strength.

In applying the rock mass strength methods explained earlier:

- Rock Mass Strength (RMS) and DRMS: account for water by reducing the rating as the water flow rate (25 litres/min) or condition (dry, moist, etc) changes;
- Other rock mass rating systems:
    - Q system: accounts for water by reducing the rating as the 'head' of water increases;
    - RMR system: accounts for water by reducing the rating as the water flow rate (25 litres/min) or condition (dry, moist, etc) changes;

NOTE

'Head' of water refers to the height of the water column above a certain point and is directly related to water pressure. Since the stress / pressure created in water by the weight of the water is hydrostatic. This means that pressures in water are similar in all directions and depth dependent only. If a 'head' of water exists above a point on a joint, the pressure exerted by the water on that joint surface can easily be determined by:

$$\text{Pressure} = \rho g h$$

where g is the gravitational acceleration, h is the head of water and ρ is the density of water.

EXAMPLE

What is the water pressure on a joint if a 20 water head exists above the joint?

$$\begin{aligned} \text{Pressure} &= \rho g h \\ &= (1\ \text{tonne.m-3})(9.81\text{m.s-2})(20\text{m}) \\ &= 196.2\ \text{kPa}. \end{aligned}$$

REFERENCE

- Page 97 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 70 Chapter 11 - Practical rock engineering, 2006, E Hoek

LEARNING OUTCOME 6.1.7

DESCRIBE AND EXPLAIN THE GENERAL RELATIONSHIP BETWEEN STRENGTH AND SEISMIC WAVE VELOCITY IN A ROCK MASS.

There exists no direct relationship between rock strength and seismic wave velocity, but indirectly, the relationship suggests that a rock mass with low intact strength and several discontinuities with infilling and water present (i.e. low strength rock mass) would also suggest a 'soft' rock mass due to the presence and character of the joints within the rock mass.

In a homogeneous isotropic elastic medium the compression (1st arrival wave, also called the P wave)seismic wave has been shown propagates with velocity

$$\alpha = \sqrt{\frac{\lambda + 2G}{\rho}} = \sqrt{\frac{E(1-v)}{(1+v)(1-2v)\rho}}$$

where E = Young's modulus and v= Poisson's ratio and where ρ = density of the material

CONNECTION 50

Outcome 3.3 in Paper 2 guide

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At the same time, the shear (2nd arrival wave, also called the S wave) seismic wave has been shown topropagate at a velocity given by

$$\beta = \sqrt{\frac{G}{\rho}} = \sqrt{\frac{E}{2(1+2v)\rho}}$$

In both cases, an increase in rock mass stiffness (indirectly an indication that fewer joints etc are present in the rock mass and that it should be 'stronger') would result in an increase in seismic wave velocities.

Barton also succeeded to determine an empirical relationship between the Q Index and the seismic P wave velocity:

$V_p \approx 3.5 + Log_{10} Q_c$ (km.s^{-1}) where
$Q_c = (Q.\sigma_c)/100$

EXAMPLE

If we assume the following:

ρ = 2750 kg/m^3,

E = 91GPa and

v = 0.15, the values

α= 5900 m/s and

β = 3800 m/s

can be calculated from the equations shown above.

α = [((91GPa)(1-0.15))/((1+0.15)(1-0.3)2750kg.m-3)]0.5
= [77.35GPa/2.214kg.m-3]0.5
= [34,940,711]0.5
= 5911 m.s^{-1}

β = [91GPa/(2(1+0.15)2750kgm-3]0.5
= [91GPa/6325]0.5
= [14,387,351]0.5
= 3793 m.s-1

REFERENCE

- Page 321 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 139 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

ROCK MASS CLASSIFICATION.

LEARNING OUTCOME 6.2.1

APPLY BARTON, BIENIAWSKI AND LAUBSCHER'S GEOMECHANICS CLASSIFICATIONSYSTEMS TO CLASSIFY ROCK MASSES

REFERENCE

- Page 40 Chapter 3 - Practical rock engineering, 2006, E Hoek
- Page 76 - Handbook on hard rock strata control, 1995, AJS Spearing

NOTE

To apply any of the rock mass rating systems discussed later in this outcome, data on the rock mass to be rated is required. To assist, the first section of this outcome will provide an example of data gathering that can be followed to allow appropriate values to be selected when applying the appropriate rock mass rating system. The data collection information provided in this section is based mainly on the process followed during geotechnical logging of borehole cores.

1. Geotechnical interval
   Before geotechnical core logging can commence, the borehole should be divided into geotechnical intervals. These intervals will allow the allocation of appropriate values and ratings to a number of

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measuring points that fall in an interval of material that is geotechnically 'similar'. Dividing the borehole into geotechnical intervals is based on:

- the geology (change in rock type)
- the joint sets (total number of joints, their spacing and number of joint sets) and fracturing
- the joint conditions
- the presence of veining, shearing fault/weakness zones
- the strength of the rock
- the state of weathering

Generally, if there is a potential interval that is less than 50cm in length, it should be added to the most geotechnically representative interval above or below it.

2. 'Depth From' and 'Depth To'
Start by noting down the depth at the top of the interval to the bottom of the interval. It allows allocated ratings or values to be presented at its correct spatial position, i.e. depth below surface, distance ahead of face, etc. It is important to ensure that you check your measurements by:

- measuring forwards off of the nearest core block at the top of the interval
- measuring backwards of the core block at the bottom of the interval and
- ensuring that core loss has been accounted for.

This exercise takes a great deal of practice but is much easier if the metre markings have been measured with a high level of accuracy, as you can then go by the metre markings rather than the core blocks to save time.

3. Total Core Recovery/Drilled Length (TCR)
The Total Core Recovery (TCR) is determined from:Drilled Length or Total Core Recovery (TCR) = 'Depth To' – 'Depth From'

4. Solid Core Recovery (SCR)
The Solid Core Recovery (SCR) is measured by estimating the intact core length. If there is any broken material in the geotechnical interval, the broken material has be reconstructed into an intact piece of core, i.e. how long would it be if it had been intact? The SCR should be less than the TCR.

The main difference between the TCR and the SCR is that the TCR is the total amount of core drilled, regardless of whether or not the material is there. The SCR is a measurement of what core is actually physically present or how much core was lost.

5. Percentage Core Recovery (%CR)
The percentage Core Recovery (%CR) is simply:%CR = (SCR / TCR) x 100

6. Rock Quality Designation as a percentage (RQD)
The Rock Quality Designation (RQD) is the sum of lengths of complete cylindrical core that are longer than 10cm, i.e. the smallest distance between fractures should be 10cm or greater. Only natural

fractures should be considered in the measurement, not drilling, handling or induced fractures. The RQD must be less than or equal to the % CR.
%RQD = (The Sum of Core > 10cm) / TCR x100

7. Rock type
Either a specific rock type or a description of the rock type such as weathered, faulted, sheared, interbedded, intercalated etc can be included.

8. Weathering
This should be estimated using the one or more of the following terms.
   - Unweathered: No visible signs of alteration in the rock material but discontinuity planes may be stained.
   - Slightly: Discontinuities are stained or discoloured and may contain a thin filling of altered material. Un-weathered rock colour generally preserved. Discolouration may extend into the rock from the discontinuities.
   - Moderately: Slight discolouration extends from discontinuities for a distance greater than 20% of their spacing (i.e. generally greater part of the rock). Discontinuities may contain filling of altered material. The surface of the core is not friable (except in the case of poorly cemented sedimentary rocks) and the original fabric of the rock has been preserved. Partial opening of grain boundaries may be observed.
   - Highly: Discolouration extends throughout the core. The surface of the core is friable and usually pitted due to washing out of highly altered minerals by water. The original fabric of the rock has mainly been preserved but separation of grains has occurred. Not easily indented with a knife, does not slake in water.
   - Completely: Residual soil. The core is totally discoloured though internally the rock fabric is partly preserved but grains have completely separated. Easily indented with a knife, slakes in water.

9. Hardness
This should be estimated using the one or more of the following terms.

   - Very Soft (1 – 34 MPa)
   - Soft (35 - 84 MPa)
   - Hard (85 – 104 MPa)
   - Very Hard (145 – 165 MPa)
   - Extremely Hard (165 - +185 MPa)

10. Joint orientation
Joint orientation is described through the Dip angle (α - angle) and Dip Direction (β–angle) as measured from the drill core.

Unorientated drill core: During core drilling, runs of core (commonly approximately three metres long) are extracted from the core barrel and placed in core boxes. The extraction process rotates the core randomly so that once the core is laid out in core boxes; its original orientation is lost, although the orientation of the core axis is generally still known. Unorientated core provides information of joint dip angles but cannot adequately assist to provide the dip directions.

Orientated drill core: Orientated core provides sufficient information to allow analysis of discontinuity orientations (dip direction can be determined). The need for the joint orientation data makes it essential that an orientation line is drawn accurately on the core. Failure to extend this line accurately could render efforts in the drilling and logging processes useless.

Joint orientation measurement conventions used are:

- Alpha angle (α–angle): the acute angle (Figure 11) between the core axis and the long axis of the ellipse (0-90°);
- Beta angle (β– ngle): the angle (Figure 11) between a reference line along the core and the ellipse apical trace measured in a clockwise sense (0-360°). In 'oriented core', the reference line is the 'orientation mark' or 'bottom mark' and the beta angle of the apical trace of the ellipse is measured clockwise from this line.

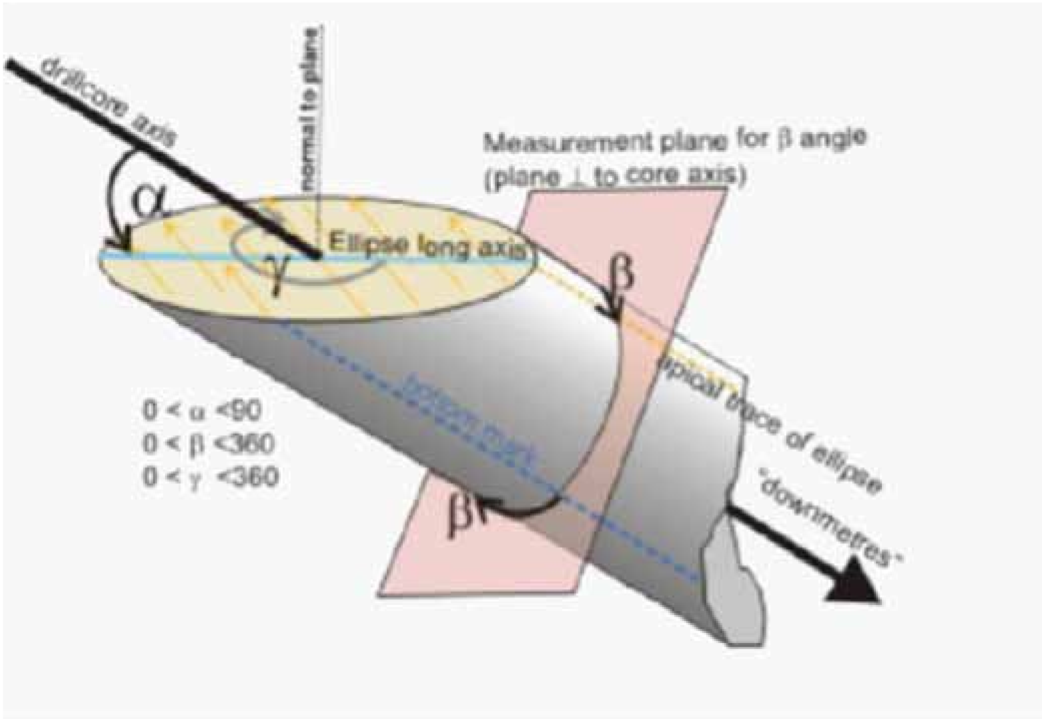

Figure 11: Joint orientation measurement convention

11. Fracture Frequency (FF)
    The Fracture Frequency (FF) indicates joint spacings and is determined by counting the fractures / joints within a specific interval. However, FF does not recognise core recovery and the counted FF must be adapted if there is some core loss. The adjustment is done by dividing the number of fractures in an interval of core (FF/SCR) by the core recovery and multiplying the quotient by 100, therefore FF = ((Total Number of Joints / SCR) / CR) x100

12. Joint Surface Condition (JC) (Figure 12)
    Micro (Mi): Assesses the micro-joint roughness (less than 10cm scale) with one or more of the following ratings:

- Polished
- Smooth Planar
- Rough Planar
- Slickensided Undulating
- Smooth Undulating
- Rough Undulating
- Slickensided Stepped
- Smooth Stepped
- Rough Stepped/Irregular

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The 'rough' and 'smooth' prefixes generally refer to the grain size of the host rock but characterise the nature of the joint surface between the 'steps' or 'undulations'.

13. Joint roughness profiles

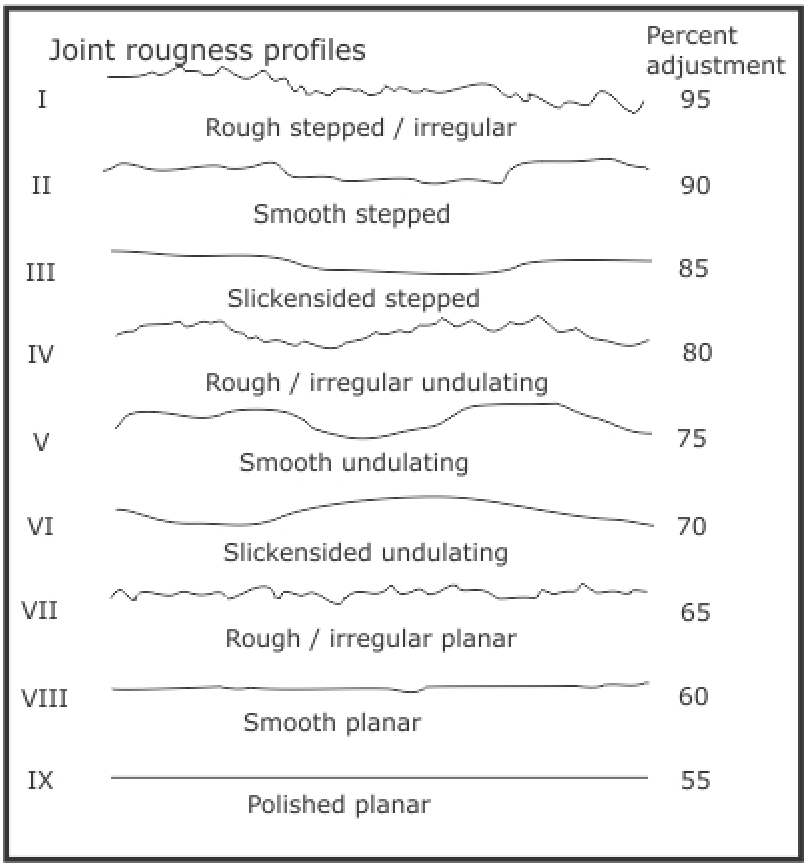

Figure 12: Joint roughness profiles

Macro (Ma): The macro joint roughness (1 m scale) is difficult to determine from core fracture surfaces, due to the limited exposure. A value is required and estimation is made from the core from one of the following ratings:

- Planar
- Undulating
- Rough Planar
- Curved
- Irregular

14. Infill type
The fill on the fracture surfaces must be characterised by one or more of the following ratings:

- Gouge Thickness > Amplitude of Irregularities
- Gouge Thickness < Amplitude of Irregularities
- Soft Sheared Material - Fine
- Soft Sheared Material - Medium
- Soft Sheared Material - Coarse
- Non-softening Material - Fine
- Non-softening Material - Medium
- Non-softening Material - Coarse
- Clean

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The 'Gouge' implies soft sheared material and not a solid vein. 'Soft sheared' implies friable material that can be broken between the fingers. 'Non-softening' implies a non-friable material such as calcite, quartz etc that would typically have closed the joint, it is softer than the host rock.

15. Wall alteration
This is an indication of the fill material on the joint surfaces. One of the following ratings should be chosen:

    - Wall = Rock Hard
    - Wall > Rock Hard
    - Wall < Rock Hard

16. Comments
Any information on core loss, faults, fracture zones, drilling fractures, closed joints or joint sets etc that can assist in estimating rock mass quality can be included.

BARTON, BIENIAWSKI AND LAUBSCHER'S GEOMECHANICS CLASSIFICATION SYSTEMS TO CLASSIFY ROCK MASSES

Rock mass classifications have been developed empirically (in practice / with experience / experimental) for over 100 years and are continually improved upon to ensure safer mining environments.

NOTE

Rock engineers should familiarise themselves with improvements on these systems on a regular basis.

Barton's classification system (Q Index)

INTERESTING INFO

Barton et al (1974)) developed the Q Index system from civil engineering case histories in which all of the components of the engineering geological character of the rock mass were included. The classification system and support recommendations made by him were made from 212 case studies. The original parameters in the Q rating system have not been changed, but the stress reduction factor (SRF) has been altered by Grimstad and Barton in 1993.

The parameters rated in the Q system consist of:

- RQD
- Number of joint sets
- Joint alteration
- Joint water conditions
- Stress reduction factor.

The numerical value of the Q Index varies on a logarithmicscale (do not calculate an average value for a rock mass but find the 50th percentile value assuming a normal distribution to the calculated Q Index values) from 0.001 to a maximum of 1,000 and is defined by:

$$Q = \frac{RQD}{J_n} x \frac{JR}{J_a} x \frac{J_w}{SRF}$$

where: RQD is the Rock Quality Designation (see notes below), J_n is the joint set number, J_r the joint roughness, J_a the joint alteration, Jw the joint water and SRF a 'stress reduction factor'and for which ratings are allocated from Barton's Classification of individual parameters used in the Tunnelling Quality Index Q to calculate the appropriate Q Index.

CONNECTION 51

Barton's classification system (figure 13)

The rock tunneling quality Q can be considered to be a function of only three parameters which are measures of:

- The first quotient (RQD/J_n), represents the structure of the rock mass. This is a measure of the block or particle size within the rock mass;
- The second quotient (J_r/J_a) represents the roughness and frictional characteristics of the joint walls or filling materials indicating inter-block shear strength and
- The third quotient (J_w/SRF) consists of two stress parameters

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The parameter J_w is a measure of water pressure, which affects the shear strength of joints due to a reduction in the effective normal stress. SRF is a measure of the 'loosening load' in when excavating through shear zones, 'rock stress' in competent rock and 'squeezing loads' in plastic, incompetent rocks. This quotient can be regarded as a 'total stress parameter' and is often described as the 'active stress'.

On completion of the Q Index calculations, the rock mass can be classified in terms of Figure 14 as Exceptionally Poor to exceptionally Good according to the Q Index value:

Q	Group	Classification
10-40 40-100 100-400 400-1000	1	Good Very Good Extremely Good Exceptionally Good
0.10-1.0 1.0-4.0 4.0-10.0	2	Very Poor Poor Fair
0.001-0.01 0.01-0.1	3	Exceptionally Poor Extremely Poor

Figure 14: Classification of rock mass based on Q-values (Barton)

EXAMPLE

A 15 m wide workshop for an underground mine is to be excavated in quartzite at a depth of 2500 m below surface. Two sets of rough, planar and unaltered joints traverse the workshop. Laboratory tests on core samples of intact rock provided an average uni-axial compressive strength of 210 MPa. The principal stress directions are approximately vertical and horizontal and the magnitude of the horizontal principal stress is approximately 0.5 times that of the vertical principal stress. The rock mass is moist but there is no evidence of flowing water. RQD values obtained from core logging were calculated as 90%. Determine the Q Index rating for the rock mass exposed in the excavation.

Determining input values for the Q value calculation.

The numerical value of RQD is used directly in the calculation of Q andthe value of 90% will be used :

RQD = 90

From Figure 13 part 2 (See Connection 51) follows that, for two joint sets, the joint set number is

$J_n = 4$

For rough or irregular joints which are undulating, Figure 13 part 3 (See Connection 51) gives a joint roughness number of

$J_r = 1.5$

Figure 13 part 4 (See Connection 51) gives the joint alteration numberfor unaltered joint walls with surface staining only as

$J_a = 1.0$

Figure 13 part 5 (See Connection 51) shows that for an excavation with minor water inflow, the joint water reduction factor is

$J_w = 1.0$

For a depth below surface of 2500 m the overburden stress is approximately 66MPa(2700kg/m^3) yielding a major vertical principal stress σ_1= 66MPa. Since the uni-axial compressive strength of thequartzite is given as210MPa, this gives a ratio of σ_c/σ_1 of 3.18.

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Figure 13(part 6) shows that for competent rock with rock stress problems, this value of σc/σ1can be expected to produce mild rock burst conditions and that the value of SRF lie between 5 and 10.A value of

$$SRF = 6.3$$

A fast rating can be obtained by selecting the ratings closest to the parameter values. However, it is suggested that the user allocate proportionate ratings. Rating to be used is determined by determining the ratio of the calculated σ_c/σ_1 value to the lower value stipulated in Figure 13 (namely 2.5). This ratio provides a factor of 1.27 (Example: 3.18/2.5=1.27). Multiplication of the 1.27 factor with the lower rating from Figure 13 (namely 5) gives the allocated value of 6.3.

The process above will obviously improve the accuracy of any rock mass rating comparisons made between rock mass or areas within a rock mass. The user must determine whether this level of accuracy in an empirical method will provide the results required or whether it is unnecessary to go into such detail differences.

Using these values gives:

$$Q = \frac{RQD}{J_n} x \frac{JR}{J_a} x \frac{J_w}{SRF}$$

$$Q = (90/4)x(1.5/1)x(1/6.3)$$

$$\underline{Q = 5.357}$$

In relating the value of the Q Index to the stability and support requirements ofundergroundexcavations, Barton et al (1974) defined an additional parameter called the Equivalent Dimension, D_e, of the excavation. This dimension is obtained bydividing the span, diameter or wall height of the excavation by a quantity called theExcavation Support Ratio, ESR.

$$D_e = \frac{\text{(Excavation span,diameter or height (m))}}{\text{(Excavation Support Ratio (ESR))}}$$

The value of ESR is related to the intended use of the excavation and to the degree ofsecurity which is demanded of the support system installed to maintain the stability of theexcavation.

Barton et al (1974) suggest the values for ESR as shown in Figure 15:

Excavation category		ESR
A	Temporary mine openings.	3-5
B	Permanent mine openings, water tunnels for hydro power (excluding high pressure penstocks), pilot tunnels, drifts and headings for large excavations.	1.6
C	Storage rooms, water treatment plants, minor road and railway tunnels, surge chambers, access tunnels.	1.3
D	Power stations, major road and railway tunnels, civil defence chambers, portal intersections.	1.0
E	Underground nuclear power stations, railway stations, sports and public facilities, factories.	0.8

Figure 15 ESR values recommended by Barton et al. (1974)

The equivalent dimension, D_e, plotted against the value of Q can be used to define a numberof support categories from a standard chart (Figure 16) published by Barton et al (1974).

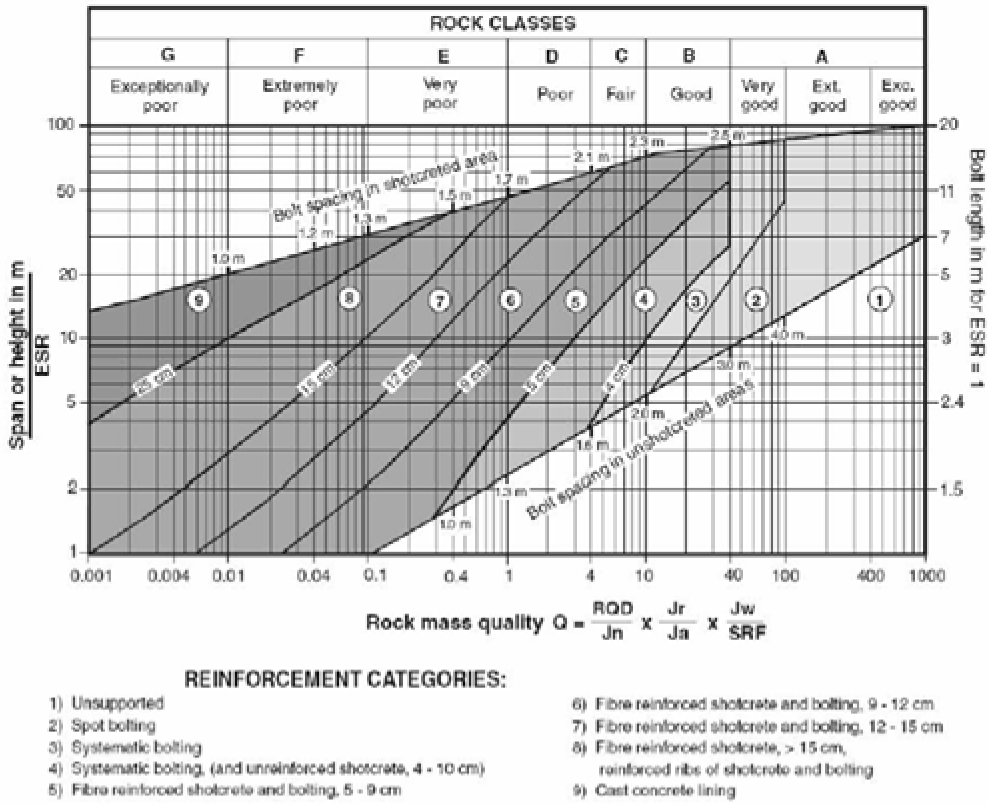

Figure 16: Estimated support categories based on the tunneling quality index Q (Grimstad and Barton, 1993)

The length L of rockbolts can also be estimated from the excavation width B and the Excavation Support Ratio ESR:

$$L = 2+0.15/ESR$$

EXAMPLE

Estimate support requirements for the workshop in the previous example.

The workshop discussed in the example falls into the category of permanent mine openings and is assigned an excavation support ratio of 1.6.

Hence, for an excavation span of 15m, the equivalent dimension

$D_e = 15/1.6$

$D_e = \underline{9.4}$

EXAMPLE

Answer the questions that follow with reference to the summarised results of geotechnical logging of 42mm diameter drill core provided in the table below to the right.

Hole ID	Q Ratings			Distance to slickensided-hangingwall horizontal parting (m)	Orebody Point load strength (kN)
	Hangingwall	Orebody	Footwall		
A1	3.86	9.32	29.41	1.54	5.2
A2	3.54	8.86	28.42	1.23	5.4
A3	4.10	7.64	30.02	1.63	5.7
A4	3.32	8.32	27.64	1.67	5.3
A5	4.54	8.96	31.01	1.24	5.6
A6	3.56	9.43	30.23	1.54	5.4

Convert and tabulate the orebody Q-ratings into adjusted MRMR values using the following information:

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MRMR = 9 ln(Q) + 44

Adjustments of 82% for weathering, 85% for joint orientation, and 80% for blasting effects

Hole ID	Ore body	MRMR - unadjusted	Weathering	Joint orientation	Blasing	MRMR - adjusted
A1	9.32	64.09	0.82	0.85	0.80	35.74
A2	8.86	63.63	0.82	0.85	0.80	35.48
A3	7.64	62.30	0.82	0.85	0.80	34.74
A4	8.32	63.07	0.82	0.85	0.80	35.17
A5	8.96	63.73	0.82	0.85	0.80	35.54
A6	9.43	64.20	0.82	0.85	0.80	35.80
AV	8.76	63.50	Adjustments			35.41

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- Page 25,46,102 - Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page
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- Page 78 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown
- Page 3, 57 - The comlete ISRM suggested methods for rock classification, testing and monitoring, 2007, Ulusag, R & Hudson, JA

Bieniawski's Rock Mass Rating (RMR) classification system.

Bieniawski first published the details of a rock mass classification called the Geomechanics Classification or the Rock Mass Rating (RMR) system in 1976.

NOTE

An updated version RMR^{89} (published in 1989) will be used in this Outcome.

The following six parameters are used to classify a rock mass using the RMR system:

1. Uni-axial compressive strength of rock material.
2. Rock Quality Designation (RQD).
3. Spacing of discontinuities.
4. Condition of discontinuities.
5. Groundwater conditions.
6. Orientation of discontinuities.

CONNECTION 52

Rock Mass Rating system (Figure 17)

The Rock Mass Rating system (see connection 52) gives the ratings for each of the six parameters listed above, where after the individual ratings are summed to give a value ofRMR.

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EXAMPLE

A tunnel is to be driven through slightly weathered granite with a dominant joint set dipping at 60o against the direction of the drive. Index testing and logging of diamond drilled core give typical Point-load strength index values of 8 MPa and average RQD values of 70%. The slightly rough and slightly weathered joints with a separation of < 1mm, are spaced at 300 mm. Tunnelling conditions are anticipated to be wet.

The RMR value for the example under consideration is determined as follows (from Figure 17 (see connection 52)):

Table	Item	Value	Rating
A1	Point Load Index	8MPa	12
A2	RQD	70%	13
A3	Spacing of discontinuities	300mm	10
E4	Condition of discontinuities	Note 1	22
A5	Groundwater	Wet	7
B	Adjustment for joint orientation	Note 2	-5
Total			59

NOTE

For slightly rough and altered discontinuity surfaces with a separation of < 1 mm, Figure 17 (A4) (see connection 52) *gives a rating of 25. When more detailed information is available on the joints, Figure 17(E)* (see connection 52) *can be used to obtain a more refined rating. In this case, the rating is the sum of: 4 (1-3 m discontinuity length), 4 (separation 0.1-1.0 mm), 3 (slightly rough), 6(no infilling) and 5 (slightly weathered) = 22.*

Figure 17(F) (see connection 52) *gives a description of 'Fair' for the conditions assumed where thetunnel is to be driven against the dip of a set of joints dipping at 60°. Using thisdescription for 'Tunnels and Mines' in Figure 17(B,)* (see connection 52) *gives an adjustment rating of -5.*

A fast rating can be obtained by selecting the ratings closest to the parameter values. However, it is suggested that the user allocate proportionate ratings. Using the example data, the Point Load Index rating for 8MPa can simply be selected from Figure 17 (see connection 52) *as '12' OR by using proportionate ratings, the following process can be followed:*

- *Subtract the actual strength (8MPa) and the lower value of the category within which this value falls (4MPa) from each other (8MPa – 4MPa = 4MPa);*
- *This 4MPa difference is 66% of the full category UCS value of 6MPa (10MPa – 4MPa in Figure 20);*
- *Determine 66% of the full category Rating of 5 (12-7 in Figure 20) which is (66% x 5 = 3.3);*
- *Add this value to the lower category Rating value of 7 (3.3 + 7) to get the final rating of 10.7.*

The process above will obviously improve the accuracy of any rock mass rating comparisons made between rock mass or areas within a rock mass. The user must determine whether this level of accuracy in an empirical method will provide the results required or whether it is unnecessary to go into such detail differences.

Bieniawski (1989) published a set of guidelines for the selection of support in tunnels inrock for which the value of RMR has been determined. These guidelines are reproducedin Figure 18. Note that these guidelines have been published for a 10 m span horseshoeshaped tunnel, constructed using drill and blast methods, in a rock mass subjected to avertical stress < 25 MPa (equivalent to a depth below surface of <900 m).

For the case considered earlier, with RMR = 59, Figure 18 suggests:

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- That a tunnel could be excavated by top heading and bench, with a 1.5 to 3 m advance in the top heading.
- The value of RMR of 59 indicates that the rock mass is on the boundary between the 'Fair rock' and 'Good rock' categories. In the initial stages of design and construction, it is advisable to utilise the support suggested for fair rock. If the construction is progressing well with no stability problems, and the support is performing very well, then it should be possible to gradually reduce the support requirements to those indicated for a good rock mass. In addition, if the excavation is required to be stable for a short amount of time, then it is advisable to try the less expensive and extensive support suggested for good rock. However, if the rock mass surrounding the excavation is expected to undergo large mining induced stress changes, then more substantial support appropriate for fair rock should be installed. This example indicates that a great deal of judgement is needed in the application of rock mass classification to support design.
- Support should be installed after each blast and the support should be placed at amaximum distance of 10 m from the face. Systematic rock bolting, using 4 m long 20mm diameter fully grouted bolts spaced at 1.5 to 2 m in the crown and walls, isrecommended. Wire mesh, with 50 to 100 mm of shotcrete for the crown and 30 mm ofshotcrete for the walls, is recommended.

Rock mass class	Excavation	Rock bolts (20 mm diameter, fully grouted)	Shotcrete	Steel sets
I - Very good rock *RMR*: 81-100	Full face, 3 m advance.	Generally no support required except spot bolting.		
II - Good rock *RMR*: 61-80	Full face , 1-1.5 m advance. Complete support 20 m from face.	Locally, bolts in crown 3 m long, spaced 2.5 m with occasional wire mesh.	50 mm in crown where required.	None.
III - Fair rock *RMR*: 41-60	Top heading and bench 1.5-3 m advance in top heading. Commence support after each blast. Complete support 10 m from face.	Systematic bolts 4 m long, spaced 1.5 - 2 m in crown and walls with wire mesh in crown.	50-100 mm in crown and 30 mm in sides.	None.
IV - Poor rock *RMR*: 21-40	Top heading and bench 1.0-1.5 m advance in top heading. Install support concurrently with excavation, 10 m from face.	Systematic bolts 4-5 m long, spaced 1-1.5 m in crown and walls with wire mesh.	100-150 mm in crown and 100 mm in sides.	Light to medium ribs spaced 1.5 m where required.
V – Very poor rock *RMR*: < 20	Multiple drifts 0.5-1.5 m advance in top heading. Install support concurrently with excavation. Shotcrete as soon as possible after blasting.	Systematic bolts 5-6 m long, spaced 1-1.5 m in crown and walls with wire mesh. Bolt invert.	150-200 mm in crown, 150 mm in sides, and 50 mm on face.	Medium to heavy ribs spaced 0.75 m with steel lagging and forepoling if required. Close invert.

Figure 18: Guidelines for excavation and support of 10 m span rock tunnels in accordancewith the RMR system (Bieniawski 1989)

NOTE

It should be noted that Figure 18 is a guideline only, based on the limitations and experiences out of which the table was drafted. The application of this table's suggestions should also take into account all other factors for support design.

REFERENCE

- Page 22,171 - Underground excavations in rock, 1980, E Hoek and ET Brown
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- Page 26,109 - Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page

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- Page 366 - A textbook on rock mechanics for tabular hard rock mines, 2002, JA Ryder and AJ Jager
- Page 27 - Best practice rock engineering handbook for other mines, 2001, TR Stacey
- Page 47 Chapter 3 - Practical rock engineering, 2006, E Hoek

Laubscher's classification system.

Laubscher developed the Mining Rock Mass Rating (MRMR) system in 1975 (modified by Laubscher and Taylor in 1976) and determines the rock mass quality based on allocated ratings for 4 parameters, namely intact rock strength, RQD, joint spacing and joint condition.The range of the MRMR system lies between 0 and 100 and rock quality is divided into five classes and ten sub-classes (Figure 19).

The MRMR system also includes adjustments for joint orientations relative to excavations, the effect of blasting, stress and weathering to determine an 'adjusted MRMR'. Laubscher utilised the MRMR to indicate 'cavability' of the rock mass and found that when rock mass material stability is seriously affected due to deteriorating with time, stress levels, joint orientation or where blasting affect the rock mass stability, the use of the adjusted MRMR is more appropriate.

NOTE

The MRMR and adjustments for joint orientations relative to excavations, the effect of blasting and weathering can be used to determine a rock mass strength (RMS) or design rock mass strength (DRMS).

Class	1		2		3		4		5	
Rating	100-81		80-61		60-41		40-21		20-0	
Description	Very good		Good		Fair		Poor		Very poor	
Sub-classes	A	B	A	B	A	B	A	B	A	B

Figure 19: Classification MRMR values for jointed rock mass (Laubscher 1984)

CONNECTION 52

"Learning Outcome 6.2.1" on page 200

The rating tables utilised to determine the MRMR for a specific area and rock mass and include:

- Intact rock strength rating – Figure 20
- RQD rating – Figure 20
- Joint spacing – Figure 21
- Joint condition and groundwater – Figure 22. Adjustments for the joint condition and groundwater parameter are cumulative. Example: A dry, straight joint with a smooth surface and fine soft-sheared joint filling would have a minimum rating of (40 x 70% x 60% x 60%) = 10.1

The MRMR value is determined by adding the ratings achieved in the 4 parameters discussed above.

	Parameter	Range of Values										
1	RQD	100-97		96-84	83-71	70-56	55-44	43-31	30-17	16-4	3-0	
	Rating (= RQD x 15/100)	15		14	12	10	8	6	4	2	0	
2	UCS (MPa)	185	184-165	164-145	144-125	124-105	104-85	84-65	45-64	44-25	24-5	4-0
	Rating	20	18	16	14	12	10	8	6	4	2	0

Figure 20: Intact UCS and RQD ratings for MRMR

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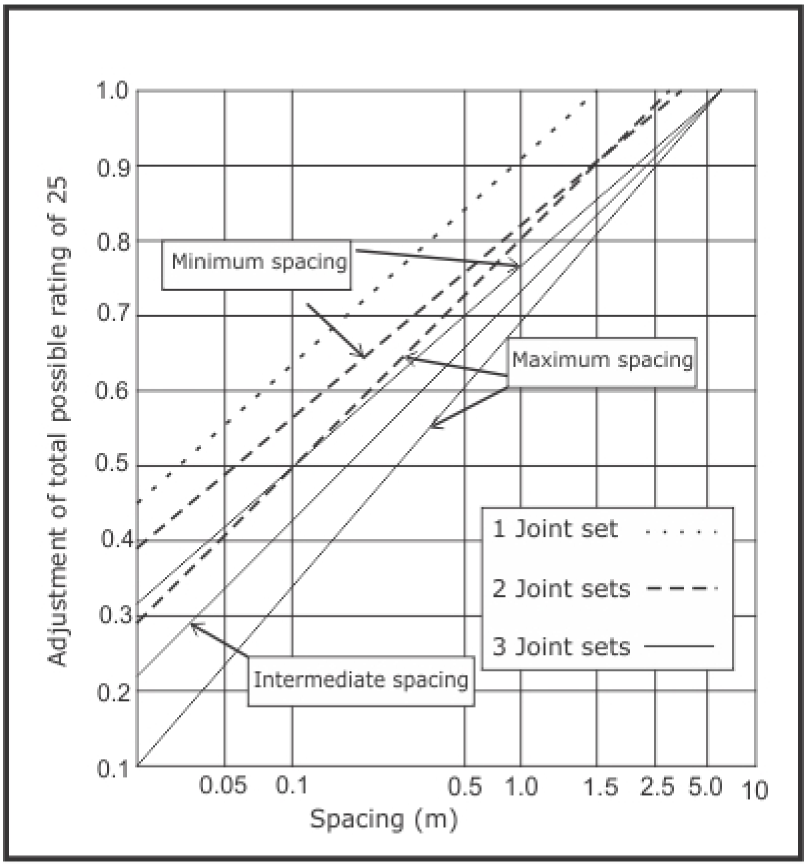

Figure 21: Ratings for joint spacing for MRMR

EXAMPLE

Determine the MRMR for a rock mass with the following properties: UCS 180MPa, RQD 68%, 2 Joint sets with Set showing an average spacing of 0.1m and Set 2 an average of 0.5m. The joints are moist, curved joints with very smooth surfaces and no infilling.

Parameter	Description		Dry Condition	Wet Conditions		
				Moist	Moderate pressure 25-125 1/mm	Severe Pressure >125 1/min
A Joint Expression (large scale irregularities)	Wavy	Multi-Directional	100	100	100	95
		Uni-Directional	95 90	95 90	90 85	80 75
	Curved		89 80	85 75	80 70	70 60
	Straight		79 70	74 65	60	40
B Joint Expression (small scale irregularities or roughness	Very rough		100	100	95	90
	Striated or rough		99 85	99 85	80	70
	Smooth		84 60	80 55	60	50
	Polished		59 50	50 40	30	20
C Joint Wall Alteration Zone	Stronger than wall rock		100	100	100	100
	No alteration		100	100	100	100
	Weaker than wall rock		75	70	65	60

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D Joint Filling	No fill – surface staining only		100	100	100	100
	Non softening and sheared material (clay or talc free)	Coarse Sheared	95	90	70	50
		Medium Sheared	90	85	65	45
		Fine Sheared	85	80	60	40
	Soft sheared material (eg talc)	Coarse Sheared	70	65	40	20
		Medium Sheared	65	60	35	15
		Fine Sheared	60	55	30	10
	Gouge thickness <amplitude of irregularity		40	30	10	
	Gouge thickness <amplitude of irregularity		20	10	Flowing material 5	

Figure 22: Ratings for joint conditions and groundwater for MRMR

A fast rating can be obtained by selecting the ratings closest to the parameter values. However, it is suggested that the user allocate proportionate ratings. Using the example data, the UCS rating for 180MPa can simply be selected from Figure 20 as '18' OR by using proportionate ratings, the following process can be followed:

- *Subtract the actual rock strength (180MPa) and the lower value of the category within which this value falls (165MPa) from each other (180MPa – 165MPa = 15MPa);*
- *This 15MPa difference is 79% of the full category UCS value of 19MPa (184MPa – 165MPa in Figure 20);*
- *Determine 79% of the full category Rating of 2 (18-16 in Figure 20) which is (79% x 2 = 1.6);*
- *Add this value to the lower category Rating value of 16 (1.6 + 16) to get the final rating of 17.6.*

The process above will obviously improve the accuracy of any rock mass rating comparisons made between rock mass or areas within a rock mass. The user must determine whether this level of accuracy in an empirical method will provide the results required or whether it is unnecessary to go into such detail differences.

UCS: 18
RQD: 10
Joint spacing: 25 x 0.55 = 13.8
Joint conditions: (40 x 75% x 55% x 100%)
= 16.5
MRMR: = 18+10+13.8+16.5
= 58.3

Adjustments to determine the $MRMR_{adjusted}$ are shown in the section below and include adjustments for:

- Weathering – Figure 23. A weathering adjustment to relevant rock mass is susceptible to deterioration over time. Example: If the rock will be completely weathered in 6 months, the adjustment is 54%; if slightly weathered after 1 year, the adjustment is 90%.

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- Joint orientation – Figure 24. The orientations of the joints have a significant effect on excavation stability. The joint orientation adjustment depends on the orientations of the joints with respect to the vertical axis of the block created by the intersection of the joints.

NOTE

If a shear zone or a fault is intersected that has a significant influence on stability, adjustments with respect to the dip of the feature relative to the development, are 0^0 to 15^0 – 76%; 15^0 to 45^0 – 84%; 45^0 to 75^0– 92%.

- Stress – The stresses acting around the mining excavations have an effect on the stability of those excavations. The adjustments for mining induced stresses are based on judgement and should be applied carefully. 'Good' confinement enhances stability at a maximum positive adjustment of 120% while 'poor' confinement (associated with numerous, closely spaced joint sets), does not promote stability and has a maximum negative adjustment of 60%.
- Blasting – Figure 25. The quality of blasting has an influence on the creation of blast fracturing as well as loosening of the rock mass.

Rate of weathering and adjustments (%)					
Description of weathering extent	6 months	1 year	2 years	3 years	4 + years
Fresh	100	100	100	100	100
Slightly	88	90	92	94	96
Moderately	82	84	86	88	90
Highly	70	72	74	76	78
Completely	54	56	58	60	62
Residual soil	30	32	34	36	38

Figure 23: Weathering adjustment ratings to determine MRMRadjusted

Number of joints defining the block	**Adjustment (%)** Number of block faces inclined away from the vertical				
	70	75	80	85	90
3	3	-	2	-	-
4	4	3	-	2	-
5	5	4	3	2	1
6	6	5	4	3	2 or 1

Figure 24: Joint orientation adjustment ratings to determine MRMR adjusted

EXAMPLE

If the MRMR value was 72, after adjustments of 90% for weathering, 70% for joint orientation, 100% for mining induced stresses, and 94% for blasting effects, the adjusted MRMR is:

MRMR Adjusted
= 72 x 0.9 x 0.7 x 1.0 x 0.94
= 42.6

Excavation Technique	**Adjustment (%)**
Boring	100
Smooth wall blasting	97
Good conventional blasting	94
Poor blasting	80

Figure 25: Blasting adjustment ratings to determine MRMRadjusted

The above adjustments are all cumulative and are multiplied with the MRMR to determine the MRMRadjusted.

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- Page 11 Chapter 1 - Practical rock engineering, 2006, E Hoek
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LEARNING OUTCOME 6.2.2

DETERMINE ROCK MASS 'M' AND 'S' PARAMETERS FOR THE HOEK AND BROWN CRITERION, BASED UPON ROCK CLASSIFICATION

CONNECTION 53

"LEARNING OUTCOME 4.1.14" on page 121

This section describes and explain how the 'm'-parameter for intact rock required for the Hoek and Brown criterion may be obtained, has dealt with most of the issues relative to this outcome. Values for m and s parameters used in the Hoek and Brown criterion can be calculated from laboratory tri-axial tests on core samples taken from a specific rock mass. Various publications give values for the m_i of different rock types that can be used in the absence of actual test results.

EXAMPLE

The generalised and modified Hoek-Brown criteria for estimating the field strength of jointed rock masses has become increasingly popular in determining input parameters for numerical modelling and limit equilibrium analyses in rock mechanics. Answer the following questions related to the Hoek-Brown criteria.Provide basic definitions for the following Hoek-Brown input parameters and typical values assigned for weak, average and good rock mass conditions: GSI, D and mi.

- Geological Strength Index (GSI): The concept of the Geological Strength Index (GSI) is an alternative to the Bieniawski RMR value. It had become increasingly obvious that Bieniawski's RMR is difficult to apply to very poor quality rock masses and also that the relationship between RMR and the Hoek-Brown 'm' and 's' parameters is no longer linear in these very low ranges. It was also felt that a system based more heavily on fundamental geological observations and less on 'numbers' is needed. Weak ≤ 25, average 25-50, Good 50-75
- Disturbance factor D is a factor which depends upon the degree of disturbance to which the rock mass has been subjected by blast damage and stress relaxation. It varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses.Weak 0.7-1.0 Average 0.3-0.7 Good 0-0.3
- m_i valueis a material constant based on the rock type. Weak 1-10 Average 10-20 Strong 20-30

EXAMPLE

What is the typical range of σ_3 values used to statistically derive the rock mass σ_{ci} and mivalues.

Stress range $\sigma t < \sigma 3 < \frac{1}{4}\ \sigma c$

REFERENCE

- Page 15 Chapter 11 - Practical rock engineering, 2006, E Hoek
- Page 513 - Underground excavations in rock, 1980, E Hoek and ET Brown
- Page 135 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown

EXAMPLE

How would one apply Bieniawski's RMR_{76} and RMR_{89} rating values to obtain GSI values.

- GSI_{89} = [(RMR_{89} values>23)–5] provided dry conditions are assumed in the ratings and no adjustments are made for joint orientation.
- GSI_{76} = [(RMR_{76} values>18)] provided dry conditions are assumed in the ratings and no adjustments are made for joint orientation.

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EXAMPLE

Under what rock mass conditions should caution be exercised when using the Hoek-Brown criteria?

- When working in rocks with high degrees of schistocity which leads to transverse isotropic conditions.
- Whenthe behaviour of the rock mass is believed to be anisotropic, i.e. different responses is possible in different directions.
- For very weak rock masses, GSI < 25
- When the frequency of jointing on an excavated face is lower than the dimensions of the face (i.e. very few joints cutting face) and the out of plane direction probably control stability.

EXAMPLE

List the equivalent Mohr-Coulomb parameters obtained as an output from a Hoek-Brown analysis

LEARNING OUTCOME 6.2.3

CONNECTION 54

"LEARNING OUTCOME 4.1.13" on page 118

APPLY THE HOEK AND BROWN CRITERION TO ESTIMATE THE STRENGTH OF A ROCK MASS

Outcome 4.1.13, dealing with "Describe and explain how to apply the Hoek and Brown criterion to predict rock failure" fully address this outcome.

LEARNING OUTCOME 6.2.4

IDENTIFY, DESCRIBE AND EXPLAIN THE LIMITATIONS OF THE HOEK AND BROWN CRITERION

The Hoek-Brown criterion does not make any assumptions about the failure mechanisms that are responsible for the yield of rock under stress. It is a fairly general criterion that can be used for most materials ranging from soils to hard rock in most stress conditions.

The points listed below explain the limitations of the Hoek-Brown criterion:

- It is a purely empirical criterion. It ignores the effect of the intermediate principal stress on the rock strength and assumes ($\sigma_2 = \sigma_3$) and includes plain strain situations. This may result into 30-50% errors in results.
- The Hoek-Brown criteria's formula fits the laboratory low-stress regimes reasonably well, but sometimes tends to over-estimate strengths at high confining stresses. Many hard rocks show very little downward curvature at high confining stresses ($\sigma_3 > 0.5\ \sigma_c$).
- Use of the suggested (1982) tabulations of 'm' and 's' values can lead to greatly overstated 'downgrading' of the in-situ strength. Practitioners using the Hoek-Brown criterion should carry out their own back-analyses of in-situ strengths and corresponding estimates of appropriate 'm' and 's' values, wherever possible.
- Hoek and Brown have recognized that their criterion is applicable only to either
  - intact rock, or
  - in-situ rock that is transected by many closely-spaced joint sets of multiple angles or rock which is otherwise heavily and isotropically damaged. In-situ rock which is transected by, for example, just one set of joints is anisotropic and cannot be described by theHoek-Brown criterion - other techniques in which the discrete joints are directly modeled, need to be used under these

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conditions (Figure 26).

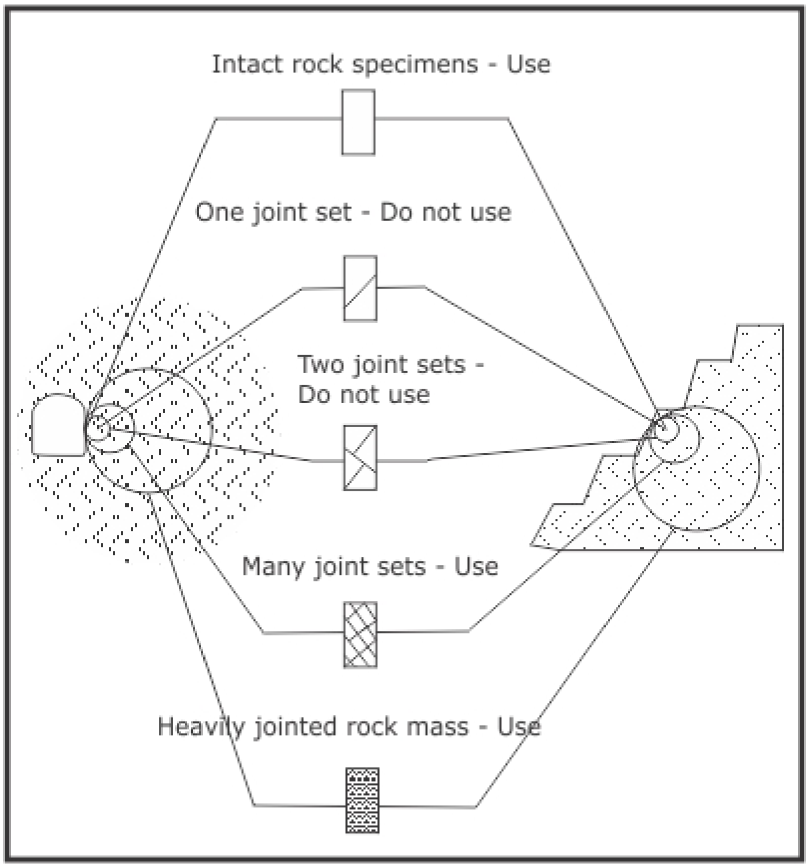

Figure 26: Hoek-Brown limitations

REFERENCE

- Page 17,130 - Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page

LEARNING OUTCOME 6.2.5

ESTIMATE ROCK MASS DEFORMABILITY FROM JOINT STIFFNESS AND ROCK MASS CLASSIFICATION RESULTS.

Engineering behaviour of the rock mass is affected by the presence and characteristics / properties of discontinuities/joints. In such cases the determination of deformabilityis expressed in terms of Modulus of Deformation (E_m). The deformability is notably sensitive to scale effect because of the presence of discontinuities. Modulus of Deformation (E_m) is defined as the ratio of stress (σ) to corresponding strain (ε) during loading of the rock mass.

There are large variations in the In-situ values of modulus of deformation obtained from different project sites as the moduli are highly anisotropic and vary non-linearly with applied stress (increasing depth).(Figure 27)

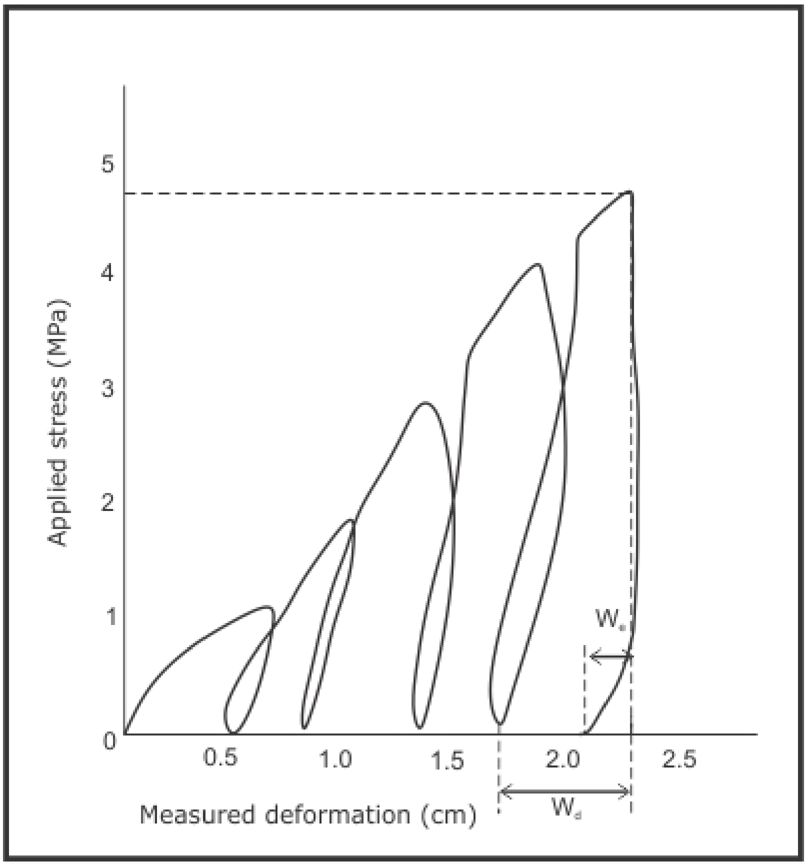

Figure 27: A typical stress vs deformation curve of an in-situ rock mass. (Goodman et al)

The rock mass deformation modulus can be determined from:

- Bienawski's RMR: For RMR > 55, the following relationship has been shown to provide approximate rock mass deformation moduli

  $E_m = 2(RMR) - 100GPa$

  Improvement to this relationship, specifically for Em ranges between 1GPa and 10GPa is given by

  $E_m = 10^{((RMR-10)/40)} GPa$

- Barton's Q Index: Further scrutiny of data allowed the following relationship to be derived

  $E_m = 10Q_c^{0.33} GPa$ where
  $Q_c = (Q\ \sigma_c)/100$

- GSI:Hoek and Brown suggested the following relationship

  $$E_m = \left(1 - \frac{D}{2}\right)\sqrt{\frac{\sigma_c}{100}.10^{\frac{(GSI-10)}{40}}}\ GPa$$

  Where RMR ≈ GSI and for RMR > 25 and D is the blasting damage adjustment rating.

- Joint stiffness: For a rock mass where a single set of discontinuities is present, a set of elastic constants for a transversely isotropic medium can be calculated. If the rock material is isotropic with elastic constants E and v, and the joints in the rock mass have stiffness properties K_s (shear stiffness) and K_n (normal stiffness) with joint spacing 'S', the equivalent elastic constants for the rock mass are given by:

$$\frac{1}{E_2} = \frac{1}{E} + \frac{1}{K_n S}$$

$v_2 = E_2/E.v$ and

$$\frac{1}{G_2} = \frac{1}{E} + \frac{1}{K_s S}$$

Where E,v and G are the elastic constants for the rock material and E_2,v_2 and G_2 the elastic constants for the rock mass.

Joint stiffness represent the strain behaviour of joints under stress where K_n is the normal stiffness and therefore represents how the joint strains under the application of normal stress (perpendicular to the joint). At the same time, K_s is the shear stiffness, indicating how the joint strains under application of a shear stress (parallel to the joint surface).

- Page 281,369 - Fundamentals of rock mechanics, 2007, JC Jaeger, NGW Cook and RW Zimmerman
- Page 87,135 - Practical handbook for underground rock mechanics, 1986, TR Stacey and CH Page
- Page 22 Chapter 11 - Practical rock engineering, 2006, E Hoek
- Page 202 - Cable bolting in underground mines, 1996, DJ Hutchinson and MS Diederichs
- Page 173 - Underground excavations in rock, 1980, E Hoek and ET Brown
- Page 136 - Rock mechanics for underground mining, 2006, BHG Brady and ET Brown
- Insitu deformability characteristics of rock mass by Goodman Jack

BRIDGING MATERIAL

CONTENTS

GREEK ALPHABET

Name	Lower case	Upper case	Name	Lower case	Upper case
alpha	α	Α	nu	ν	Ν
beta	β	Β	xi	ξ	Ξ
gamma	γ	Γ	omicron	ο	Ο
delta	δ	Δ	pi	π	Π
epsilon	ε	Ε	rho	ρ	Ρ
zeta	ζ	Ζ	sigma	σ	Σ
eta	η	Η	tau	τ	Τ
theta	θ	Θ	upsilon	υ	Υ
iota	ι	Ι	phi	φ	Φ
kappa	κ	Κ	chi	χ	Χ
lambda	λ	Λ	psi	ψ	Ψ
mu	μ	Μ	omega	ω	Ω

CONSTANTS

G	Gravitational constant	6.6742 *10-11 Nm2/kg2
RE	Earth radius (average)	6.376 * 106 m
g	gravitational acceleration (sea level)	9.806 m/s2
k	Coulomb-force constant	8.988 * 109 Nm2/C2

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c	Speed of light in vacuum	299 792 458m/s
e	Euler's number	2.71828
π	Pi	3.14159
π2	Pi squared	9.86960...
√2		1.41421...
√3		1.73205...
1°		0.01745... rad
1 rad		57.29577... °

METRIC PREFIXES

tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hector	h	10^{2}
deka	da	10^{1}
-	-	10^{0}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

MEASURING UNITS

Length	metre [m]
Mass	kilogram [kg]
Time	second [s]
Angle	radian [rad], degree [°]
Force	Newton [N] = {kgm/s2}
Energy	Joule [J] = [Nm]
Pressure	Pascal [Pa] = [N/m2]
Frequency	Hertz [Hz] = [1/s]
Current	Ampere [A] = [C/s]
Electric potential	Volt [V]
Stress	Pascal [Pa] = [N/m2]
Strain	- [m/m]

SIGNS & SYMBOLS

= equals
≈ equals approximately

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~ is in the order of
≠ is not equal to
is defined as
> is greater than
< is less than
≥ is greater than or equal to
≤ is less than or equal to
± plus or minus
is proportional to
∑ is the sum of

GLOSSARY

Accelerometer	A seismic transducer that produces an electric signal proportional to ground acceleration. Accelerometers are suitable for the detection of very small seismic events (micro-seismicity).
Aftershock	A seismic event that follows soon after a larger seismic event and originates at or near the focus of the first event. Aftershocks are generally at least 1,5 magnitude units smaller than the main shock.
Amplifier	An electronic device to increase the level of an analogue signal.
Amplitude (wave)	The maximum height of a wave, or depth of a trough, as measured from the equilibrium particle position.
Analogue to digital converter (a / d)	An electronic device that converts an electrical signal to numbers suitable for use in a digital computer.
Apparent stress	A model independent measure of the stress change at the seismic source. It is defined as: σ A = GE / Mo where G = modulus of rigidity, E = seismic energy, and Mo = moment
Apparent volume	Measures co-seismic inelastic deformation.
Attenuation	Reduction in the amplitude of a wave as the distance to the source increases.
Bedding plane	Visible planar discontinuity in sedimentary or metamorphic rock, which may be sufficiently weak to form a separation.
Blocky	Refers to roughly equi-dimensional fragments of rock formed between steeply intersecting joints, induced fractures, bedding planes or other discontinuities.
Body waves	P-waves and S-waves as opposed to surface waves. S-waves cannot propagate in liquids.
Brune's model	One of the most widely used models of the seismic source. It allows the displacement (D) and average static stress drop(□ S) over the area of rupture to be calculated according to: D = Mo / (6.28GRs2) where Rs = source radius
Burst fracture	Steep-dipping shear fracture sub-parallel to the line of the face, invariably comminuted or shattered within a zone of up to several centimetres wide. Small secondary or "feather" fractures usually form at an acute angle pointing in the direction of relative displacement. Primary displacement can have magnitudes of up to a few centimetres.
Choked	Descriptive of a working place that is almost completely filled with fragmented rock that has fallen or been thrust into the opening.
Cleavage surface	Can split through grains or crystals. Sometimes it has curved or fan-like traces usually starting at a single point, which may indicate rapid exclusion of a slab by spilling. This surface results from indirect tensile failure or extension fracture.
Closure	The amount by which the cross-sectional dimension is actually reduced by the combined effect of convergence and additional inelastic movements, such as bed-separation, dilation, bulking, footwall heave etc.
Clustering	Expresses the proximity in time and space between events in a group. Clustering proportional to the product of mean time difference and average distance between events.

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Coefficient of friction	Constant of proportionality relating normal stress and corresponding critical shear stress at which sliding starts between surfaces.
Coda	That part of a seismogram that follows the source pulse, either P-waves or S-waves. Coda waves originate from scattering of body waves by inhomogeneities.
Cohesion	Shear resistance at zero normal stress.
Comminution	White, finely-ground rock material (rock-flour) characteristic of burst fractures. May be indicative of secondary movement on induced fractures.
Compressive stress	Normal stress tending to shorten a body in the direction in which it acts.
Conglomerate	Sedimentary rock containing (semi-) rounded pebbles
Convergence	The amount by which the cross-sectional dimension of an excavation (usually the working height of a stope) is reduced as a result of elastic relaxation in the surrounding rock continuum.
Corner frequency	The frequency at which the spectrum of ground velocity of P- or S-waves reaches a maximum. The corner frequency is generally assumed inversely proportional to the source dimension.
Creep	Time-dependant deformation without seismic energy release.
Cross bedding	Internal bedding oblique to main bedding planes.
Crushed	Descriptive of rock that has been reduced to small whitened fragments (sometimes elongated and "splintery"), where cohesion has been reduced to the extent that the rock can be removed by hand.
Dip	Angle at which a stratum or planar feature is inclined from the horizontal (positive down).
Dispersion	The distortion of a wave form due to a change of velocity with frequency.
Dodlomite	Limestone containing magnesium carbonate and calcium carbonate.
Dome	Approximately circular, convex geological feature.
Dyke	Tabular, near-vertical intrusion.
Dynamic stress drop	Difference between initial stress and the kinetic friction level on a fault.
Earthquake	A sudden motion in the earth caused by the abrupt release of accumulated strain of tectonic origin. When associated with mining, earthquakes are labelled "seismic events".
Energy index	Ratio of seismic energy of a given event to the average energy of events of the same seismic moment; indicator of the stress level in the rock in which the event took place.
Epicentre	The point on the earth's surface directly above the focus of a seismic event.
Fast fourier transform (fft)	A rapid numerical technique used for determining the frequency content of a wave.
Fault	Discontinuity where one block of the rockmass has moved relatively to the adjacent block.
First motion	The initial swing in a wave radiated from a seismic event as recorded at a given station.
Fluctuation	Change over time, variation.
Focus	The initial rupture point within a seismic source at which strain energy is first converted to elastic energy.
Foreshock	A seismic event that precedes a larger seismic event and that originates at or near the hypocentre of the main shock.
Fracture	Break in the continuity of a body of rock due to mechanical failure as a result of excessive stress. Fractures may be open or closed, but not "bonded" together.
Geophone	A sensor converting ground velocity into an electric signal. Geophones record seismic events in a wide range of frequencies.
Graben	Elongated block of the rock mass that has dropped down between the two adjacent blocks.
Hazmag	Estimate of the largest magnitude event to be expected in an area. Usually within 0.2 to 0.3 of the largest event recorded in the past. Based on the cum. moment of all events in a group converted to an equivalent magnitude.
Head waves	Waves critically refracted along a lithological boundary with a velocity contrast.

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Hypocentre	Idealised point source of a seismic event, defined in 3D-space.
Joint	A usually smooth-walled discontinuity with no displacement. Often shows slight alteration, sometimes bonded. It is occasionally difficult to distinguish from an induced fracture.
Joint-laminations	Closely-spaced jointing that cuts rock into slices.
Lamination	Fine layering of rock due to closely-spaced bedding.
Limestone	Layered sedimentary rock consisting mainly of calcium carbonate ($CaCO_3$).
Local magnitude	Measure of the strength of a seismic event; based on energy release, source size, and forces acting at the source.
Magnitude	An arbitrarily defined quantity characteristic of the total energy released by a seismic event. The Richter magnitude scale is related to the logarithm of an observed maximum displacement on a calibrated instrument and to its distance from the epicentre. The Gutenberg-Richter formula is an empirically determined relationship between energy and magnitude: $E (\ln MJ) = 10^{1.5M-1.2}$ Local magnitude as it is used in mine seismology is calculated from seismic energy and seismic moment.
Mechanism (Source mechanism)	The physical process of ground disturbance at the seismic source.
Micro-seismic event	A very small seismic event that is normally detectable only by means of sensitive instruments and that is unlikely to cause damage, but whose effects may extend into the audible range of 'rock noise' underground.
Mine seismology	Application and extension of seismological techniques to seismic events associated with mining. It includes the study of the relationship between seismicity and mining-related parameters.
Moment tensor (seismic)	3D distribution of forces inside the seismic source volume that is associated with inelastic deformation
Origin time	The initiation time of a seismic event. The origin time is usually determined simultaneously with the focus when an event is located.
Particle velocity (ground velocity)	The velocity with which a particle is displaced by a seismic wave.
Pegmatite	Coarse-grained, vein-like intrusion consisting of quartz, feldspar and mica.
Precursor	A phenomenon that precedes a seismic event.
Prediction	Anticipation of the time or magnitude or position (or all three) of a seismic event.
P-wave	The first seismic body wave emanating from a seismic source. Particle displacement is in the direction of wave propagation.
Pyroxene	Group of silicates found in igneous rocks, rich in Mg, Fe, Ca;
Pyroxenite	Coarse-grained igneous rock consisting mainly of pyroxene.
Quartzite	Common metamorphic rock consisting of silica (SiO_2) grains.
Quick magnitude	Quick estimate of the local magnitude of a seismic event; available from the seismic system within seconds after the event.
Rayleigh wave	A type of surface wave propagated along a free surface of a semi-infinite medium. The particle motion takes place in a vertical plane parallel to the direction of propagation.
Reflection	A type of ray-bending in which part of the energy is reflected from the velocity contrast.
Refraction	Change in direction of wave propagation when waves pass obliquely from one region of velocity to another.
Rock type	Igneous = originates from deep parts of the earth's crust and cools slowly at shallow depth (plutonic, coarse grained); or erupts onto surface and cools rapidly (volcanic, fine grained); Sedimentary = result of sedimentation process of rock fragments, of organic or of chemical materials; compacted over time forming hard rock; Metamorphic = Igneous or sedimentary rocks transformed by high heat and/or pressure into rocks of different mineral content and texture.

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Rockburst	A sudden and violent disruption of rock or disturbance of excavation walls in mines that is caused by, or accompanied by, a 'shock' or tremor. Injury to persons (or damage to equipment) resulting from an isolated fall of rock or from flying fragments does not necessarily indicate the occurrence of a rockburst, although a tremor from a distant event can cause the fall of an unstable piece of rock. Such an occurrence might be denoted as "fall of ground associated with a seismic event".
Sampling rate	The rate at which an analogue-to-digital converter samples the analogue signal. In seismology a constant sampling rate is used.
Sandstone	Sedimentary rock made up of visible sand grains.
Scaling	The non-violent formation of slabs that have become completely separated from the excavation surface.
Seismic deborah number (S.Relaxation time / flow time)	The ratio of elastic to viscous processes in seismic deformation expresses stability of a system; low values correspond to low stability.
Seismic energy	That part of total strain energy that is converted to elastic waves and radiated away from a seismic source. The energy released by a tremor can be calculated directly from a seismogram. It is usually assumed to have spread out in a spherical wave front from the source. The energy of the passing seismic waves is calculated at each seismometer site and is used to estimate the total radiated seismic energy.
Seismic event	Sudden failure due to stresses exceeding the strength of the rock mass or a discontinuity. The resulting emission and radiation of kinetic energy in the form of ground vibrations cause a noticeable 'shock' or tremor.
Seismic hazard	Hazard (probability of loss) associated with seismicity.
Seismic hazard assessment	Seismic hazard, in its basic form, is expressed as the probability and magnitude of the largest event expected. More complex assessments can include event frequency, expected PPV, probability of occurrence of large events, etc.
Seismic moment	Seismic moment is a measure of the earthquake size and is related to the leverage of the forces (couples) across the area of slip. Unlike magnitude, the seismic moment is not a scalar but a tensor (i.e. its value is different depending on the direction). This characteristic can be used to determine other source parameters, such as slip direction. Scalar seismic moment is the product of the modulus of rigidity (G), the rupture area (A), and the displacement or "ride" (D): Mo = GAD (in Nm).
Seismic network	System for data collection and analysis of seismic data utilising a number of sensors, data transmission and storage facilities.
Seismic potency	A measure of earthquake size or strength. P is equal to seismic moment divided by shear modulus G.
Seismic parameters	A set of variables that are used to describe seismic events or the seismicity of a confined area.
Seismic response to production=srp (Σva / σvm)	Ratio of cumulative Apparent Volume VA in a given time and space window over volume mined VM
Seismic relaxation time \ (S.Viscosity / rigidity)	Measures predictability of the seismic process: short relaxation time indicates a quick system response, i.e. low predictability;
Seismic schmidt number (S.Viscosity / diffusivity)	Expresses the degree of complexity of seismic processes in time and space; unpredictable systems with quick response time have low Schmidt number values
Seismic source	A volume of rock that fails suddenly, deforms, and releases energy of various types.
Seismic strain (Σmo / gδv)	Measures seismic moment density in rock volume ΔV: Proportional to ratio of cum. seismic moment / ΔV
Seismic stress (Cum. Energy / cum. Moment)	Indicates areas of high stress by measuring relative seismic energy release per unit of seismic moment
Seismology	A geophysical science, which is concerned with the study of earthquakes and measurements of the elastic properties of rock.
Seismometer	Contains A/D converter, data storage, and communication devices. Forms part of a seismic network.
Shale	Fine-grained, layered sedimentary rock.

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Shattered	a) Of rock – disruption into fragments so small that it is not easy to determine whether these are basically slabs, slices or blocks. b) Of support packs – timber component severely compressed and splintered and concrete bricks fragmented.
Shear-wave splitting	Result of differing horizontal and vertical S-wave velocities in a layered medium; visible in seismograms.
Sill	Shallow-dipping tabular intrusion, parallel to the bedding of the intruded strata.
Slabbing	The non-violent formation of slabs that are not detached from the free surface.
Slicken-siding	Polished, linear features or grooves on discontinuities of geological age, such as faults.
Slip	Used to denote a minor fault or joint with very slight displacement.
Source dimension	Measure of the linear dimension of seismic source, inversely proportional to the corner frequency.
Source mechanism	The 3-dimensional distribution of forces and resulting movements within the seismic source.
Source pulse	The part of a seismogram resulting from waves that radiate directly from the source.
Source volume	Volume of inelastic deformation of a seismic source.
Spalling	The violent formation of slabs that separate from a strained surface. If the force is sufficient for the slab to be ejected from the surface, this would constitute one form of strain burst.
Spitting	Small strain burst that causes splinters of rock to "explode" from a strained surface.
Strain burst	A commonly used term denoting sudden rock failure at the lower end of the magnitude spectrum of violent events. The failure is associated with the surface of the excavation and involves only a few hundred kilograms of rock.
Stress drop	The decrease in shear stress acting on that part of a fault that slips during a seismic event.
Stress-induced fracture	Generally clean, "cleavage type" discontinuity parallel to stope face with no primary displacement.
Striations	Linear markings in a comminuted surface or recent source region indicating direction or movement.
Surface wave	Wave that propagates along a free surface (e.g. the Earth's surface or perhaps the stope hanging wall or footwall). Types of surface waves are Rayleigh and Love waves.
S-wave	Distortional, equi-voluminar, secondary or shear wave. Particle motion in a plane normal to direction of wave propagation.
Talking	The noise associated with small failures of rock occurring beyond the exposed surface and not causing any fall of rock fragments.
Time history	The plotting of seismic parameters vs. time; used to detect trends in seismic activity.
Wave velocity (phase velocity)	The speed with which an individual wave of a certain frequency advances.

MATHS LIBRARY

Co-ordinate Systems

The commonly used **Cartesian co-ordinate** system is formed by two perpendicular axes in a plane, one horizontal and one vertical, which intersect at their origins. Each axis carries a scale of real numbers so that any point in the plane can be uniquely identified by a pair of numbers, the point's coordinates (P(x,y)). The coordinate axes divide the plane into four quadrants I to IV. The arrows on the coordinate axes show the direction in which the numbers increase.

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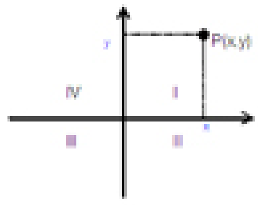
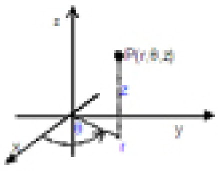
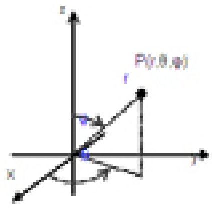


2D Cartesian coordinates



Cylindrical coordinates



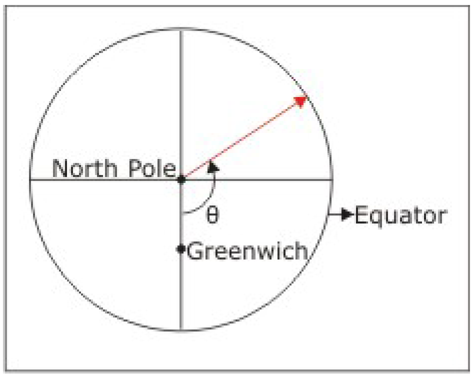
Spherical coordinates

Cartesian coordinate systems are suitable for planar or, if a third orthogonal axis is added, 3-dimensional mathematical problems that involve rectangular shaped objects and straight line trajectories. In 3D, the position of point P would be fully described by P(x,y,z).

Polar co-ordinates or cylindrical co-ordinate systems are more suitable. for problems involving spherical or cylinder shaped objects and curved paths. The geographical coordinates describing locations on the Earth's surface are an example of such spherical coordinates:
Longitude is measured in degrees east or west from the Prime Meridian defined by the location of the Royal Observatory in Greenwich, East of London, UK. In terms of the sketch above θ ranges between -180 and +180 degrees. Each of the 360 degrees is divided into 60 minutes, each minute into 60 seconds.

Latitude is measured in degrees North or South of the equator, the plane that stands normal on the Earth's axis of rotation and intersects Earth's centre of gravity.

The Greek letter Φ is used to denote latitude, so that $\Phi=(90°- \varphi)$ in the above sketch of spherical co-ordinates, which is the angle between the x-y plane and the line with length r. Thus, latitude Φ is in the range of -90° to +90°.



Looking down onto Earth from above the North pole: The definition of longitude θ.
In many cases, mine coordinate systems are a special case of a 3D Cartesian coordinate system whose origin is defined by a geographical location. Their axes refer to geographical directions such as South/West/Up (SWU), but the definition of South=0, West=0 and Up=0 is often specific for a single mine or group of neighbouring mines.
SWU refers to x increasing South, y increasing West and z increasing Up. Some mines choose mean sea level for zero elevation, others a historic datum line 1818.297 above mean sea level.

South/West/Up is a left-handed system while Up/West/South is a right-handed system, depending on whether the first three fingers of the right or left hand can open up the coordinate space: If the thumb, first digit and middle finger of the right hand – in that specific order - are able to align with the coordinate axes the system is called right-handed, as is the case with Up/West/South. Else, it is called left-handed.

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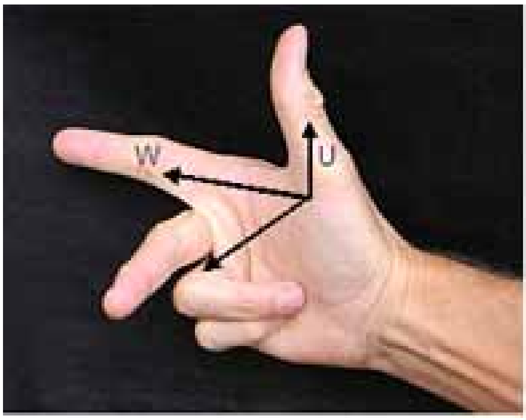
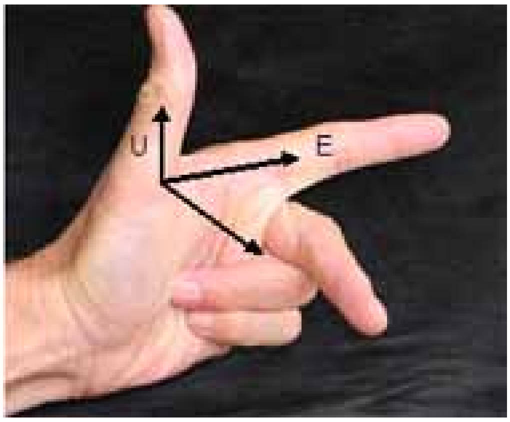
BRIDGING MATERIAL



Right-handed system,
(for example Up/West/South)



Left-handed system,
(for example Up/East/South)

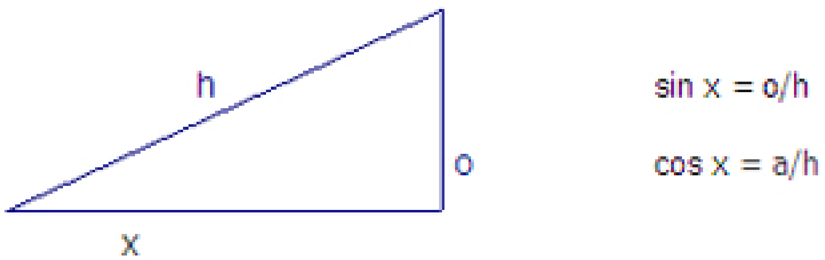
Examples: Are NWU, ESD and SED left- or right-handed?

Many South African mines use survey co-ordinates based on the LO27 system, but with certain modifications. The origin of the LO27 system is at the intersection of 27° longitude with the equator; x increases to the South and y increases to the West. The problem of very large x-values – Johannesburg is approximately 2 900km below the equator – is resolved by dropping the first two digits from the x-coordinates bringing x-values for West Wits mines to around 20 000m and y-values to around -50 000m.

Quadrants and trigonometry signs in quadrants

The most commonly used trigonometric functions are sine and cosine: sin(x) and cos(x) respectively. Both of these are periodic meaning that they are fully described for values of x between 0° and 360° (or 0 and 2π rad), and repeat themselves for x values outside of this range. For the simplified case of amplitude and angular velocity being equal to 1, and the initial phase being zero, the two functions take on the form f(x)=sin(x) and f(x)=cos(x).

The properties of sin(x) and cos(x) originate from their geometric definitions, which are based on a triangle that includes a right angle of 90°. This definition sets sin(x) as the length of the opposite side o divided by the hypotenuse h of the triangle. Cos(x) is defined as the side adjacent side a divided by h.



Example: Consider a car travelling up an inclined road (h) for 2km. The angle to the horizontal x of the incline is 16°. By how much has the car's elevation o increased? Correct answer: 551m

For growing angles x, sin(x) will increase up to 90°, then decline to zero when x reaches 180°, while cos(x) is initially equal to 1, then declines for

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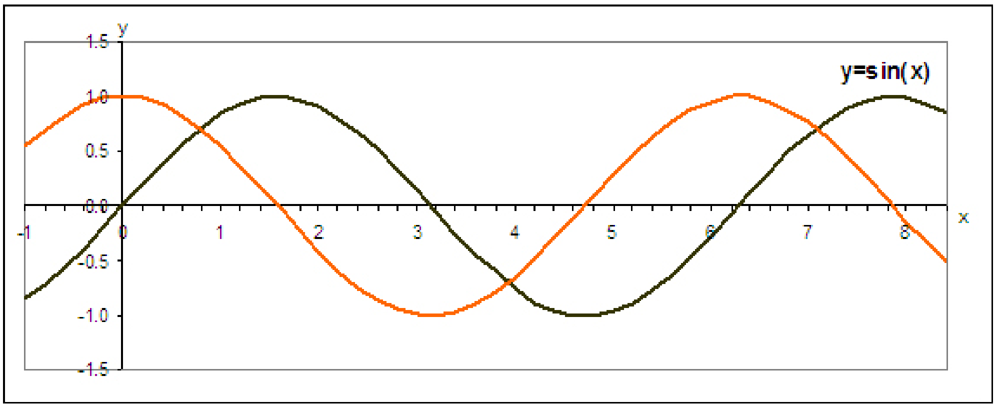
growing x until reaching zero for x=90°. This is the shape and behaviour of the two functions in the graph below. The graphs of sine and cosine are periodic:



y=sin(x) is periodic, equals 0 when x=0 and maximum/minimum at odd multiples of $\pi/2$;
y=cos(x) is periodic, equals 1 when x=0 and maximum/minimum at even multiples of $\pi/2$.

The properties of sine and cosine give rise to the following relationships:

$$\sin 2x + \cos 2x = 1$$
$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \pm \sin(\alpha)\sin(\beta)$$

Cubic Equation

No info available as yet.

Absolute Values

In mathematics, the absolute value |a| of a number 'a' is the numerical value of 'a' without regard to its sign. So, for example, the absolute value of 3 is 3, and the absolute value of -3 is also 3. The absolute value of a number may be thought of as its distance from zero.

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Stress and Strain

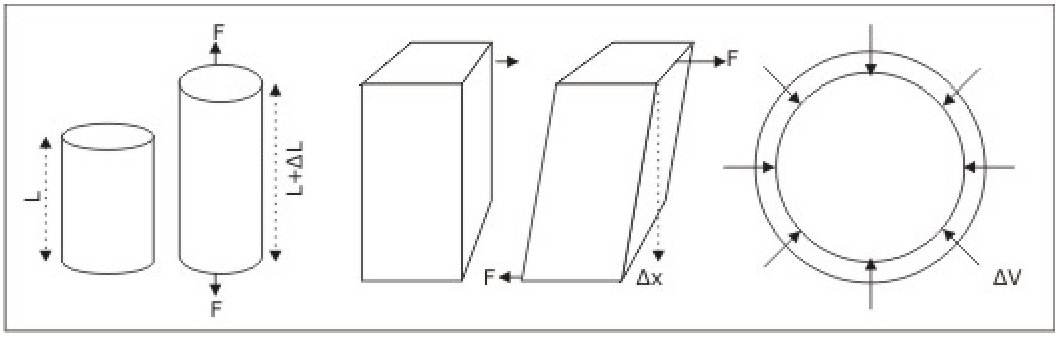

Figure A Figure B Figure C

Three simplified ways in which forces act on areas, each resulting in stress. In all cases is the body deformed as a result of the stress.

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Figure A: A cylinder subjected to tensile stress stretches by an amount ΔL, its diameter is reduced; Figure B: A brick-shaped body subjected to shearing stress is deformed by an amount Δx; Figure C: A sphere is subjected to hydraulic stress, shrinking in volume as a result. For demonstration purposes
the deformation in the sketches is greatly exaggerated.

Stress is defined as force per unit area, the resulting strain is defined as deformation per unit length. For many solid materials stress is proportional to
strain, the proportionality factor is called the modulus of elasticity (Hooke's law):

stress = modulus * strain

Tension and compression:
The modulus for compressive and tensile stresses is the Young's Modulus E. The stress is determined by the force F acting
perpendicular to the area A (end area of the cylinder in Figure A):

$$F/A = E*\Delta L/L$$

The strain $\Delta L/L$ is a dimensionless quantity and is often in the range of <<1%. E has the same unit as stress: [N/m^2].

Shear:
In the case of shearing the force vector acts parallel to the surface of the object. The corresponding modulus is the shear modulus G, also called or rigidity:

$$F/A = G*\Delta x/L$$

The strain $\Delta x/L$ is dimensionless. G has the same unit as stress: [N/m^2].

Hydraulic stress:
The object in Figure C is subject to hydraulic compression. Its volume decreases by ΔV due to the pressure P, a force per unit area.
The strain is defined as the relative volume change $\Delta V/V$. The relevant modulus is the bulk modulus B of the material:

$$P = B*\Delta V/V$$

In deep level rock engineering applications the volume V consists of rock and the pressure P is due to the weight of the overburden. P is determined by the depth and the density of the overlying rock mass, usually between 2 000 and 3 000kg/m3 for most rock types in South African mines.

Energy

kinetic energy:
Energy is a fundamental concept in physics. Energy takes on many different forms and is part of almost all physical process. On a basic level, it can be defined as 'stored work', meaning energy can be converted to work: The kinetic (motion related) energy in flowing water can drive a saw mill; on impact, a moving hammer can drive a nail into wood.

In a closed process, energy can only be converted from one form into another: Energy is conserved and can neither be gained nor lost. One of the most common forms of energy that is frequently converted into other forms is kinetic energy, E_{kin}, which is proportional to the mass m of the moving body:

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$$E_{kin}=1/2mv^2$$

where v is the velocity of the body.

The unit of energy is Joule [J=Nm]

Because $E_{kin} \propto v^2$, a car travelling at 100km/h has four times as much kinetic energy as a car travelling at 50km/h. This means, it will cause four times as much damage on impact than the slower car!

In rock mechanics, E_{kin} is of interest when choosing suitable support types for areas which experience falls of ground. The support unit must be able to absorb the kinetic energy of ejected rock and convert this energy into other harmless forms of energy, such as deformation and heat.

Potential (gravitational) energy:

In falling objects, potential energy is converted to kinetic energy. The potential energy E_{pot} is due to gravity: An object at higher altitude (further away from the Earth's centre) has a greater potential energy due to gravity than an object of equal mass at lower altitude. Potential energy is proportional to mass, height and gravitational acceleration:

$$E_{pot}=mgh$$

where h is the difference in height.

The unit of energy is Joule [J=Nm].

Strictly speaking and in most cases, E_{pot} means a change in potential energy due to a change in height rather than an absolute energy value. For a pumped storage scheme to generate electricity, it is only relevant how far up the water is pumped between a lower and a higher altitude dam, than at which altitude the scheme operates. The amount of electricity generated by the scheme depends on the height difference between the two storage dams.

Conservation of energy

The conservation of energy law states that the total energy in any process is constant:

Energy may change in form or be transferred from one part of a process to another, but the sum of all energy forms stays the same.

Frequently converted forms of energy common to processes in a mining environment are chemical (explosives), thermal (ventilation), strain (deformation in rocks and support units), electric (pumps, drills, fans) and gravitational (hoist).

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